Abstract—This paper describes the design essentials of an outside rotor Permanent Magnet Halbach array motor with a hybrid magnetic bearing, intended for kinetic energy storage. The theoretical estimations are compared to Finite element analysis as well as an approximated model using standard shape magnets. A dipole Halbach array produces a uniform flux distribution inside the cylindrical stator, where straight windings on an Ironless stator are placed near the inner boundary. The position of the windings inside this field does not affect the efficiency of the motor. When the motor is operated continuously, these variations become insignificant.

The objective of this exercise was to develop and evaluate the viability of a unit economical enough for rural area and stand alone applications in the developing world.

2. MOTOR DESIGN

The heart of the motor, with an outside rotor design, is a dipole Halbach array, establishing a uniform flux within the stator area, as shown in figure 1. A uniform flux is needed for operating the motor on a magnetic bearing, where slight movement of the stator inside the air gap is inevitable. Halbach [3] describes the flux density for a dipole with the equations:

\[ B = B_{\text{rem}} \cdot \log \left[ \frac{r_2}{r_1} \right] \cdot \kappa \]

\[ \kappa = \sin \left( \frac{2 \pi}{M} \right) \frac{2 \pi}{M} \]

Where:

- \( B \) = Resultant uniform flux
- \( B_{\text{rem}} \) = Remnant flux in permanent magnets
- \( r_2 \) = outer radius
- \( r_1 \) = inner radius
- \( M \) = number of poles

Inspection of (1) reveals that the wall thickness of the array has a direct impact on the resultant flux density. Plotting the flux density versus the wall thickness results in figure 2. This wall thickness is limited by commercial magnet dimensions, strength considerations due to centrifugal forces, cost of materials and dimensional constraints.

When the number of magnets approaches infinity in (1), it has a limit value of:

\[ \lim_{M \to \infty} \sin \left( \frac{2 \pi}{M} \right) = 1 \]

This result is not practical, unless one such magnet could be manufactured. To use a large number of magnets is costly, because each magnet has to be magnetised in a unique direction, the wedges become smaller and assembly becomes increasingly cumbersome.

The above designs are both 1 kWh units, suspended on magnetic bearings. The Steinmier [2] model used a hybrid magnetic bearing with an active bearing backup. Both incorporate filament wound multiple rim graphite composite de-
An eight magnet array gives an acceptable result, shown in figure 3, which was used for this model.

In order to reduce the cost of manufacture, the segments may be replaced by commercially available rectangular magnets, embedded in a steel matrix, as shown in figure 4.

Fig. 6: Finite Element results of an approximated 8-segment Halbach array

The three phase coils consists of copper windings wound onto an iron-less stator to eliminate iron losses. With the permanent magnetic field moving, eddy currents will be generated in magnetic and conductive circuits in the vicinity of the
The torque generated is directly proportional to the ampere turns and radius from the rotational centre in a Halbach array.

The armature windings are shown as single wires as opposing pairs, as shown in figure 7. When a current is flowing in a phase, it exerts a force on the winding and the magnetic field, which are equal and opposite. Having a conductor in a uniform flux, the air gap between the coils and the rotor inside the perimeter is of no significance for this motor’s efficiency. The torque generated by the motor is proportional to the ampere turns and radius from the rotational centre in a Halbach array and are described by [4]:

$$ T = B \cdot i \cdot r $$

(5)

$F$ = force, $B$ = magnetic flux, $L$ = length of conductor, $i$ = current, $r$ = radius and $T$ = torque.

A full bridge converter acts as a solid state commutation device in this brushless DC machine, shown in figure 8.

![Fig. 7: Motor winding position](image)

By switching the MOSFETS / IGBTS of a full bridge converter, figure 8, in the sequence shown in table 1 results in a square wave voltage fed to the 3-phase windings. The induced back EMF in the windings is sinusoidal. With the rotor being a fixed magnetic field and the windings on an iron-less core, the voltage equation for a DC generator, on the DC side of the converter, applies [4] i.e.:

$$ E = K \Phi \omega + I \cdot R $$

(6)

$E$ = induced motor EMF, $\Phi$ = flux density, fixed for a permanent Halbach array, $\omega$ = angular velocity in radians per second, $I$ = winding current, $R$ = resistance measured over the MOSFETS and the windings and $K$ = motor constant.

The induced voltage can be estimated by the induction law according to Fitzgerald [4]:

$$ V_i = N \frac{d\Phi}{dt} $$

(7)

$V_i$ = induced voltage, $N$ = number of turns and $\frac{d\Phi}{dt}$ = rate of change of magnetic flux.

For a conductor loop the induced voltage becomes:

$$ V_i = \omega \Phi_{\text{max}} \sin(\omega t) $$

(8)

$\omega$ = angular velocity in radians per second and $\Phi_{\text{max}}$ = magnetic flux given by the relationship [4]:

$$ \Phi_{\text{max}} = B A $$

(9)

$B$ = calculated flux density in the Halbach array and $A$ = area enclosed within a conductor loop.

