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D,Q Reference Frames for the Simulation of Induction Motors

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SUMMARY

This paper presents the equations of three preferable reference frames for use in the simulation of induction machines when using the d,q 2-axis theory. It uses case studies to demonstrate that the choice of the reference frame depends on the problem to be solved and the type of computer available (analog or digital).

1. INTRODUCTION

Induction motors are being used more than ever before in industry and individual machines of up to 10 MW in size are no longer a rarity. During start-up and other severe motor operations the induction motor draws large currents, produces voltage dips, oscillatory torque and can even generate harmonics in the power system [1 - 4]. It is therefore important to be able to model the induction motor in order to predict these phenomena. Various models have been developed, and the d,q or two-axis model for the study of transient behaviour has been well tested and proven to be reliable and accurate.

It has been shown [1] that the speed of rotation of the d,q axes can be arbitrary although there are three preferred speeds or reference frames as follows:

(a) the stationary reference frame when the d,q axes do not rotate;

(b) the synchronously rotating reference frame when the d,q axes rotate at synchronous speed;

(c) the rotor reference frame when the d,q axes rotate at rotor speed.

Most authors [2 - 4] use one or other of these reference frames without giving specific reasons for their choice. Whilst either the stationary [3] or the synchronously rotating [2] reference frames are most frequently used, the particular reference frame should be chosen in relation to the problem being investigated and the type of computer (analog or digital) that is used. The purpose of this paper is therefore to provide guidelines (by means of case studies) in order to choose the most suitable reference frame. It also shows that there are instances when the rotor reference frame, which appears to have been avoided by most authors, is the best choice.

2. THEORY

Figure 1 shows a schematic diagram of a 3-phase induction motor with the d,q axes superimposed. The q-axis lags the d-axis by 90°. A voltage v_{as} is applied to stator phase A while the current flowing through it is i_{as}. Phases B and C are not shown on the diagram in an attempt to maintain clarity. In the d,q model, coils DS and QS replace the stator phase coils AS, BS and CS, while coils DR and QR replace the rotor phase coils AR, BR and CR.

Although the d,q axes can rotate at an arbitrary speed, there is no relative speed between the four coils DS, QS, DR and QR. The physical significance of showing the D,Q coils in Fig. 1 is to illustrate that in effect the 3-phase induction motor with its six coils is replaced by a new machine with four coils. In order to predict the mechanical and electrical behaviour of the original machine correctly, the original ABC variables must be transformed into d,q variables, but this transformation depends on the speed of rotation of the D,Q coils, hence each reference frame has its own transformation.

In general, for any arbitrary value of θ', the transformation of stator ABC phase variables $[F_{ABC}]$ to d,q stator variables $[F_{0dq}]$ is carried out through Park's transform as follows:

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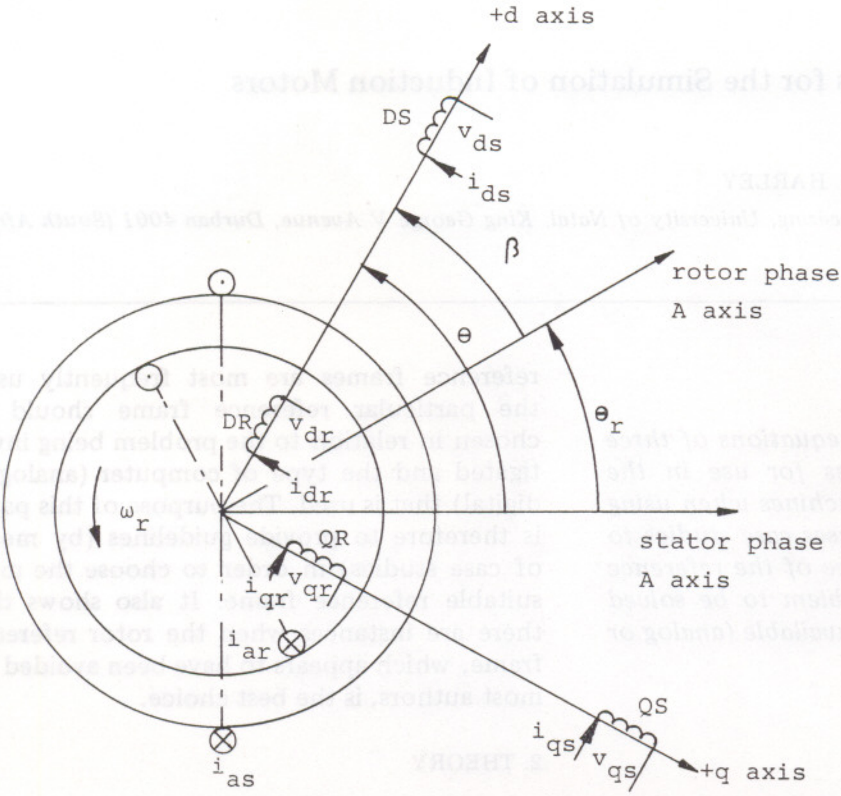

Fig. 1. D,Q axes superimposed onto a three-phase induction motor.

$$[F_{0\mathrm{dq}}] = [P_\theta][F_{\mathrm{ABC}}] \tag{1}$$

where

$$[P_\theta] = 2/3 \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ \cos\theta & \cos(\theta-\lambda) & \cos(\theta+\lambda) \\ \sin\theta & \sin(\theta-\lambda) & \sin(\theta+\lambda) \end{bmatrix} \tag{2}$$

A new variable called the zero sequence component is included with the d,q variables in order to handle unbalanced voltages and to invert Park's transform.

The transformation of rotor ABC variables to rotor d,q variables is again carried out using Park's transform, but this time the angle β in Fig. 1 is used instead of θ.

