

# Sizing Ultracapacitors For Hybrid Electric Vehicles

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**Abstract** – An efficient energy storage medium is essential in all hybrid electric vehicles. The advances in double layer electrolytic capacitor technology have opened new areas to complement batteries as a storage medium. In this paper we will review some of the present applications of ultracapacitors as well as to provide guidelines for sizing ultracapacitors for minimal mass in hybrid electric vehicles. Equations for both constant current as well as constant power discharge are discussed. An iterative method for determining the minimum number of ultracapacitor cells is introduced. The effects of ultracapacitor sizing on the rating of interface power electronics are examined.

**Keywords** – Ultracapacitors, hybrid electric vehicles, energy storage

## I. INTRODUCTION

Ultracapacitor technology has been commercially available for over the past decade. They can store much more energy than conventional capacitors and are available in sizes up to 4000F with voltage ratings of up to 3V per cell. They can be discharged or charged faster than batteries and can deliver 10-20 times more power e.g. ultracapacitors typically have 10 times the specific power (W/kg) as well as a much lower charge time when compared to lead acid batteries. They also offer 10 to 100 times the energy density (Wh/Kg) of conventional capacitors. In terms of energy and power density, ultra capacitors can therefore be placed between batteries and conventional capacitors [1-2]. A comparison of conventional storage technologies is shown in Table 1.

Ultracapacitors can be used as a temporary power source for backup purposes. Batteries were first used for this application, however the large energy storage capability of ultracapacitors makes them an alternative to batteries. In these applications the ultracapacitor is charged from the mains and provides temporary power during a power failure. [3]

The second application of ultracapacitors is important in the field of energy storage in hybrid electric vehicles. Ultracapacitors can be used to provide the short bursts of energy needed by hybrid electric vehicles during acceleration. Ultracapacitors do not need regular replacement like batteries because they are not as adversely affected during repetitive deep charging and discharging. This also implies that ultracapacitors are more environmentally friendly since they don't need to be frequently discarded. Ultracapacitors can be fully charged from a total discharge within a few seconds. This makes them ideal for use in regenerative braking systems. Ultracapacitors are not a

prone to temperature effects as are batteries and can operate in temperatures as low as  $-40^{\circ}\text{C}$  [3].

Ultracapacitors cannot replace batteries completely, however they can be used to complement each other. In the hybrid electric vehicle, the ultracapacitor can be used during vehicle operation. During standstill a downsized battery can be used to power the auxiliary systems. A slow charge time can be used to charge the batteries and thus extending their lifetime. Although batteries cannot deliver large currents at extremely low temperatures, they can deliver enough current to slowly charge an ultracapacitor, which can be used to start the vehicle. This could prompt the use of longer lasting batteries rather than large batteries designed to deliver hundreds of amps during a cold start.

Available Performance	Lead Acid Battery	Ultra Capacitor	Conventional Capacitor
Charge Time	1 to 5 hrs	0.3 to 30 s	$10^{-3}$ to $10^{-6}$ s
Discharge Time	0.3 to 3 hrs	0.3 to 30 s	$10^{-3}$ to $10^{-6}$ s
Energy ( Wh/kg )	10 to 100	1 to 10	< 0.1
Cycle Life	1000	> 500 000	> 500 000
Specific Power ( W/kg )	< 1000	< 10 000	< 100 000
Charge / Discharge efficiency	0.7 to 0.85	0.85 to 0.98	> 0.95

Table 1. A Comparison of conventional storage technologies [1].

## II. THE DEFICIENCIES OF BATTERIES

Although progress has been made in hybrid electrical vehicle control, engine and motor design, very little has been done to improve the passive energy storage device, namely the battery. Cold weather adversely affects the operation of batteries. Batteries have a very limited life cycle under extreme discharging conditions and need to be continuously replaced throughout the lifetime of the vehicle. The cost associated with the purchase of new batteries and the disposal of old ones is cumulative. Batteries are not environmentally friendly and thus cannot be easily disposed [3].