For this motor running at 48000 rpm with a flux density of 0.39 Tesla and a mean area 0.022 X 0.050 m per winding, the induced Voltage reduces to: $V_i = 2.43 N \sin(\omega t)$ per winding, with $N$ = number of turns. The RMS value for a sinusoidal wave shape has a value of [5] $\frac{3}{\sqrt{2}} = 1.7$ volts per phase per winding at maximum speed.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>BRIDGE CONVERTER SWITCHING SEQUENCE FOR 3-PHASE OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle</td>
<td>0</td>
</tr>
<tr>
<td>Positive</td>
<td>$T_a$</td>
</tr>
<tr>
<td>Negative</td>
<td>$T_b$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>ESTIMATED VOLTAGES FOR DIFFERENT NUMBERS OF TURNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Windings</td>
<td>$V_{L-L}$ (Delta Connected)</td>
</tr>
<tr>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>34.3</td>
</tr>
<tr>
<td>25</td>
<td>42.9</td>
</tr>
</tbody>
</table>

Substituting $K\Phi$ in (6), we get:
\[ E = \frac{K_v}{900} \omega + I \cdot R \]  

(10)

\( K_v \) = maximum rated voltage of the motor and \( R = \) combined resistance of the semiconductor switches and the windings. The constant, "900", is the maximum speed of the machine in Hertz.

3. FLYWHEEL DESIGN

Benham et al. [6] defines the hoop and radial stresses in a rotating cylinder by:

\[ \sigma_\theta = \frac{3 + \nu}{4} \rho \omega^2 \left[ (r_o)^2 + \frac{1 - \nu}{3 + \nu} (r_i)^2 \right] \]

(11)

\[ \sigma_r = \frac{3 + \nu}{8} \rho \omega^2 (r_o - r_i)^2 \]

(12)

\( \sigma_\theta \) = maximum hoop stress, \( \sigma_r \) = maximum radial stress, \( \rho \) = density of the cylinder, \( r_o, r_i \) = cylinder outer and inner radii, \( \omega \) = angular speed and \( \nu \) = poisons ratio.

By maximising the specific kinetic energy, it was shown that an optimum is reached when points \( p_3, p_4 \) and \( p_5 \) coincide, resulting in a profile disk as depicted in figure 10.

Berger and Porat [7] have shown that it is possible to obtain much higher specific energies when using a combination of piecewise differential disk profiles, as shown in Fig 10.

By maximising the specific kinetic energy, it was shown that an optimum is reached when points \( p_3, p_4 \) and \( p_5 \) coincide, resulting in a profile disk as depicted in figure 10.

Philips [8] describes the strength of a short fibre composite. This depends on the critical fibre length, i.e. the length at which fibre and the matrix fails at the same strain. This length is calculated by:

\[ l_c = \frac{\sigma_f d}{\sigma_{xy}} \]

(14)

\( l_c \) = critical length, \( d \) = fibre diameter and \( \sigma_{xy} \) = shear strength of the matrix.

For a short fibre composite with fibres longer than the critical length, the material strength can be estimated by [8]:

\[ \sigma = f \sigma_f \left( l - \frac{l_c}{2l} \right) + (1 - f) \sigma_m \]

(15)

\( l \) = average fibre length.
The resulting flywheel parameters are listed in table 3.

<table>
<thead>
<tr>
<th>Outer diameter</th>
<th>210 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>160 mm</td>
</tr>
<tr>
<td>Safe operating speed</td>
<td>48000 rpm</td>
</tr>
<tr>
<td>Safety factor</td>
<td>1.5 (on the outer rim)</td>
</tr>
<tr>
<td>Total Weight</td>
<td>6.234 kg</td>
</tr>
<tr>
<td>Capacity</td>
<td>157.42 W-h</td>
</tr>
<tr>
<td>Inertia</td>
<td>$26.55 \times 10^{-3}$ kg m/s</td>
</tr>
<tr>
<td>Specific energy</td>
<td>25.25 W-h/kg</td>
</tr>
</tbody>
</table>

4. HYBRID MAGNETIC BEARING

Steinmier et al. [2] describes the operation of a stable hybrid magnetic bearing system. This system is constructed of two passive radial magnetic bearings (figure 12) and a journal thrust bearing, shown in figure 13. Steinmier [2] calculated the rotational frictional moment at the point of contact as:

$$T = 0.9 \pi r^2 F \mu$$

where $T$ = Torque, $r$ = Ball diameter, $F$ = vertical force and $\mu$ = frictional coefficient.

Yonnet [9] shows that opposing ring and disk shaped NdFeB permanent magnets yield a stable radial bearing, but unstable in the axial direction, shown in figure 12. Likewise, opposing disk magnets will be stable in the axial direction, but not in the other five degrees of freedom. Using two pairs of ring and disk magnets removes the remaining degrees of freedom except for two i.e. one in the direction of the shaft and the rotation thereof. The radial stiffness in this arrangement is approximately linear with distance.