The voltage balance equations for the d,q coils are as follows [5]:

$$v_{\mathrm{ds}} = R_{\mathrm{s}} i_{\mathrm{ds}} + p\psi_{\mathrm{ds}} + \psi_{\mathrm{qs}} p\theta \tag{3a}$$

$$v_{\mathrm{qs}} = R_{\mathrm{s}} i_{\mathrm{qs}} + p\psi_{\mathrm{qs}} - \psi_{\mathrm{ds}} p\theta \tag{3b}$$

$$v_{\mathrm{dr}} = R_{\mathrm{r}} i_{\mathrm{dr}} + p\psi_{\mathrm{dr}} + \psi_{\mathrm{qr}} p\beta \tag{3c}$$

$$v_{\mathrm{qr}} = R_{\mathrm{r}} i_{\mathrm{qr}} + p\psi_{\mathrm{qr}} - \psi_{\mathrm{dr}} p\beta \tag{3d}$$

where

$$\begin{bmatrix} \psi_{\mathrm{ds}} \\ \psi_{\mathrm{dr}} \\ \psi_{\mathrm{qs}} \\ \psi_{\mathrm{qr}} \end{bmatrix} = \begin{bmatrix} L_{\mathrm{ss}} & L_{\mathrm{m}} & 0 & 0 \\ L_{\mathrm{m}} & L_{\mathrm{rr}} & 0 & 0 \\ 0 & 0 & L_{\mathrm{ss}} & L_{\mathrm{m}} \\ 0 & 0 & L_{\mathrm{m}} & L_{\mathrm{rr}} \end{bmatrix} \begin{bmatrix} i_{\mathrm{ds}} \\ i_{\mathrm{dr}} \\ i_{\mathrm{qs}} \\ i_{\mathrm{qr}} \end{bmatrix} \tag{4}$$

At this stage θ has not been defined and is quite arbitrary. However, certain simplifications in the equations occur if θ is restricted to be one of the following three angles:

(a) Stationary reference frame

Now $\theta = 0$; this is called the stationary reference frame because the d,q axes do not rotate. In addition, the +d-axis is chosen to coincide with the stator phase A axis. Hence, from Fig. 1,

$$\omega = p\theta = 0 \tag{5}$$

$$\beta = \theta - \theta_{\mathrm{r}} = -\theta_{\mathrm{r}} \tag{6}$$

$$p\beta = -p\theta_{\mathrm{r}} = -\omega_{\mathrm{r}} \tag{7}$$

Substituting eqns. (5) - (7) into eqn. (3),

$$v_{\mathrm{ds}} = R_{\mathrm{s}} i_{\mathrm{ds}} + p\psi_{\mathrm{ds}} \tag{8a}$$

$$v_{\mathrm{qs}} = R_{\mathrm{s}} i_{\mathrm{qs}} + p\psi_{\mathrm{qs}} \tag{8b}$$

$$v_{\mathrm{dr}} = R_{\mathrm{r}} i_{\mathrm{dr}} + p\psi_{\mathrm{dr}} - \omega_{\mathrm{r}}\psi_{\mathrm{qr}} \tag{8c}$$

$$v_{qr} = R_r i_{qr} + p\psi_{qr} + \omega_r \psi_{dr} \tag{8d}$$

Replacing the flux linkages in eqn. (8) by currents (using eqn. (4)),

$$[v] = [R][i] + [L]p[i] + \omega_r [G][i] \tag{9}$$

where

$$[v] = [v_{ds},\ v_{dr},\ v_{qs},\ v_{qr}]^T \tag{10}$$

$$[i] = [i_{ds},\ i_{dr},\ i_{qs},\ i_{qr}]^T \tag{11}$$

and the other matrices appear in Appendix A.

Equation (9) can be rearranged in state space form for solution on a digital computer as follows:

$$p[i] = [B]\{[v] - [R][i] - \omega_r [G][i]\} \tag{12}$$

where $[B]$ is the inverse of the inductance matrix and is given in Appendix A. $[B]$ is a constant matrix and needs to be inverted only once during the entire simulation.

If the busbar voltages are

$$v_{as} = V_m \cos(\omega_s t + \gamma) \tag{13a}$$

$$v_{bs} = V_m \cos(\omega_s t + \gamma - \lambda) \tag{13b}$$

$$v_{cs} = V_m \cos(\omega_s t + \gamma + \lambda) \tag{13c}$$

and the motor terminals are connected directly to the busbars, then using Park's transform of eqn. (2),

$$v_{ds} = V_m \cos(\omega_s t + \gamma)$$
$$v_{qs} = -V_m \sin(\omega_s t + \gamma) \tag{14}$$

Note that v_{ds}, the voltage applied to the stator d-axis coil, is the same as the stator phase A voltage. This means that the stator d-axis current i_{ds} is exactly equal to the phase A current i_{as}, and it is not necessary to compute i_{as} separately at each step of the integration process through the inverse of Park's transform. This saves computer time and hence is an advantage of the stationary reference frame.

If there are compensating capacitors connected between the busbar and the motor, then for the case when series capacitors are present in the line (but shunt capacitors are absent (Appendix B)):

$$v_{ds} = V_m \cos(\omega_s t + \gamma) - v_d^c - R_L i_{ds} - L_L p i_{ds} \tag{15}$$

$$v_{qs} = -V_m \sin(\omega_s t + \gamma) - v_q^c - R_L i_{qs} - L_L p i_{qs} \tag{16}$$

The voltage components v_d^c and v_q^c across the series capacitor are found by integrating the following two differential equations:

$$p v_d^c = i_{ds}/C_s \tag{17}$$

$$p v_q^c = i_{qs}/C_s \tag{18}$$

When there are shunt capacitors connected across the terminals of the motor but the series capacitors are absent,

$$p i_{dL} = \{V_m \cos(\omega_s t + \gamma) - R_L i_{dL} - v_{ds}\}/L_L \tag{19}$$

$$p i_{qL} = \{-V_m \sin(\omega_s t + \gamma) - R_L i_{qL} - v_{qs}\}/L_L \tag{20}$$

where

$$p v_{ds} = (i_{dL} - i_{ds})/C \tag{21}$$

$$p v_{qs} = (i_{qL} - i_{qs})/C \tag{22}$$

It is possible to formulate the equations where shunt and series capacitors are connected simultaneously to the motor. However, such a system is rather unlikely in practice and is therefore not considered here.