Batteries are the secondary power source in hybrid electric vehicles. They provide short bursts of supplementary power during acceleration, cold starting and even braking. However, not one battery type delivers all the power requirements needed by hybrid electric vehicles. Lithium

iron batteries can provide a small amount of current over a long time but cannot provide large bursts of power over a short time. They also have difficulty in absorbing energy during regenerative braking [3].

Lead acid and Ni-MH batteries are capable of delivering the short bursts of power, however operating them under the extreme discharging conditions dramatically reduces their lifetime. As a solution to this problem, manufacturers parallel many batteries to increase the power characteristics of the battery pack. This practice increases the weight and cost of the vehicle. Since the weight has been increased, the vehicle will demand higher currents from the batteries and therefore once again more batteries are needed. Clearly this is not the best solution to the energy storage problem.

## II. MODELING OF ULTRACAPACITORS

When sizing the ultracapacitor, it is necessary to understand the implications of the various factors that not only affect the capacitor, but will also affect the design of the interface power electronics.

These factors affecting the choice of capacitor include:

- the peak capacitor voltage,
- allowable maximum percentage discharge,
- peak current flowing through the capacitor,
- capacitor time constant ( $\tau$ ),
- capacitance per cell,
- cell voltage,
- number of cells needed,
- mass of the cell array,
- cost of the cell array.

An ultracapacitor can be modeled in a similar manner to conventional capacitors. The circuit schematic in Figure 1 represents the first-order model for an ultracapacitor. It's comprised of four ideal circuit elements: a capacitance  $C$ , a series resistor  $R_s$ , a parallel resistor  $R_p$ , and a series inductor  $L$ .  $R_s$  is called the equivalent series resistance (ESR). This resistance contributes to the energy loss component of the ultracapacitor during charging or discharging. The parallel resistor,  $R_p$ , models the leakage current found in all ultracapacitors. This leakage current varies from a few milliamps to tens of milliamps in large ultracapacitors. The series inductance,  $L$ , is usually very small and is neglected for constant current charging / discharging applications. However, in many applications where there is a repetitive charging / discharging, this inductance plays a significant role especially at high frequencies.

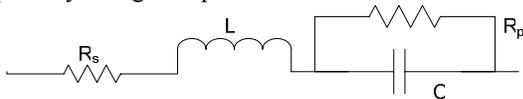


Figure 1. First order model of an ultracapacitor [5].

The voltage rating of ultracapacitors is typically less than 4V. When higher voltage ratings are required, many

ultracapacitors are connected in series to form a string of cells. Ultracapacitors, like conventional electrolytic capacitors, suffer from large deviations in their capacitance. These deviations can be as high as 20% from the nominal cell capacitance value. When connected in series, a common current flows through all the cells and since the cell capacitances differ, the cell voltages will vary. This could result in individual cells being overcharged and thus exceeding their voltage limits. For this reason, an active or passive voltage balancing circuit is employed to regulate the cell voltage.

The discharge profile for an ultracapacitor under constant current is shown in Figure 2. A constant current discharge is particularly useful when determining the parameters of the ultracapacitor. These parameters include the equivalent series resistance, specific power and specific energy.

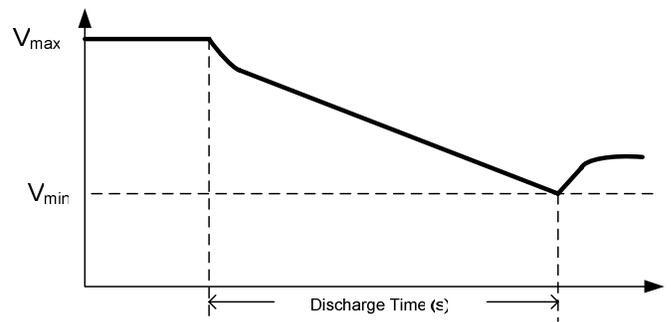


Figure 2. The voltage discharge profile for an ultracapacitor under constant current discharge [6].

