Comparison of models for predicting disturbances caused by induction motor starting

SYNOPSIS

When starting induction motors directly on line, various system disturbances occur like voltage dips, inrush currents and low lagging power factor. To counteract these disturbances, series and shunt capacitors are sometimes added but they, in turn, may adversely affect motor operation and generate harmonics into the power system. This paper uses different mathematical models for the induction machine to analyse these problems and the results of several case studies are included.

Keyword Index: Induction motor starting, disturbances.

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<tr>
<td>$\theta$</td>
<td>arbitrary angle $= \omega t$</td>
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<td>$\omega_h$</td>
<td>speed when p.u. torque = p.u. power</td>
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I Introduction

Induction motors are being used more than ever before in industry, and individual machines of up to 10 MW in size are no longer a rarity. When starting induction motors directly on line, various system disturbances occur like large inrush currents, voltage dips and low lagging power factor.

A capacitor can be connected in series with the feeder to compensate for the feeder inductance, and hence improve the voltage regulation. Series capacitors are therefore particularly well suited to radial circuits where the induction motor shares a common bus with other equipment which would be affected adversely by the voltage dips. Shunt capacitors connected in parallel with the load are mainly used to improve the receiving end power factor and voltage, by neutralising part of the lagging current in the circuit. They therefore in fact compensate for the inductance of the load and are most effective when the load draws a current that is substantially constant in magnitude and power factor. However, under certain conditions, series capacitors can cause unstable voltage and current oscillations at subsynchronous frequencies in the feeder and motor, while shunt capacitors can generate harmonics into the power system.

The purpose of this paper is to investigate the starting performance of an induction motor and its effects on the power system when series or shunt capacitors are present. It evaluates different mathematical models for the motor and presents the results of several case studies, with particular reference to voltage dips and inrush currents.

2 Theory

The following three mathematical models are available to simulate the dynamic behaviour of an induction motor:

(a) RMS equivalent circuit
(b) D, Q axis representation
(c) A, B, C phase variable representation

The RMS equivalent circuit uses RMS values of currents and voltages to predict torque. It neglects electrical transients in the machine, although slowly changing mechanical transients can be studied. The $d, q$ axis representation on the other hand simulates the transient response of the electrical circuit as well as of any mechanical condition. Moreover the $A, B, C$ representation also simulates the same transients as the $d, q$ method, but has the disadvantage that a computer is absolutely essential and that it uses more computer time because it consists of more differential equations. The rest of this paper therefore considers the RMS equivalent circuit and the $d, q$ axis methods only.

2.1 RMS equivalent circuit

The equivalent circuit of an induction motor with capacitors is shown in Fig 1. The electrical torque is given by:

\[ T_e = \frac{3}{2} \frac{L}{L_r} (i_d^* i_q - i_q^* i_d) \]

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The mechanical motion is described by:

\[ p\omega = (M - T_f) / J \]  

(2)

The voltage \( V_1 \) is given by the following equation when the shunt capacitance is omitted from Fig 1:

\[ V_1 = \dot{V}_{bus} - (R_L + j\omega L_L - j/\omega C_s)I_L \]  

(3)

When a shunt capacitor is connected across the motor terminals, and the series capacitor is omitted:

\[ V_1 = \dot{V}_{bus} - (R_L + j\omega L_L)I_L \]  

(4)

2.2 Two axis equations or d, q theory

The two axis (d, q) voltage expressions\(^{(1)}\) for an induction motor can be summarized as follows in a stationary reference frame:

\[
\begin{bmatrix}
  v_d \\
  v_q
\end{bmatrix} =
\begin{bmatrix}
  R_d & L_d \\
  L_d & R_q
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} +
\begin{bmatrix}
  \omega_L G \\
  \omega_L G
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
\]

(5)

where \( [v] = [v_d, v_q] \) and \( [i] = [i_d, i_q] \)

\[
\begin{bmatrix}
  v_d \\
  v_q
\end{bmatrix} = [B][v] - [R][i] - \omega_L [G][i]
\]

(6)

Eq (5) can be written in state space form for solution on a digital computer as follows:

\[
p[i] = [B][v] - [R][i] - \omega_L [G][i]
\]

(7)

where \( [B] \) is the inverse of the \( [L] \) matrix. Expressions for all these matrices appear in Ref [1]. The electrical torque is given by:

\[
T_e = L_m \omega_b (i_d i_q - i_q i_d) / 3
\]

(8)

The equation for mechanical motion is:

\[
p\omega = (T_e - T_f) / J
\]

(9)

The voltages \( V_{ds}, V_{qs}, V_{qr}, V_{q}, V_{q} \) are obtained from the orthogonal Park's Transform\(^{(1)}\) operating on the actual phase voltages. To obtain the phase voltages at any instant, the inverse of this transform is used. In the stationary reference frame, for a switching angle of \( \gamma \), this yields

\[
V_{ds} = \sqrt{3/2} V_m \cos(\omega t + \gamma)
\]

(10)

where \( V_{ds} = 0 \) for a cage rotor.