(b) Synchronously rotating reference frame

Now $\theta = \omega_s t$; this is called the synchronously rotating reference frame because the d,q axes rotate at synchronous speed. From Fig. 1,

$$\omega = p\theta = \omega_s \tag{23}$$

$$\beta = \theta - \theta_r = \omega_s t - \theta_r \tag{24}$$

$$p\beta = \omega_s - \omega_r \tag{25}$$

Substituting eqns. (23) - (25) into eqn. (3),

$$v_{ds} = R_s i_{ds} + p\psi_{ds} + \omega_s \psi_{qs} \tag{26a}$$

$$v_{qs} = R_s i_{qs} + p\psi_{qs} - \omega_s \psi_{ds} \tag{26b}$$

$$v_{dr} = R_r i_{dr} + p\psi_{dr} + (\omega_s - \omega_r)\psi_{qr} \tag{26c}$$

$$v_{qr} = R_r i_{qr} + p\psi_{qr} - (\omega_s - \omega_r)\psi_{dr} \tag{26d}$$

Following the same procedure as for the stationary reference frame, the state space equation for simulation on a digital computer is

$$p[i] = [B]\{[v] - [R][i] - \omega_s [F][i] - (\omega_s - \omega_r)[G_s][i]\} \tag{27}$$

where the $[v]$ and $[i]$ vectors are the same as in eqns. (10) and (11), and the other matrices appear in Appendix A.

Without any capacitors, the terminal voltages in eqn. (14) become

$$v_{ds} = V_m \cos\gamma$$
$$v_{qs} = -V_m \sin\gamma \tag{28}$$

This means that the stator d,q voltages are DC quantities and this has advantages in the field of feedback controller design when the motor equations are linearized around a steady operating point. It is also possible to use a larger step length in the digital integration routine when using this frame, since the variables are slowly changing DC quantities during transient conditions.

If there are compensating capacitors connected to the motor, then for the case when only series capacitors are present (Appendix B), the terminal voltage eqns. (15) and (16) become:

$$v_{ds} = V_m \cos\gamma - v_d^c - R_L i_{ds} - L_L p i_{ds} - \omega_s L_L i_{qs} \tag{29}$$

$$v_{qs} = -V_m \sin\gamma - v_q^c - R_L i_{qs} - L_L p i_{qs} + \omega_s L_L i_{ds} \tag{30}$$

The voltage components v_d^c and v_q^c across the series capacitor are found by integrating the following two differential equations:

$$p v_d^c = i_{ds}/C_s - \omega_s v_q^c \tag{31}$$

$$p v_q^c = i_{qs}/C_s + \omega_s v_d^c \tag{32}$$

When there are shunt capacitors connected across the terminals of the motor (without series capacitors), eqns. (19) - (22) become

$$p i_{dL} = (V_m \cos\gamma - R_L i_{dL} - v_{ds} - \omega_s L_L i_{qL})/L_L \tag{33}$$

$$p i_{qL} = (-V_m \sin\gamma - R_L i_{qL} - v_{qs} + \omega_s L_L i_{dL})/L_L \tag{34}$$

$$p v_{ds} = (i_{dL} - i_{ds})/C - \omega_s v_{qs} \tag{35}$$

$$p v_{qs} = (i_{qL} - i_{qs})/C + \omega_s v_{ds} \tag{36}$$

(c) Rotor reference frame

Now $\theta = \int \omega_r \, dt$; this is called the rotor reference frame because the d,q axes rotate at the rotor speed, but in addition the d-axis position coincides with the rotor phase A axis. Hence, from Fig. 1,

$$\omega = p\theta = \omega_r \tag{37}$$

$$\beta = \theta - \theta_r = 0 \tag{38}$$

$$p\beta = 0 \tag{39}$$

Substituting eqns. (37) - (39) into eqn. (3),

$$v_{ds} = R_s i_{ds} + p\psi_{ds} + \omega_r \psi_{qs} \tag{40a}$$

$$v_{qs} = R_s i_{qs} + p\psi_{qs} - \omega_r \psi_{ds} \tag{40b}$$

$$v_{dr} = R_r i_{dr} + p\psi_{dr} \tag{40c}$$

$$v_{qr} = R_r i_{qr} + p\psi_{qr} \tag{40d}$$

Following the same procedure as for the stationary reference frame, the state space equation for simulation on a digital computer is

$$p[i] = [B]\{[v] - [R][i] - \omega_r [G_r][i]\} \tag{41}$$

where the $[v]$ and $[i]$ vectors are the same as in eqns. (10) and (11), and the other matrices appear in Appendix A.

Without any capacitors, the terminal voltage eqn. (14) becomes

$$v_{ds} = V_m \cos(s\omega_s t + \gamma) \tag{42}$$

$$v_{qs} = -V_m \sin(s\omega_s t + \gamma) \tag{43}$$

The d,q voltages are therefore of slip frequency and the d-axis rotor current behaves exactly as the phase A rotor current does. This means that it is not necessary to compute the phase A rotor current at each step of the digital integration process through the inverse of Park's transform. This saves computer time and hence is an advantage of the rotor reference frame when studying rotor quantities.