The equation relating the capacitance to the voltage, current and discharge time can be expressed as follows [6]:

$$C = \frac{I}{V_{\max} - V_{\min}}(t + \tau) \quad (1)$$

where,

$C$  is the capacitance,

$I$  is the average current,

$V_{\max}$  is peak capacitor voltage,

$V_{\min}$  is the lowest voltage after discharge,

and  $\tau$  is the product of the equivalent series resistance and the capacitance.

Although equation (1) gives an insight to the properties of the ultracapacitor, it should be noted that this equation should not be used to size ultracapacitors for constant power applications. The capacitor voltage (under constant power discharge) is described by equation (2). For simplicity the equivalent series resistance is neglected but will be analyzed later.

$$V(t) = \sqrt{V_{\max}^2 - \frac{2pt}{C}} \quad (2)$$

where

$p$  is the constant power,

$t$  is the time,  
 $C$  is the total capacitance of a string of capacitors,  
 $V_{max}$  is the initial voltage,  
 $V(t)$  is the capacitor voltage at time  $t$ .

Since the power being delivered is constant, the minimum voltage can be determined based on the current conducting capabilities of the ultracapacitor. Equation (2) can then be rewritten as

$$V_{min} = \sqrt{V_{max}^2 - \frac{2pT}{C}} \quad (3)$$

where,  
 $p$  is the constant power,  
 $T$  is discharge the time,  
 $C$  is the total capacitance of a string of capacitors,  
 $V_{max}$  is the initial voltage,  
 $V_{min}$  is the final voltage.

Similarly, the maximum current occurs at the minimum voltage and the expression for the current can be derived as follows.

$$i(t) = \frac{p}{\sqrt{V_{max}^2 - \frac{2pt}{C}}} \quad (4)$$

$$I_{max} = \frac{p}{\sqrt{V_{min}^2 - \frac{2pT}{C}}} \quad (5)$$

where,  
 $p$  is the constant power,  
 $T$  is discharge the time,  
 $C$  is the total capacitance of a string of capacitors,  
 $V_{max}$  is the initial voltage  
 $I_{max}$  is the final current at time  $T$ .

Equation (3) describes the voltage discharge profile for a specified constant power,  $p$  and time,  $T$ . The variables in the equations are therefore the initial voltage  $V_{max}$  and the capacitance  $C$ . It is common to make the mistake of choosing a specific voltage and thus calculating the capacitance required, which often results in an over sizing of the ultracapacitors. The aim when sizing any string of capacitors is to minimize the mass, which implies using the least amount of capacitors. The voltage function,  $V(t)$ , and current waveform,  $I(t)$ , is shown in Figure 3.

It is important to note that the variables  $V_{max}$  and  $C$  are in fact related by the common variable  $n$ , which is the number of cells. The assumption is that the capacitors will never be charged above the combined maximum voltage rating of all the cells. We can therefore introduce this relationship with the following equations.

$$V_{max} = nV_{cell} \quad (6)$$

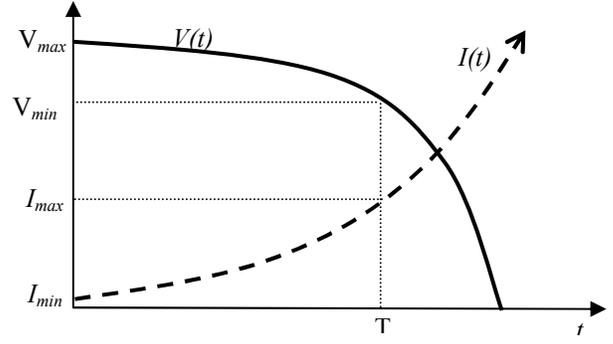


Figure 3. The voltage and current waveforms for a constant power discharge.

$$C = \frac{C_{cell}}{n} \quad (7)$$

where  
 $n$  is the number of cells,  
 $V_{cell}$  is the maximum cell voltage,  
 $C_{cell}$  is the individual cell capacitance.

