Modeling of Permanent Magnet Motor Drives

PRAGASAN PILLAY, MEMBER, IEEE, AND R. KRISHNAN, MEMBER, IEEE

Abstract—**Recent research has indicated that the permanent magnet motor drives, which include the permanent magnet synchronous motor (PMSM) and the brushless dc motor (BDCM) could become serious competitors to the induction motor for servo applications. The PMSM has a sinusoidal back emf and requires sinusoidal stator currents to produce constant torque while the BDCM has a trapezoidal back emf and requires rectangular stator currents to produce constant torque. The PMSM is very similar to the wound rotor synchronous machine except that the PMSM that is used for servo applications tends not to have any damper windings and excitation is provided by a permanent magnet instead of a field winding. Hence the *d*, *q* model of the PMSM can be derived from the well-known model of the synchronous machine with the equations of the damper windings and field current dynamics removed. Because of the nonsinusoidal variation of the mutual inductances between the stator and rotor in the BDCM, it is also shown in this paper that no particular advantage exists in transforming the *abc* equations of the BCDM to the *d*, *q* frame. Hence the solution of the original *abc* equations is proposed for the BDCM.**

NOMENCLATURE

B	damping constant, N/rad/s (in Newtons per radian per second)
e_a, e_b, e_c	a, b, and c phase back emfs (in volts)
E_p	peak value of back emf (in volts)
i_a, i_b, i_c	a, b, and c phase currents, (in amperes)
i_d, i_q	d, q axis stator currents (in amperes)
J	moment of inertia, $kg - m^2$
L_a, L_b, L_c	self inductance of a, b, and c phases (in henrys)
L_{ab}	mutual inductance between phases a and b (in henrys)
L_d, L_q	d, q axis inductances (in henrys)
p	derivative operator
P	number of pole pairs
R	stator resistance (in ohms)
T_e	electric torque (in newton meters)
T_l	load torque, (in newton meters)
v_a, v_b, v_c	a, b, and c phase voltages (in volts)
v_d, v_q	d, q axis voltages (in volts)
V_{dc}	dc bus voltage (in volts)
λ_{af}	mutual flux due to magnet (in webers)
λ_d, λ_q	d, q axis flux linkages (in webers)
ω_r	rotor speed (in radians per second)
ω_s	synchronous speed, (in radians per second)
θ_r	angle between stator phase A and the rotor (in radians)
*	superscript indicating reference value

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The authors are with the Electrical Engineering Department, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.
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I. INTRODUCTION

RECENT research [1]–[3] has indicated that the permanent magnet motor drives, which include the permanent magnet synchronous motor (PMSM) and the brushless dc motor (BDCM) could become serious competitors to the induction motor for servo applications. The PMSM has a sinusoidal back emf and requires sinusoidal stator currents to produce constant torque while the BDCM has a trapezoidal back emf and requires rectangular stator currents to produce constant torque. Some confusion exists, both in the industry and in the university research environment, as to the correct models that should be used in each case. The PMSM is very similar to the standard wound rotor synchronous machine except that the PMSM has no damper windings and excitation is provided by a permanent magnet instead of a field winding. Hence the *d*, *q* model of the PMSM can be derived from the well-known [4] model of the synchronous machine with the equations of the damper windings and field current dynamics removed.

As is well known, the transformation of the synchronous machine equations from the *abc* phase variables to the *d*, *q* variables forces all sinusoidally varying inductances in the *abc* frame to become constant in the *d*, *q* frame. In the BDCM motor, since the back emf is nonsinusoidal, the inductances do not vary sinusoidally in the *abc* frame and it does not seem advantageous to transform the equations to the *d*, *q* frame since the inductances will not be constant after transformation. Hence it is proposed to use the *abc* phase variables model for the BDCM. In addition, this approach in the modeling of the BDCM allows a detailed examination of the machine's torque behavior that would not be possible if any simplifying assumptions were made.

The *d*, *q* model of the PMSM has been used to examine the transient behavior of a high-performance vector controlled PMSM servo drive [5]. In addition, the *abc* phase variable model has been used to examine the behavior of a BDCM speed servo drive [6]. Application characteristics of both machines have been presented in [7]. The purpose of this paper is to present these two models together and to show that the *d*, *q* model is sufficient to study the PMSM in detail while the *abc* model should be used in order to study the BDCM. It is therefore tutorial in nature and summarizes previously published work on the PMSM and BDCM.

The paper is arranged as follows: Section II presents the mathematical model of the PMSM. Section III presents the mathematical model of the BDCM. Section IV uses these models to present some key results to illustrate the use of these models to study both transient and steady state behavior of these drive systems. Section IV has the conclusions.