If there are compensating capacitors connected to the motor, then for the case when only series capacitors are present, the terminal voltage eqns. (15) and (16) become

$$v_{ds} = V_m \cos(s\omega_s t + \gamma) - v_d^c - R_L i_{ds} - L_L p i_{ds} - \omega_r L_L i_{qs} \tag{44}$$

$$v_{qs} = -V_m \sin(s\omega_s t + \gamma) - v_q^c - R_L i_{qs} - L_L p i_{qs} + \omega_r L_L i_{ds} \tag{45}$$

where

$$p v_d^c = i_{ds}/C_s - \omega_r v_q^c \tag{46}$$

$$p v_q^c = i_{qs}/C_s + \omega_r v_d^c \tag{47}$$

When there are shunt capacitors connected across the terminals of the motor (without series capacitors), eqns. (19) - (22) become

$$p i_{dL} = \{V_m \cos(s\omega_s t + \gamma) - R_L i_{dL} - \omega_r L_L i_{qL} - v_{ds}\}/L_L \tag{48}$$

$$p i_{qL} = \{-V_m \sin(s\omega_s t + \gamma) - R_L i_{qL} + \omega_r L_L i_{dL} - v_{qs}\}/L_L \tag{49}$$

$$p v_d^c = (i_{dL} - i_{ds})/C - \omega_r v_{qs} \tag{50}$$

$$p v_q^c = (i_{qL} - i_{qs})/C + \omega_r v_{ds} \tag{51}$$

Thus far, the voltage equations for three reference frames have been presented with their advantages. The electrical torque is given by the following expression which is independent of the reference frame:

$$T_e = \omega_b L_m (i_{dr} i_{qs} - i_{qr} i_{ds})/2 \qquad (52)$$

The mechanical motion is described by

$$p\omega_r = (T_e - T_L)/J \qquad (53)$$

3. RESULTS

In order to evaluate the usefulness of each frame of reference, the equations of §2 are used to predict the transient behaviour of a 22 kW motor; its parameters appear in Appendix C.

3.1. Frames of reference

Figure 2 shows the predicted start-up characteristics against no load, using each one of the above three reference frames.

The electrical torque (Fig. 2(b)) exhibits the familiar initial 50 Hz oscillations, and full speed is reached within 0.2 s. Without any soft-start control being used, and considering the worst case, the motor starting current reaches 9.5 p.u. Despite the fact that the behaviour of the physical variables as predicted by each reference frame is identical, the d,q variables in the respective frames of reference are different; this difference can be exploited by careful matching of the reference frame to the problem being solved, as illustrated in the following sections.

3.2. Stationary reference frame

In this reference frame the d-axis is fixed to and thus coincident with the axis of the stator phase A winding. This means that the mmf wave of the stator moves over this frame at the same speed as it does over the stator phase A windings. This reference frame's stator d-axis variables therefore behave in exactly the same way as do the physical stator phase A variables of the motor itself. Figures 3(a) and (b) show the identical nature of the stator phase A current and the stator d-axis current. It is therefore not necessary to go through the inverse of Park's transform to compute the phase A current, thus saving in valuable computer time.

The calculated rotor d-axis variables of this reference frame are, however, also transformed at 50 Hz frequencies and are therefore not the same as the actual rotor phase A variables which are at slip frequency. Figure 3(c) and (d) show this difference between the rotor phase A current and the rotor DR current.

It would therefore be an advantage when studying transient occurrences involving the stator variables to use this reference frame.

3.3. Rotor reference frame

Since in this reference frame the d-axis of the reference frame is moving at the same relative speed as the rotor phase A winding and coincident with its axis, it should be expected from the considerations of §3.2 that the behaviour of the d-axis current and the phase A current would be identical.

Figures 4(c) and (d) show how the rotor phase A current and the rotor d-axis current are initially at 50 Hz, when the rotor is at standstill, but gradually change to slip frequency at normal running speed. If this reference frame is used for the study of rotor variables, it is therefore not necessary to go through the inverse Park's transform to compute the actual rotor phase A current. Since the rotor d-axis variables are at slip frequency, this reference frame is useful in studying transient phenomena in the rotor.

The transformation of the stator variables at slip frequency in this frame clearly shows its unsuitability for studying stator variables, which are at 50 Hz frequencies.

3.4. Synchronously rotating reference frame

When the reference frame is rotating at synchronous speed, both the stator and rotor are rotating at different speeds relative to it. However, with the reference frame rotating at the same speed as the stator and rotor space field mmf waves, the stator and rotor d,q variables are constant quantities, whereas the actual variables are at 50 Hz and slip frequencies respectively. Figure 5(b) shows how the stator d-axis current gradually reduces from 50 Hz frequency at the instant of switching to the equivalent of a steady DC when at rated speed.

The steady nature of this stator d-axis current makes this reference frame useful when an analog computer is used in simulation