Determining the stiffness can be done analytically with great difficulty. Generally the Finite element method is used and is verified experimentally. Steinmeier et al. [2] determined a linear radial stiffness of 600 N/m for a disk and annular magnet pair, shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Steinmier model [2]</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>ØINNER [mm]</td>
<td>44.5</td>
<td>30</td>
</tr>
<tr>
<td>ØOUTER [mm]</td>
<td>50.8</td>
<td>60</td>
</tr>
<tr>
<td>Ødisk [mm]</td>
<td>19.2</td>
<td>22</td>
</tr>
<tr>
<td>Height [mm]</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Stiffness [N/m]</td>
<td>600</td>
<td>3000</td>
</tr>
</tbody>
</table>

A 5 mm stainless steel ball on a stainless steel surface will give a frictional moment of approximately $1 \times 10^4$ Nm per kilogram of flywheel. For this flywheel it amounts to a rundown time of about 200 days, excluding any other losses, from a speed of 48000 rpm.

For a flywheel exerting 60 N on a single ball the frictional losses will be as indicated in Table 5 [10].
### Table 5

**Friction Values for Bearing Materials**

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient of Friction [10]</th>
<th>Torque [Nm]</th>
<th>Power @ 50000 rpm [mW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ruby / sapphire</td>
<td>0.1</td>
<td>$1.5 \times 10^{-4}$</td>
<td>152</td>
</tr>
<tr>
<td>Hard Steel</td>
<td>0.42</td>
<td>$6.0 \times 10^{-4}$</td>
<td>628</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.15</td>
<td>$2.2 \times 10^{-4}$</td>
<td>230</td>
</tr>
<tr>
<td>Teflon Steel</td>
<td>0.04</td>
<td>$6.0 \times 10^{-5}$</td>
<td>63</td>
</tr>
</tbody>
</table>

3. **Mechanical Losses of the Flywheel System**

To determine the losses in the hybrid magnetic bearing, a rundown curve was obtained while the containment was kept under vacuum. The rundown curve is shown in figure 15.

At the resonant frequencies, especially the first, a large deceleration is visible. The largest vibrations occurs at the first resonant frequencies.

![Rundown Curve in Vacuum](image)

**Fig. 15: Rundown Curve in a Vacuum**

At these amplitudes the moving parts makes contact with the stationary parts, resulting in large mechanical losses.

The almost linear sections between the natural frequency and multiples thereof i.e. ±900, 1800 etc. rpm, shows that the only frictional force is the contact on the axial support, confirming that the air friction has become negligible.

The motor was run at a constant current, which represents a torque, and allowed sufficient time to reach balance with the frictional torque of the system. This speed which was reached for a constant input power is depicted in figure 16.

To find the maximum speed of this curve, the mechanical friction in (16) is equated to the electrical resistive loss, giving:

$$0.9\pi r^2 F \cdot \mu \cdot \omega = I^2 R$$  \hspace{1cm} (17)

The theoretical maximum speed from the motor can now be rewritten as:

$$\omega = \frac{I^2 R}{0.9\pi r^2 F \cdot \mu}$$  \hspace{1cm} (18)

Applying this to the prototype flywheel results in a frictional coefficient of: $\mu = \frac{(8^2 \times (2 * 0.055 + 0.516)) / (0.9\pi \times (0.01)^2 \times 60) \times 261)}{0.16}$, using 2500 rpm as the maximum speed (see figure 16) and a current of 8 Amperes.

This value correlates with the coefficient of friction of Cast Iron on Cast Iron and lies in between those of hard steel on hard steel and hard steel on Teflon. The slight increase over that of the steel on Teflon lubricated steel surface, used in this prototype, is due to the steel surfaces not being entirely smooth, flat and aligned. This causes the ball to slide over the surface, in small circular movements, instead of rotating on one point.

5. **System Overview**

Shown in figure 17 is a schematic, illustrating a solar array application of a kinetic energy storage system.
tors the voltage, representing the speed of the motor.

The 3-phase, full bridge converter is synchronised with the motor by an optical encoder.

When the voltage of the solar array is higher than that of the DC bus, current flows into the flywheel system which accelerates the flywheel. When a consumer starts demanding power, which the solar panel may not be able to supply at the time, the DC bus voltage reduces slightly and the current starts flowing out of the electromechanical storage system, slowing down the flywheel.

6. CONCLUSION

The design procedure of a high efficiency motor / generator with a hybrid bearing system, built with off-the-shelf components, has been described. High efficiencies were obtained, using standard manufacturing techniques available in South Africa, for a kinetic energy storage application.

It is concluded that the manufacture of efficient, low cost kinetic energy storage devices is not only possible, but should be pursued in order to develop an affordable and competitive commercial unit for rural and isolated applications, as competitor to the electrochemical battery.

7. REFERENCES