Substituting these equations into (3) and (5) results in the following expression.

$$V_{cell}^2 n^2 - \frac{2pT}{C_{cell}} n - \frac{p^2}{I_{max}^2} = 0 \quad (8)$$

In equation (8), the only variable is  $n$  since the cell voltage,  $V_{cell}$ , and cell capacitance  $C_{cell}$ , is known. The maximum current,  $I_{max}$  is chosen based on the rating of the individual cells. The variable  $n$  therefore represents the amount of capacitors required to deliver the constant power over the time period.

Solving for  $n$  thus yields,

$$n = \frac{\frac{2pT}{C_{cell}} \pm \sqrt{\frac{4p^2 T^2}{C_{cell}^2} + 4V_{cell}^2 \frac{p^2}{I_{max}^2}}}{2V_{cell}^2} \quad (9)$$

The voltage discharge ratio is defined as the ratio of the final voltage over the initial voltage. It is common to choose a value of 50%, resulting in 75% of the energy being utilized. Equation (9) does not assume that the ratio is 50%, in fact it can be greater than or less than 50% depending on the maximum cell voltage and cell capacitance. This equation holds for applications where constant power is needed at a very low current as well as capacitors with extremely low equivalent series resistances, because the voltage drop across the equivalent series resistance is negligible. However in applications where high currents are drawn, the effect of the equivalent series resistance has to be taken into account. The energy dissipated in the equivalent series resistance as well as the cabling and connectors could result in an under sizing of the number of capacitors required. For this reason an

alternative approach to the capacitor sizing is necessary.

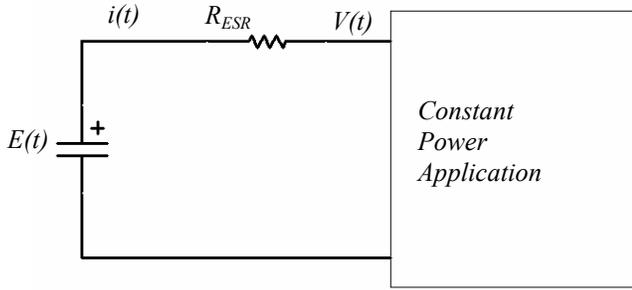


Figure 4. Equivalent circuit including the equivalent series resistance.

The system depicted in Figure 11 is governed by equations (10-15).

$$i(t) = -C \frac{dE(t)}{dt} \quad (10)$$

$$E(t) = -\frac{1}{C} \int i(t) dt \quad (11)$$

$$E(t) = i(t)R_{ESR} + V(t) \quad (12)$$

$$V(t)i(t) = p \quad (13)$$

$$E(t) = nE_{cell}(t) \quad (14)$$

$$C = \frac{C_{cell}}{n} \quad (15)$$

where,

$E(t)$  is the internal capacitor voltage,

$V(t)$  is the terminal voltage,

$I(t)$  is the current,

$p$  is the constant power,

$R_{ESR}$  is the equivalent series resistance.

The solution to the set of equations is

$$\frac{dE(t)}{dt} = \frac{-CE(t) \pm \sqrt{C^2 E(t)^2 - 4R_{ESR} C^2 p}}{2RC^2} \quad (16)$$

An iterative numerical method is needed in order to find the number of capacitors to deliver the power in the required time. The numerical method requires an initial value for the voltage  $E(t)$  and the iterative procedure needs a final target minimum voltage. The maximum current is chosen based on the rating of the capacitor. The terminal voltage,  $V(t)$ , will be a minimum when the current is a maximum. By adding the voltage drop across the equivalent series resistance, the final value of  $E(t)$  can be found. This final value is then used as the target for the numerical method employed. The magnitude of the initial voltage is iterated by the number of

cells, and the final value of each terminal voltage trajectory is compared to the target voltage. If the final value is lower than the target voltage then the number of cells is incremented until it is greater than or equal to the target voltage. In doing so, the minimum amount of cells can be found.