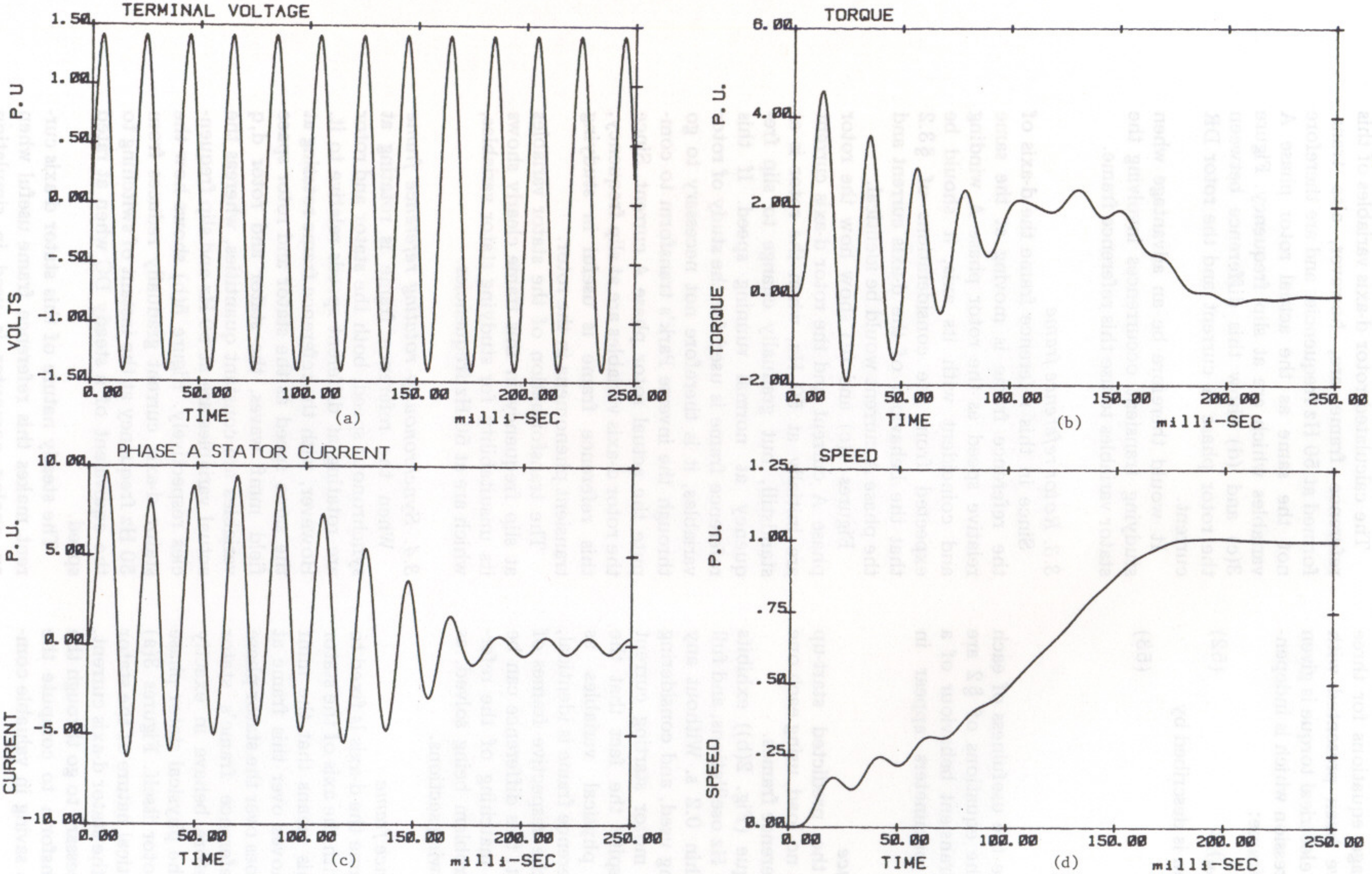

Fig. 2. Predicted starting up performance of a 22 kW induction motor.