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II. Mathematical Model of the PMSM

The stator of the PMSM and the wound rotor synchronous motor (SM) are similar. In addition there is no difference between the back emf produced by a permanent magnet and that produced by an excited coil. Hence the mathematical model of a PMSM is similar to that of the wound rotor SM. The following assumptions are made in the derivation:

1) Saturation is neglected although it can be taken into account by parameter changes;
2) The back emf is sinusoidal;
3) Eddy currents and hysteresis losses are negligible.

With these assumptions the stator d, q equations in the rotor reference frame of the PMSM are [6], [7]

$$v_d = Ri_d + p\lambda_d - \omega_r\lambda_q \tag{1}$$

$$v_q = Ri_q + p\lambda_q - \omega_r\lambda_d \tag{2}$$

where

$$\lambda_q = L_q i_q \tag{3}$$

and

$$\lambda_d = L_d i_d + \lambda_{af} \tag{4}$$

λ_{af} is the magnet mutual flux linkage.

The electric torque $T_e = 3P[\lambda_{af} i_q + (L_d - L_q) i_d i_q]/2$ (5)

For constant flux operation when i_d equals zero, the electric torque $T_e = 3\lambda_{af} i_q/2 = K_t i_q$ where K_t is the motor torque constant. Note that this torque equation for the PMSM resembles that of the regular dc machine and hence provides ease of control.

Hence in state space form

$$pi_d = (v_d - Ri_d + \omega_r L_q i_q)/L_d \tag{6}$$

$$pi_q = (v_q - Ri_q + \omega_r L_d i_d - \omega_r \lambda_{af})/L_q \tag{7}$$

$$p\omega_r = (T_e - B\omega_r - T_l)/J \tag{8}$$

$$p\theta_r = \omega_r. \tag{9}$$

v_d and v_q are obtained from v_a, v_b, and v_c through the Park transform defined below

$$\begin{bmatrix} v_q \\ v_d \\ v_0 \end{bmatrix} = 2/3 \begin{bmatrix} \cos(\theta) & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin(\theta) & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{10}$$

abc variables are obtained from d, q variables through the inverse of the Park transform defined below

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) & 1 \\ \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) & 1 \end{bmatrix} \begin{bmatrix} v_q \\ v_d \\ v_0 \end{bmatrix} \tag{11}$$

where θ is the rotor position.

The total input power to the machine in terms of abc variables is

$$\text{Power} = v_a i_a + v_b i_b + v_c i_c \tag{12}$$

while in terms of d, q variables

$$\text{Power} = 3(v_d i_d + v_q i_q)/2. \tag{13}$$

The factor 3/2 exists because the Park transform defined above is not power invariant.

From the dynamic equations of the PMSM, the equivalent circuit in Fig. 1 can be drawn. During steady state operation, the d, q axis currents are constant quantities. Hence the dynamic equivalent circuit can be reduced to the steady state circuit shown in Fig. 2. The advantage of modeling the machine using the d, q axis equations then is the subsequent ease with which an equivalent circuit can be developed.

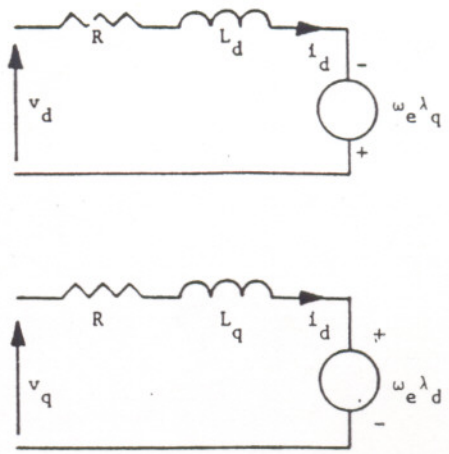

Fig. 1. PMSM equivalent circuit from dynamic equations.

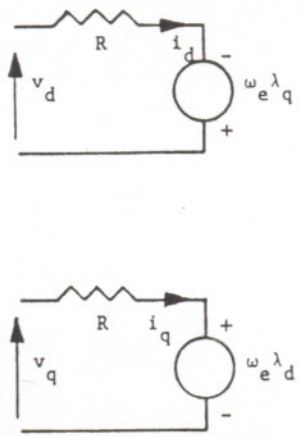

Fig. 2. PMSM equivalent circuit from steady state equations.