When the motor is supplied by a feeder with resistance \( R_L \), inductance \( L_L \), and series capacitor \( C_s \), the voltages \( v_d, v_q \) are modified as follows to allow for the volt drop along the feeder.

\[
v_{ds} = \sqrt{3/2} V_m \cos(\omega t + \gamma) - \dot{V}_{ds} - R_L i_d - L_L \dot{i}_d
\]

(11)

\[
v_{qs} = -\sqrt{3/2} V_m \sin(\omega t + \gamma) - \dot{V}_{qs} - R_L i_q - L_L \dot{i}_q
\]

(12)

The system dynamics are therefore described by the four differential equations for motor currents in Eq (7), the one for speed in Eq (9) and the two for capacitor voltages in Eqs (13). They are nonlinear and are integrated numerically on a digital computer.

When there is a shunt capacitor connected across the terminals of the motor, but with the series capacitor omitted:

\[
pv_d = i_d / C_s \quad \text{and} \quad pv_q = i_q / C_s
\]

(13)

3 Results

The two axis and equivalent circuit equations developed in section 2 are used to evaluate the starting performance of a 22 kW induction motor and its effects on the power system.

3.1 Start-up of the induction motor

Fig 2 shows the measured results of voltage, current and speed when the induction motor is started on no load. A reduced voltage is used in order to limit the starting currents and the motor therefore takes approximately 4 s to run up. There is a large volt-drop initially due to the feeder impedance. Fig 3 contains the corresponding predicted results using the \( d, q \) axis theory and these agree with the measured values in Fig 2. The torque exhibits a slip-frequency component which initially is 50 Hz and which even goes negative for the first few cycles, thus indicating momentary periods of regeneration; the slip frequency is caused by the electrical tran-
sients within the motor. The transient torque peak has approximately the same magnitude as the breakdown torque, but it is known\(^4\) that in some cases this peak could be greater than the breakdown torque.

Fig 4 shows the predicted starting-up performance based on the RMS equivalent circuit of the motor. These RMS values of current and voltage agree closely with those of the slowly changing sinusoids measured in Fig 2. The measured low frequency oscillation in the start-up current is predicted by the \(d,q\) axis method, but not by the equivalent circuit. Of greater significance though is that the equivalent circuit predicts a slowly changing average value of torque and this inability to predict the transient slip frequency electrical torque is its greatest disadvantage, especially in the presence of externally connected shunt or series capacitors as illustrated in the following sections.

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**Fig 2** Measured starting-up performance of a 22 kW induction motor

**Fig 3** Two axis predicted starting-up performance of a 22 kW induction motor
3.2 Start-up of the induction motor with shunt capacitors

Fig 5 shows the measured starting-up results in the presence of shunt capacitors while Figs 6 and 7 contain the corresponding predicted curves. The final motor terminal voltage is higher than before because of the reduction in lagging current caused by the capacitors. The initial low frequency oscillation of the current is again predicted by the \( d,q \) axis method, but not by the equivalent circuit method. Once full speed has been reached there is an overshoot or underdamped oscillatory response in voltage and current that is predicted by the \( d,q \) method. Although the magnitude of the voltage is predicted accurately, its time constant is not. This is probably due to inaccurate parameter values and also to \( r_m \) being neglected in the \( d,q \) model; \( r_m \) provides more damping and hence reduces the time constant. The motor runs up in shorter time due to the higher terminal voltage and hence higher torque available (torque is pro-
portional to voltage).

Fig 8 shows the first 50 ms of the $d, q$ predicted current and voltage together with their corresponding measured results. Good accuracy is obtained for the current and voltage and the fact that transient current and voltage harmonics are generated into the system in the presence of shunt capacitors is clearly indicated. The RMS equivalent circuit method does not predict this.

3.3 Start-up of the motor in the presence of series capacitors

Fig 9 shows the $d, q$ predicted no-load starting-up performance in the presence of series capacitors where the impedance of the series capacitors equals that of the feeder inductance. After attaining rated speed, the motor goes into a subsynchronous oscillation (known as subsynchronous resonance). During this period the tor-
aque rises to 15 p.u. (which could damage the motor's shaft) and the current increases to about 8 p.u. in spite of the motor being on no load. This large current, besides being detrimental to the motor, causes voltage fluctuations on the bus which the motor shares with other equipment, and could affect their dynamic performance and insulation strength, especially because the motor voltage momentarily rises to values which are considerably larger than the rated supply voltage. Hence although the series capacitors were installed to reduce the voltage dips, by causing the unstable motor oscillations they could in fact create a situation which is even worse.

Fig 10 shows that the equivalent circuit model on the other hand predicts none of these oscillations since it is based on RMS quantities and the capacitive reactance is simply subtracted from the inductive reactance of the
feeder. Measurements to corroborate these results would have damaged the equipment and were therefore not carried out. Similar experiments on a 2,2 kW machine\(^{(3)}\) have however clearly shown these resonances to exist in practice. Where improvement in voltage regulation is desired through the use of series capacitors, the capacitive reactance must therefore be less than the line inductive reactance (to avoid subsynchronous resonance) by an amount determined by the line and motor resistance values and would have to be evaluated for each specific case.

4 Conclusions
The equivalent circuit method can be used to predict the starting-up current and speed both with and without shunt capacitors. To obtain an accurate prediction of the torque, terminal voltage and the harmonics generated into the power system, the \(d,q\) method must be used. The equivalent circuit method gives totally incorrect results when series capacitors are present because it considers the RMS impedance of the capacitor instead of the instantaneous impedance as the \(d,q\) method does.

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6 References