When the number of cells to meet the requirements has been found, it can be used to determine the initial current. The initial current can then be used in (17) to show the current waveform. The instantaneous power dissipation in the equivalent series resistance can then be determined.

$$\frac{di(t)}{dt} = -\frac{i(t)}{C \left( R - \frac{p}{i(t)^2} \right)} \quad (17)$$

The above numerical analysis can be applied to a hybrid electric vehicle where 25kW is needed for 18s. The specifications of the capacitor used is listed in Table 2. The maximum current for the capacitor is 400A. Based on this maximum current and a power of 25kW, the minimum terminal voltage,  $v(t)$ , can be calculated to be 62.5V.

In the iterative procedure, the number of cells in the string is incremented by a single cell and a trajectory of the terminal voltage is determined. It is assumed that the voltage across the individual cells will not exceed the rated cell voltage. If the final terminal voltage falls below the target of 62.5V then the number of cells is increased. This increases the initial voltage and also decreases the overall capacitance of the string. The number of cells is increased until the final voltage of the terminal trajectory is at least equal to the required target. Since the number of cells is now known it can be used to determine the current, instantaneous power and efficiency waveforms etc, as shown in Figures 5-7. In this example the number of cells has been found to be at least 74 and the mass of the string of the capacitors will be about 37.44kg.

Cell Capacitance (F)	Weight Per cell (kg)	ESR (mΩ)	Specific Energy (Wh/kg)	Specific Power (W/kg)	Cell Voltage (V)	Time Constant (τ=RC)
2700	0.52	0.6	2.49	8929	2.5	1.6

Table 2. Specifications for the ultracapacitor used in the example.

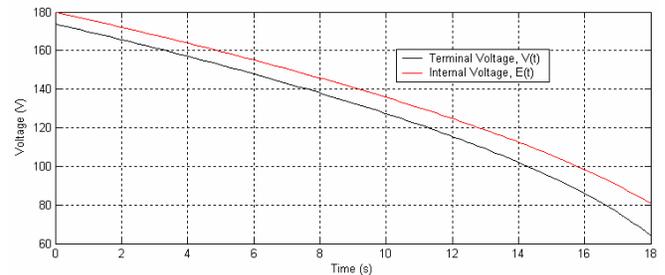


Figure 5. The terminal and internal voltage of the ultracapacitor string.

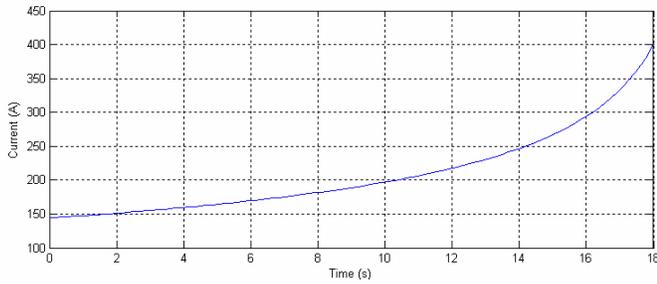


Figure 6. The capacitor current.

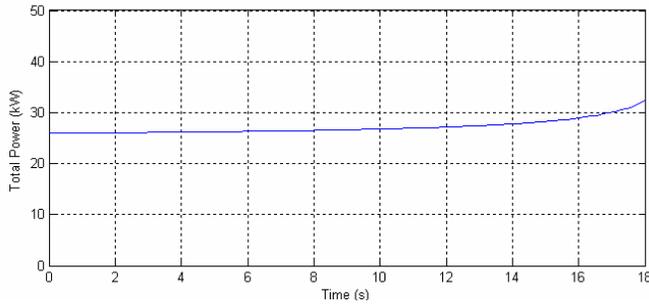


Figure 7. The total instantaneous power delivered by the ultracapacitors including the losses in the ESR.