III. Mathematical Model of the BDCM

The BDCM has three stator windings and a permanent magnet on the rotor. Since both the magnet and the stainless steel retaining sleeves have high resistivity, rotor induced currents can be neglected and no damper windings are modeled. Hence the circuit equations of the three windings in phase variables are

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + p \begin{bmatrix} L_a & L_{ba} & L_{ca} \\ L_{ba} & L_b & L_{cb} \\ L_{ca} & L_{cb} & L_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (14)$$

where it has been assumed that the stator resistances of all the windings are equal. The back emfs e_a, e_b, and e_c have a trapezoidal [6]. Assuming further that there is no change in the rotor reluctance with angle, then

$$L_a = L_b = L_c = L$$

$$L_{ab} = L_{ca} = L_{bc} = M$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} p \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (15)$$

but

$$i_a + i_b + i_c = 0. \quad (16)$$

Therefore

$$Mi_b + Mi_c = -Mi_a \quad (17)$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} p \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (18)$$

Hence in state space form we have that

$$p \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1/(L-M) & 0 & 0 \\ 0 & 1/(L-M) & 0 \\ 0 & 0 & 1/(L-M) \end{bmatrix} \left[\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} - \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \right] \quad (19)$$

and

$$T_e = (e_a i_a + e_b i_b + e_c i_c)/\omega_r. \quad (20)$$

The currents i_a, i_b, and i_c needed to produce a steady torque without torque pulsations are of 120° duration in each half cycle [6]. With ac machines that have a sinusoidal back emf, a transformation can be made from the phase variables to d, q coordinates either in the stationary, rotor, or synchronously rotating reference frames. Inductances that vary sinusoidally in the abc frame become constants in the d, q frame. Since the back emf is nonsinusoidal in the BDCM, it means that the mutual inductance between the stator and rotor is nonsinusoidal, hence transformation to a d, q reference frame cannot be easily accomplished. A possibility is to find a Fourier series of the back emf in which case the back emf in the d, q reference frame would also consist of many terms. This is considered too cumbersome hence the abc phase variable model developed above will be used without further transformation.

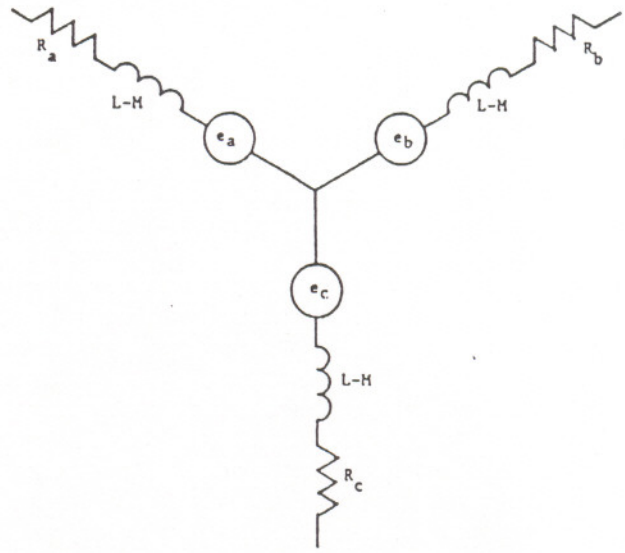

Fig. 3. BDCM equivalent circuit from dynamic equations.

The equation of motion is

$$p\omega_r = (T_e - T_l - B\omega_r)/J. \quad (21)$$

From the dynamic equations of the BDCM the circuit in Fig. 3 can be drawn. e_a, e_b, and e_c have the trapezoidal shapes characteristic of the BDCM. Because of this nonsinusoidal shape in the back emf, further simplifications in the model are difficult.

IV. Results

The models presented previously can be used to examine both the transient and steady state behavior of PM motor drive systems. The models of the current controllers and inverter switches have been presented previously [5].

Some typical results that can be generated using the above models are now given. Fig. 4 shows the results when a PMSM is started up from zero to a speed of 1750 rpm. The response is slightly underdamped in the design used here. The torque is held constant at the maximum capability of the machine while the motor runs up to speed. At 0.025 s a load of 1 pu is added. The electric torque of the machine increases in order to satisfy the load torque requirements. The sinusoidal currents required by the PMSM during the startup are shown including the voltage profile when a hysteresis band equal to 0.1 of the peak rated current is used.

Similar results are obtainable by using the dynamic model of the BDCM as shown in Fig. 5. In the BDCM, the current

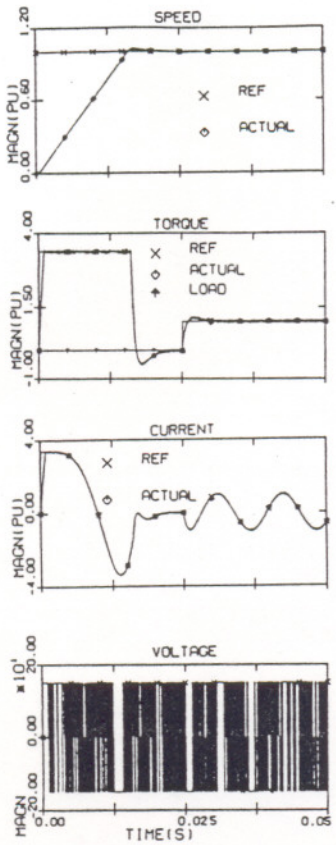

Fig. 4. PMSM transient results.