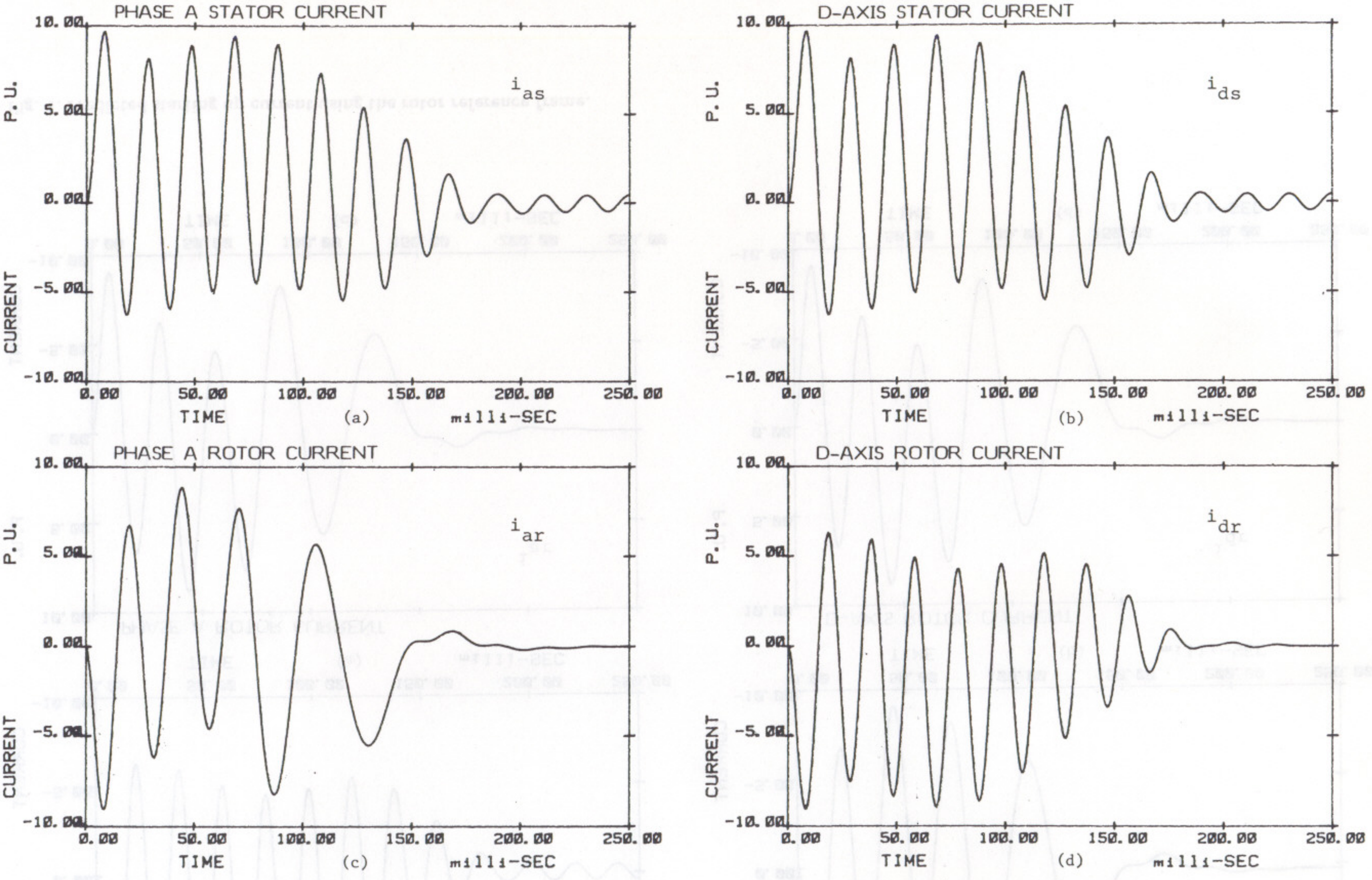

Fig. 3. Predicted starting up current using the stationary reference frame.

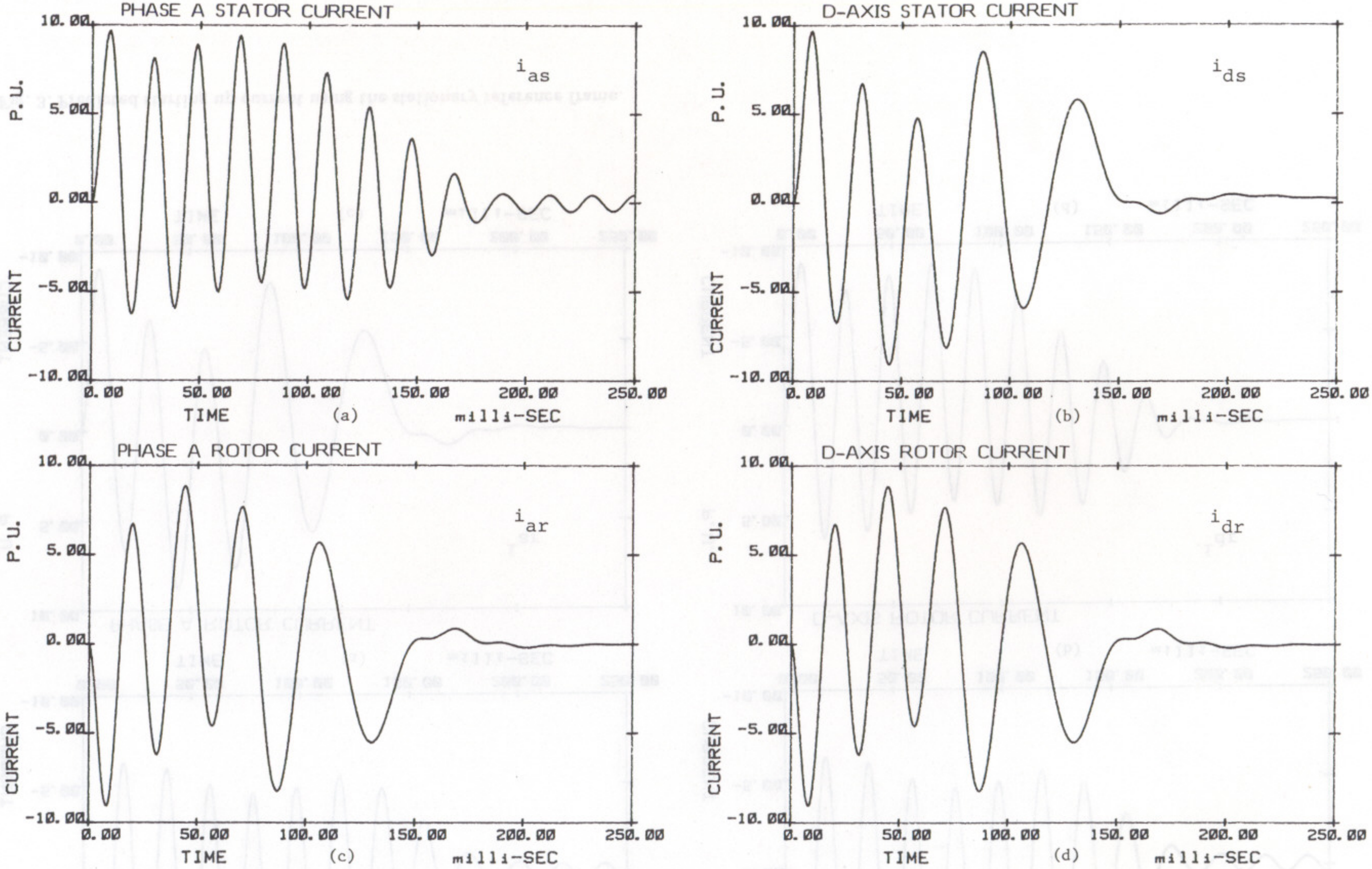

Fig. 4. Predicted starting up current using the rotor reference frame.