#### IV. THE EFFECT OF ULTRACAPACITOR SIZING ON POWER ELECTRONICS.

In the previous section we illustrated how the number of capacitors, and consequently the mass, can be determined using an iterative numerical method. This reduction in capacitor mass has implications on the rating of the power electronics used to interface between the capacitors and the constant power application. The power electronic converter needs to be rated at the maximum voltage as well as the maximum current as indicated by (18).

$$P_{dc-dc} = V_{max} I_{max} \quad (18)$$

where,

$P_{dc-dc}$  is the peak power rating of the power electronics,

$V_{max}$  is the peak capacitor voltage,

$I_{max}$  is the peak capacitor current.

In the case of the previous example the peak power rating of the converter will be 72kW. It is well documented that the current power density of automotive electronics is 5kW/kg [4]. The mass of the power electronics can therefore be estimated as 14.4kg. The total mass of the system will therefore be approximately 51.84kg. This value does not include the mass of the cabling and the auxiliary cell voltage balancing circuitry.

In the next illustration we will examine how the choice of capacitor and maximum operating current affects the sizing of the overall system. The commercially available capacitors that will be analyzed are from Maxwell, Panasonic, Montena and Ness. The latest technology in ultracapacitors are from

Jeol Ltd, Japan. These capacitors are not yet available commercially, however prototype specifications have been made available. The specifications for the ultracapacitors are shown in Table 3.

The assumption made in the calculations is that there is no limit on the current conducting capability of the ultracapacitors. Although this is not practical, knowing that all capacitors have a practical limit, it gives an insight into the sizing if manufacturers are able to design them with higher current ratings. The current limit for large ultracapacitor cells is about 400A-600A.

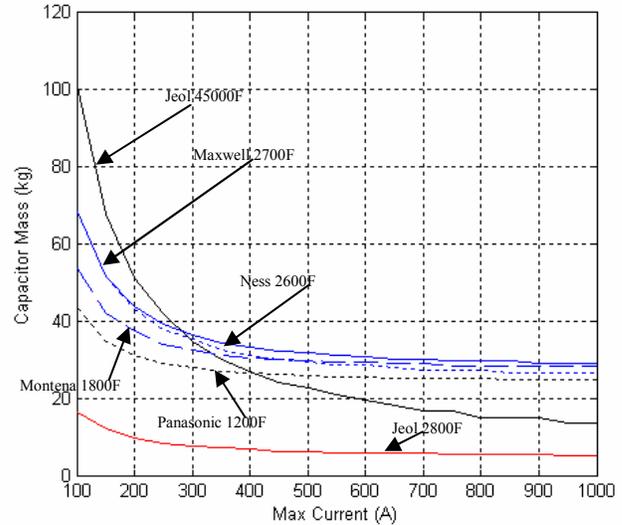


Figure 8. The mass of the capacitors versus the maximum allowable current.

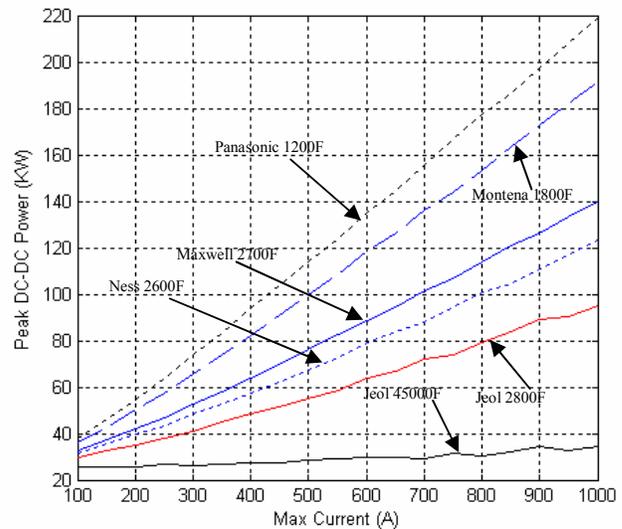


Figure 9. The rating of the power electronics versus the maximum allowable current.