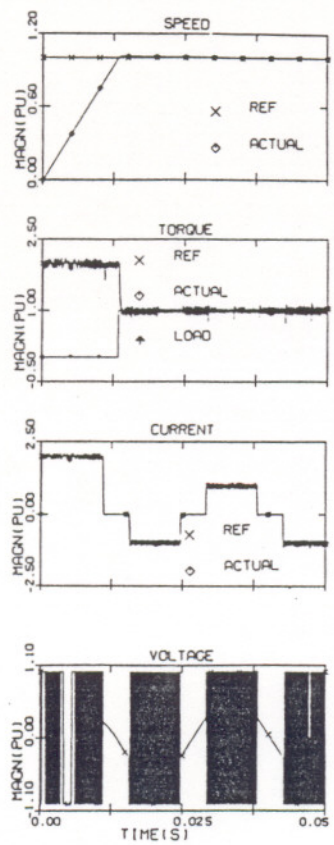

Fig. 5. BDCM transient results.

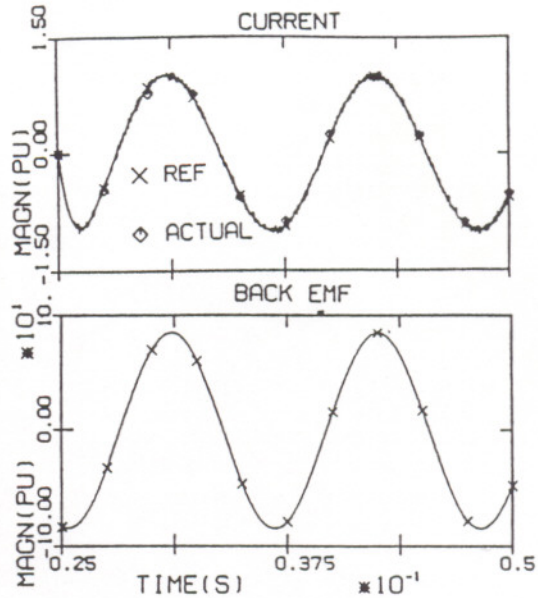

Fig. 6. PMSM steady state results.

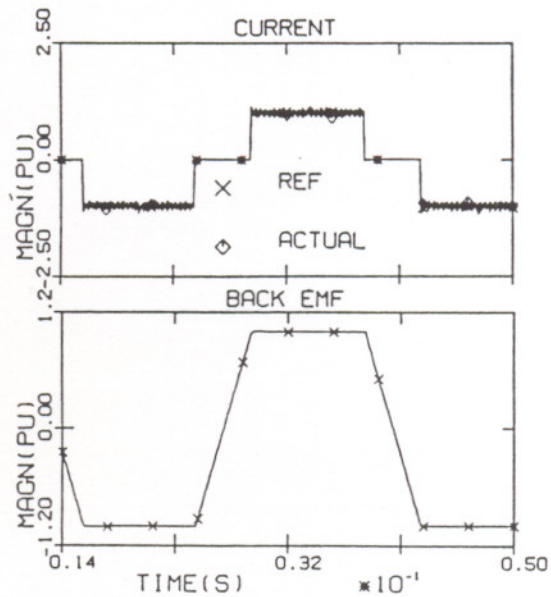

Fig. 7. BDCM steady state results.

controllers are used to force the actual current to track the rectangular shaped current references. A difference between the two drives can be seen in the voltage profile. Each phase conducts for 120° in the BDCM and remains nonconducting for 60° as shown in Fig. 5. In the PMSM on the other hand, each phase conducts continuously as shown in Fig. 4.

Steady state results of the current and back emf for example can also be studied using these models as shown in Figs. 6 and 7, respectively. Two possibilities exist. Either the transient phase can be removed in order to facilitate study of the steady state or appropriate initial conditions found so as to run the model in steady state only.

V. Conclusions

This paper has presented the dynamic models and equivalent circuits of two PM machines. It has shown that although the

PMSM and the BDCM are similar in construction, their modeling takes different forms. The d, q model of the wound rotor SM is easily adapted to the PMSM while an abc phase variable model is necessary for the BDCM if a detailed study of its behavior is needed.

Both the steady state and dynamic behavior can be studied using these models.

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