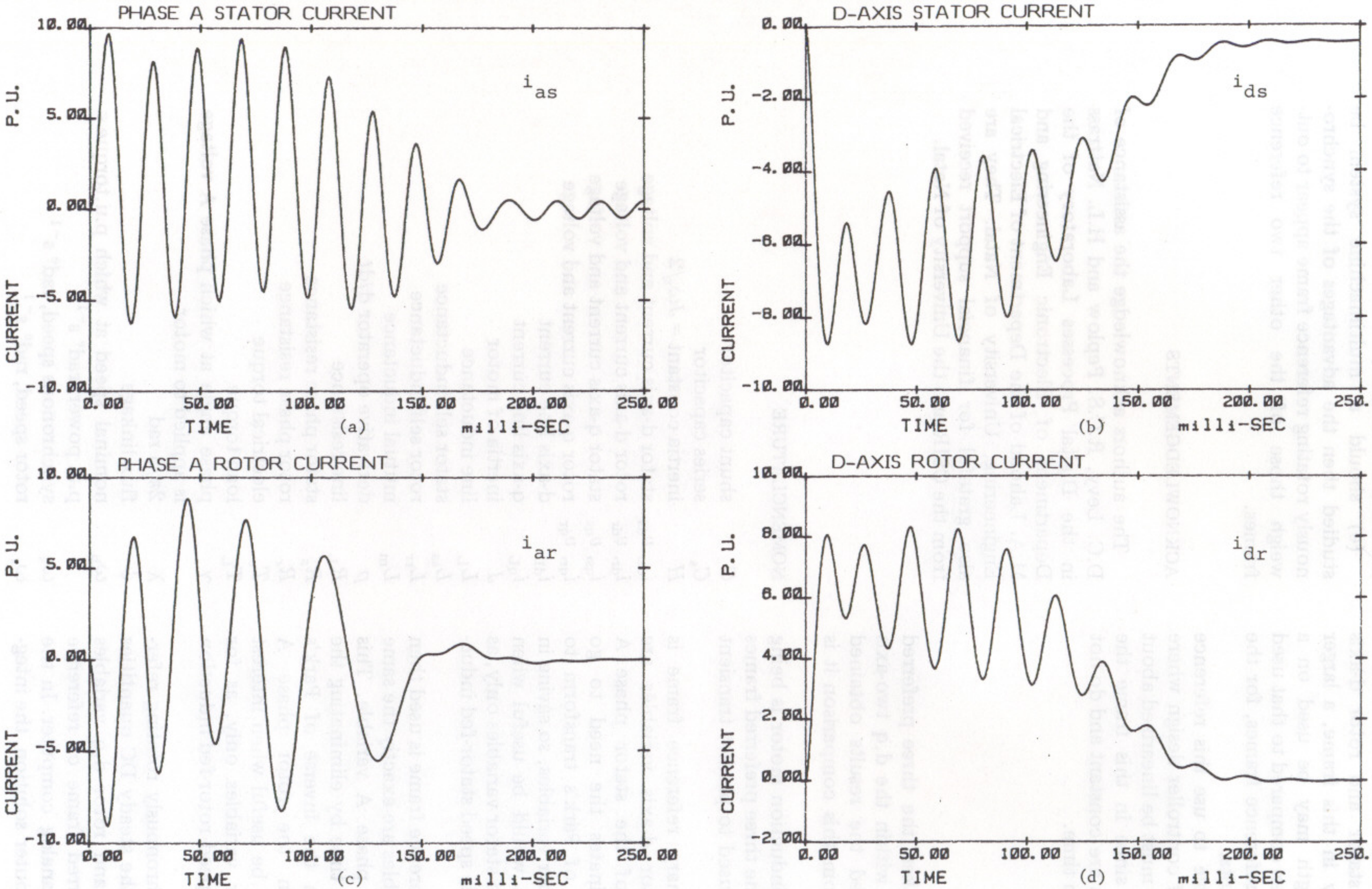

Fig. 5. Predicted starting up current using the synchronously rotating reference frame.

studies, since the input to the machine equations are constant prior to any disturbance. Also, because the stator and rotor d-axis variables vary slowly in this frame, a larger integration step length may be used on a digital computer, when compared to that used with the other two reference frames, for the same accuracy of results.

It is also preferable to use this reference frame for stability of controller design where the motor equations must be linearized about an operating point, since in this frame the steady state variables are constant and do not vary sinusoidally with time.

4. CONCLUSIONS

This paper has taken the three preferred frames of reference within the d,q two-axis theory and compared the results obtained with each frame. From this comparison it is concluded that:

(a) when a single induction motor is being studied, any one of the three preferred frames of reference can be used to predict transient behaviour;

(b) if the stationary reference frame is used, then the stator d-axis variables are identical to those of the stator phase A variables. This eliminates the need to go through the inverse of Park's transform to obtain the actual stator variables, so saving in computer time. This would be useful when interest is confined to stator variables only, as for example in variable speed stator-fed induction motor drives;

(c) if the rotor reference frame is used then the rotor d-axis variables are exactly the same as the actual rotor phase A variable. This again saves computer time by eliminating the need to go through the inverse of Park's transform to obtain the rotor phase A variables. This would be useful when interest is confined to rotor variables only, as for example in variable speed rotor-fed induction motor drives;

(d) when the synchronously rotating reference frame is used, the steady DC quantities both of the stator and rotor d,q variables make this the preferred frame of reference when employing an analog computer. In the case of a digital computer solution, the integration step length may be lengthened without affecting the accuracy of results;

(e) should a multimachine system be studied then the advantages of the synchronously rotating reference frame appear to outweigh those of the other two reference frames.