	Weight Per cell (kg)	ESR (mΩ)	Specific Energy (Wh/kg)	Specific Power (W/kg)	Cell Voltage (V)	Time Constant ( $\tau=RC$ )
Ness 2600F	0.65	0.25	2.31	7284	3	0.65
Panasonic 1200F	0.34	1	1.3	6618	3	1.2
Maxwell 2700F	0.52	0.6	2.5	8929	2.5	1.6
Montana 1800F	0.4	0.6	2.49	7812	2.7	1.8
Jeol 2800F	0.21	2.1	27	8000	3.8	6
Jeol 45000F	1.5	4.4	60	540	3.8	200

Table 3. Specifications for the ultracapacitors.

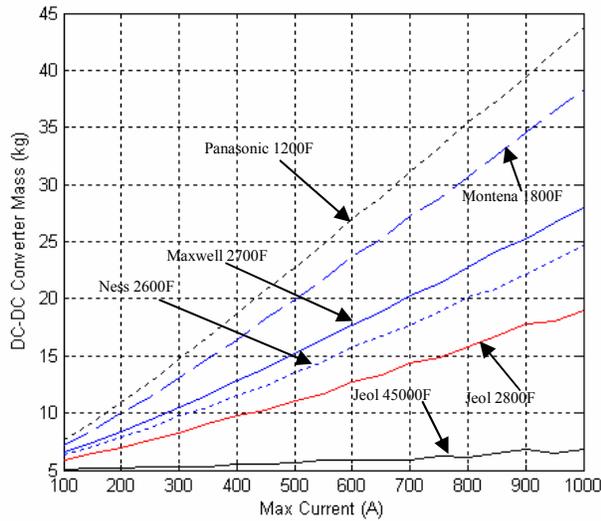


Figure 10. The mass of the power electronics versus the maximum allowable current.

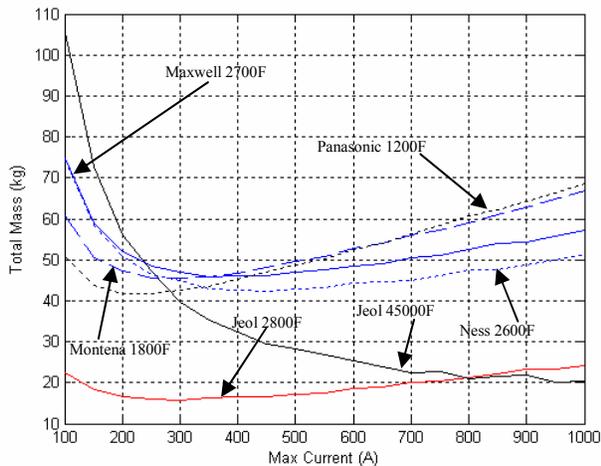


Figure 11. The mass of the system versus the maximum allowable current.

## V. CONCLUSIONS

It has been shown that the overall mass of the ultracapacitor and power electronics can be minimized by an appropriate choice of the maximum operating capacitor current. The minimum amount of capacitors and therefore the minimum mass of the ultracapacitor string can be achieved when specifying the maximum operating current to be the

rated current of the ultracapacitor. However, the maximum current affects the sizing of the power electronics and when the masses of both the ultracapacitor string and power electronics are combined, the minimum system mass can occur at currents less than the rated ultracapacitor current.

It is imperative to minimize the mass of any energy storage system in hybrid electric vehicles. The results have shown that there are reductions in mass when comparing the newer Jeol ultracapacitors to the older ultracapacitors.

The rating of the power electronic interface is influenced by the choice of operating current and voltage of the ultracapacitors. However, using different topologies for the power electronics could reduce the power rating.

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