A model for induction motor aggregation for power system studies

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Abstract

This paper proposes a new model for aggregating a group of induction motors for system studies. The aggregation method developed here is based on the transformer-type variation of the equivalent circuit. A grouping criteria is used to classify homogeneous motors into different groups. The validity of the proposed method is verified by comparing the results obtained from the aggregate motor with the sum of the individual motors. © 1997 Elsevier Science S.A.

Keywords: Induction motors; Aggregation methods

1. Introduction

The simulation of a large group of induction motors for system studies can be time consuming. In order to reduce the computation time, reduced order modelling is used to represent a group of motors with one or more aggregate motors. The accuracy of the results obtained from the aggregate motors depends in part on the assumptions made in the derivation of the aggregate motor and varies from method to method. This subject of the aggregation of induction machines for system studies has received a fair amount of attention in the literature [1-6].

In [1], to develop an aggregation model the approximate equivalent circuit is used with stator resistance being neglected. In the derivation of the model, the basic assumption is of power invariance between the aggregate motor and the sum of the individual motors. Since the skin-effect has a strong influence on the performance of induction motors, in [2] the equivalent circuit is represented by seven electrical parameters instead of five. In [3], the motor parameters are calculated from standard specifications which, together with network and load data, are incorporated into an admittance matrix. The inertia and running slip are chosen to minimize the error between the transient response of the system and its equivalent. In [4], the unknown parameters of the equivalent motor are estimated through a weighted-least squares procedure. In [5], Thévenin's theorem along with selected characteristics of the induction machine are used in the calculation of the parameters of the equivalent machine. In [6], a fictitious impedance is introduced to convert the simple equivalent circuit into a circuit with parallel elements only.

In this paper, a transformer-type equivalent circuit [7] is used to represent an induction motor. Five motors are used in the current work to obtain the aggregate motor. Results from the aggregate motor are compared with that obtained from the sum of the individual motors.

2. Development of the transformer-type equivalent circuit model for the induction motor

Usually induction motors are represented by the conventional equivalent circuit model in Fig. 1. But in this paper, an alternative type of equivalent circuit, called the transformer-type equivalent circuit, is used as shown in Fig. 2.

The two circuits are equivalent to each other. The parameters of the two circuits are related in the following way:

\[ Z_0 = Z_1 + Z_m \]  

(2.1)
\[ K = \frac{nZ_m}{Z_1 + Z_m} \quad (2.2) \]

\[ Z_3 = \frac{sn^2Z_1Z_m}{Z_1 + Z_m} \quad (2.3) \]

\[ Z_s = Z_2 + Z_3 \quad (2.4) \]

In this paper, \( s \) stands for slip of the motor and \( n \) is chosen as 1, assuming the rotor parameters of the conventional equivalent circuit model are already referred to the primary. Since \( K \) is the ratio of \( Z_m \) and \( (Z_1 + Z_m) \) and \( Z_m \gg Z_1 \), \( K \) is close to unity and \( \angle K \) is close to 0°.

### 3. Development of aggregation model

Consider the aggregation of \( N \) induction motors connected to the same bus as shown in Fig. 3. It is assumed that all \( N \) motors are represented by the transformer-type equivalent circuit model. Since all \( N \) motors are connected in parallel, they can be shown as in Fig. 3.

As a result, the aggregate motor model and parameters can be obtained as follows (Fig. 4):

\[ Z_{agg}^{s} = \frac{1}{N} \sum_{i=1}^{N} Z_i \quad (3.1) \]

\[ Z_{agg}^{s} = \frac{1}{N} \sum_{i=1}^{N} Z_i \quad (3.2) \]

The values of \( K \) and \( \angle K \) were tested for different sizes of motors and it was found that their values range from 0.96 to 0.985 and around 2° respectively. Therefore, in this paper, the values of \( K \) and \( \angle K \) were chosen as 0.98 and 0° for the aggregate motor.

In this paper, mechanical output power of the aggregate motor is assumed to be equal to the total mechanical output power delivered by the individual motors. Therefore

\[ HP_{agg} = \sum_{i=1}^{N} (HP)^{i} \quad (3.3) \]

The rotor speed of the aggregate motor was found to be

\[ \omega_{agg} = \omega_{agg} - s^{agg}\omega_{agg} \quad (3.4) \]

where \( \omega_{agg} \) is the synchronous speed of the aggregate motor. It is known from the relationship between the parameters of the transformer-type equivalent circuit and those from the conventional type of equivalent circuit that

\[ Z_s = Z_2 + Z_3 \quad (3.5) \]

For the following calculation \( Z_2 \) is considered as

\[ Z_2 = \frac{R_5}{s} + jX_2 \quad (3.6) \]