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NOMENCLATURE

C	shunt capacitor
C_s	series capacitor
H	inertia constant = $J\omega_s/2$
i_{ds}, v_{ds}	stator d-axis current and voltage
i_{dr}, v_{dr}	rotor d-axis current and voltage
i_{qs}, v_{qs}	stator q-axis current and voltage
i_{qr}, v_{qr}	rotor q-axis current and voltage
i_{dL}	d-axis line current
i_{qL}	q-axis line current
J	inertia of motor
L_L	line inductance
L_{ss}	stator self inductance
L_{rr}	rotor self inductance
L_m	mutual inductance
p	derivative operator d/dt
R_L	line resistance
R_s	stator phase resistance
R_r	rotor phase resistance
T_e	electrical torque
T_L	load torque
γ	phase angle at which phase A voltage is applied to motor
λ	$2\pi/3$ rad
ψ	flux linkage
ω_b	nominal speed at which p.u.torque = p.u. power, rad^e s^{-1}
ω_s	synchronous speed, rad^e s^{-1}
ω_r	rotor speed, rad^e s^{-1}

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APPENDIX A

D, Q axis model matrices

$$[R] = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_r & 0 & 0 \\ 0 & 0 & R_s & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix}$$

$$[L] = \begin{bmatrix} L_{ss} & L_m & 0 & 0 \\ L_m & L_{rr} & 0 & 0 \\ 0 & 0 & L_{ss} & L_m \\ 0 & 0 & L_m & L_{rr} \end{bmatrix}$$

$$[G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -L_m & -L_{rr} \\ 0 & 0 & 0 & 0 \\ L_m & L_{rr} & 0 & 0 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 0 & 0 & L_{ss} & L_m \\ 0 & 0 & 0 & 0 \\ -L_{ss} & -L_m & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[B] = \begin{bmatrix} B_s & B_m & 0 & 0 \\ B_m & B_r & 0 & 0 \\ 0 & 0 & B_s & B_m \\ 0 & 0 & B_m & B_r \end{bmatrix}$$

where

$B_s = L_{rr}/D, \quad B_r = L_{ss}/D$

$B_m = -L_m/D, \quad D = L_{ss}L_{rr} - L_m{}^2$

$$[G_s] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & L_m & L_{rr} \\ 0 & 0 & 0 & 0 \\ -L_m & -L_{rr} & 0 & 0 \end{bmatrix}$$

$$[G_r] = \begin{bmatrix} 0 & 0 & L_{ss} & L_m \\ 0 & 0 & 0 & 0 \\ -L_{ss} & -L_m & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

APPENDIX B

Derivation of expressions for voltage drops i the d, q model

The voltage drop across an inductor in AB phase variables is given by

$$[v_{ABC}]_L = Lp[i_{ABC}] \quad \text{(B-1}$$

$$= Lp[P_\theta]^{-1}[i_{0dq}] \quad \text{(B-2}$$

The voltage drop in the 0dq reference frame i given by

$$[P_\theta][v_{ABC}]_L = [P_\theta]Lp\{[P_\theta]^{-1}[i_{0dq}]\} \quad \text{(B-3}$$

$$[v_{0dq}]_L = L[P_\theta]p\{[P_\theta]^{-1}[i_{0dq}]\}$$

$$= Lp[i_{0dq}] + L[P_\theta]p[P_\theta]^{-1}[i_{0dq}] \quad \text{(B-4}$$

Now, substituting for

$$[P_\theta]p[P_\theta]^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \omega \\ 0 & -\omega & 0 \end{bmatrix} \quad \text{(B-5}$$

hence

$$v_{dL} = Lpi_d + \omega Li_q \quad \text{(B-6}$$

$$v_{qL} = Lpi_q - \omega Li_d \quad \text{(B-7}$$

The voltage drop across a capacitor in AB phase variables is given by

$$[v_{\mathrm{ABC}}]_{\mathrm{C}} = (1/C)\int [i_{\mathrm{ABC}}]\,\mathrm{d}t \tag{B-8}$$

In the 0dq reference frame the voltage drop becomes

$$[P_\theta]^{-1}[v_{\mathrm{ABC}}]_{\mathrm{C}} = (1/C)\int [P_\theta]^{-1}[i_{0\mathrm{dq}}]\,\mathrm{d}t \tag{B-9}$$

Differentiation gives

$$[P_\theta]^{-1}p[v_{0\mathrm{dq}}] + p[P_\theta]^{-1}[v_{0\mathrm{dq}}] = (1/C)[P_\theta]^{-1}[i_{0\mathrm{dq}}] \tag{B-10}$$

Multiplication by P_θ gives

$$p[v_{0\mathrm{dq}}] + [P_\theta]p[P_\theta]^{-1}[v_{0\mathrm{dq}}] = (1/C)[i_{0\mathrm{dq}}] \tag{B-11}$$

Therefore

$$p[v_{0\mathrm{dq}}] = (1/C)[i_{0\mathrm{dq}}] - [P_\theta]p[P_\theta]^{-1}[v_{0\mathrm{dq}}] \tag{B-12}$$

Hence

$$pv_{\mathrm{d}} = i_{\mathrm{d}}/C - \omega v_{\mathrm{q}} \tag{B-13}$$

$$pv_{\mathrm{q}} = i_{\mathrm{q}}/C + \omega v_{\mathrm{d}} \tag{B-14}$$

APPENDIX C

22 kW Induction motor parameters

Base time	1 s
Base speed	1 rad$^{\mathrm{e}}$ s^{-1}
Base power	27.918 kVA
Base stator voltage	220 V (phase)
Base stator current	42.3 A (phase)
Base stator impedance	5.21 Ω
Base torque	177.8 N m
Number of poles	4
Rated output power	22 kW
Stator resistance R_{s}	0.021 p.u.
Rotor resistance R_{r}	0.057 p.u.
Stator leakage reactance X_{s}	0.049 p.u. expressed
Rotor leakage reactance X_{r}	0.132 p.u. at 50 Hz
Magnetizing reactance X_{m}	3.038 p.u.