Then, using (3.5), \( Z_s \) is calculated for all the individual machines and \( Z_{agg}^{s} \) is calculated using (3.2). Again \( Z_{agg}^{s} \) is calculated using (3.2). Again \( Z_{agg}^{s} \) is the sum of \( Z_{agg}^{s} \) and \( Z_{agg}^{s} \). Therefore, equating the real parts from the relationship between \( Z_{agg}^{s} \) and \( Z_{agg}^{s} \) and \( Z_{agg}^{s} \), the following equation is found:

\[ \frac{R_{agg}}{s_{agg}} + s_{agg}K_{1R} = K_{2R} \quad (3.7) \]

where \( K_{1R} \) and \( K_{2R} \) are the real parts of \( Z_{agg}^{s} \) and \( Z_{agg}^{s} \) respectively. Rearranging (3.7), the solution of \( s_{agg} \) is obtained as follows
\[ s_{agg} = \frac{K_{3R} \pm \sqrt{(K_{3R})^2 - 4K_{1R}R_{agg}^2}}{2K_{1R}} \]  

(3.8)

Here \( R_{agg} \) is unknown and obtained from the difference between \( Z_{s_{agg}} \) and \( Z_{3_{agg}} \) calculated without considering slip.

Considering the conservation of kinetic energy, the following relation is used:

\[ J_{agg} = \sum_{i=1}^{N} J \left( \frac{\omega_i}{J_{s_{agg}}} \right)^2 \]  

(3.9)

In this paper, five different sizes of motors are considered. Conventional equivalent circuit parameters of all the five motors as well as the aggregate motor obtained are given in the Table 1.

### 4. Grouping criterion

The purpose of a grouping criterion is to identify and group homogeneous motors. The inertia and open circuit time constant are often used to classify motors. In [2], the authors have developed a grouping criterion that may be expressed as

\[ G = H\gamma \zeta \]  

(4.1)

where

\[ \gamma = \frac{(x_1 + x_2)}{(R_1 + R_2)} \]  

(4.2)

\[ \zeta = \frac{X_m}{R_2} \]  

(4.3)

The group is homogeneous if

\[ 1 \leq (G_{\max}/G_{\min}) \leq 2.5 \]  

(4.4)

Based on the above grouping criterion, the disparate sizes of motors are classified into different groups. Then aggregation is done for different motor groups separately to find aggregate motors from each group.

<table>
<thead>
<tr>
<th>HP</th>
<th>( R_s )</th>
<th>( R_L )</th>
<th>( X_{is} )</th>
<th>( X_{ir} )</th>
<th>( X_m )</th>
<th>( J )</th>
<th>rpm</th>
</tr>
</thead>
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<tr>
<td>3.0</td>
<td>4.86</td>
<td>1.84</td>
<td>2.67</td>
<td>2.67</td>
<td>84.68</td>
<td>0.09</td>
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<td>15.0</td>
<td>1.48</td>
<td>0.31</td>
<td>0.18</td>
<td>0.18</td>
<td>24.89</td>
<td>0.50</td>
<td>1765.0</td>
</tr>
<tr>
<td>30.0</td>
<td>0.73</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>14.96</td>
<td>1.00</td>
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</tr>
<tr>
<td>50.0</td>
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<td>0.14</td>
<td>0.15</td>
<td>0.15</td>
<td>9.47</td>
<td>1.66</td>
<td>1750.0</td>
</tr>
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<td>100.0</td>
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<td>0.08</td>
<td>0.10</td>
<td>0.10</td>
<td>3.97</td>
<td>2.7</td>
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<td>198.0</td>
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<td>0.035</td>
<td>0.043</td>
<td>0.041</td>
<td>2.10</td>
<td>5.96</td>
<td>1749.0</td>
</tr>
</tbody>
</table>
5. Results

In this section, results have been presented for the aggregate motor as well as the sum of the individual motors to compare the performance of the aggregate motor obtained. Table 1 presents the necessary electrical and mechanical parameters for five individual and one aggregate motor. Here, all the individual and aggregate motors are connected to a 460 V bus and the number of poles of each motor is 4.

In the simulation, the motors are first started from standstill to steady-state. Then the source is disconnected from the motors for 134 ms (8 cycles) and reclosed. The action of the circuit breaker in practice is represented by increasing the source resistance from its original value to the value of 500 Ω. This change of resistance is carried out at the zero level of each phase current to simulate the extinguishing of the current at a current zero.

Fig. 5 shows the stator phase A current of the aggregate motor while Fig. 6 shows the sum of the stator phase A currents of the individual motors. Again, Fig. 7 shows the electromagnetic torque of the aggregate motor while Fig. 8 shows the sum of the electromagnetic torques of the individual motors. A comparison between the two sets of curves shows satisfactory correlation.

6. Conclusions

In this paper, a new model has been developed to aggregate induction motors. To develop this model, a transformer-type equivalent circuit of the induction motor is used. A grouping criterion is used to classify different motors into different groups for better results. The results using the aggregate motor are compared to that obtained from the sum of the individual motors. The results are acceptable.

Acknowledgements

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References