

# Application of Wavelet Analysis in Power System Disturbance Modeling

J. Liu, P. Pillay\*

Department of Electrical & Computer Engineering  
Clarkson University  
Potsdam, NY 13699-5720

\* Adjunct Professor, University of Cape Town, S.A.

**Abstract:** This paper shows that wavelets can be used to model a variety of power system transients using a fraction of the total number of coefficients. This modeling technique paves the way for fault classification and improved protection.

## 1. Introduction

Power quality disturbances include problems caused by voltage sags, voltage swells, capacitor switching transients, voltage notches, and supply discontinuities caused by autoreclose protection. All these faults are generally transitory, i.e., of a short-term duration.

Traditionally, the identification of the transients has been based on the visual inspection of the disturbance in the time domain. This is time-consuming and also has its limitation in practical applications, especially when a large amount of transient data is to be analyzed. Additional complications arise when several kinds of transients occur at the same time. A reliable way of classifying different faults is becoming a major concern in this field, particularly with deregulation and the use of power quality based rates.

It is a well-known fact in the signal processing area that the Fourier Transform is a powerful tool for the analysis of periodic information. The drawback is that its coefficients do not have inherent time information.

Wavelets have been shown to have advantages over the Fourier Transform where time information is needed. It translates the time-domain function into a representation localized not only in frequency but also in time. The use of Wavelet transform to analyze non-stationary harmonic distortions has been proposed [1]. This approach is a more powerful technique and can better categorize and analyze, in a more effective way, many types of non-stationary voltage distortions or power quality deviations[2]. Several works have addressed its application in the power area. A dyadic-orthonormal wavelet transform analysis is used to detect and localize various types of power quality disturbances, including harmonic distortion [3]. A key

idea is to decompose a given disturbance into other signals which represent a smoothed version and a detailed version of the original signal [4]. In [5], the wavelet transform is used to detect and quantify nonsinusoidal power transients, especially voltage. In [6], a new wavelet transform based procedure for power quality analysis is presented, which is based on the multiresolution signal decomposition and reconstruction by means of the Time Discrete Wavelet Transform. The optimized decomposition of signals in frequency subbands allows the most relevant disturbances in electrical power systems to be not only detected, localized, and classified but also estimated. In [7], the ability of wavelets to reconstruct transients was demonstrated. [8] shows how wavelets can be used to accurately reconstruct non-stationary power system disturbances and the contribution of individual coefficients to the reconstruction process. In addition, it should be possible for the wavelet coefficients to be used to categorize different types of disturbances.

With the combinations of wavelet and signal processing tools, wavelets has been extensively used in a variety of power fields[9-13].

Section 2 presents the basic wavelet theory. Section 3 shows that several typical power transients can be reconstructed by a limited number of wavelet coefficients. Section 4 has the conclusion.

## 2. Wavelet Transform

The ability of the Wavelet Transform to localize both time and frequency makes it possible to simultaneously determine sharp transitions of the signals and the localization of their occurrence. In the Wavelet Transform, a set of wavelet functions is chosen as the basis, similar to the sinusoidal function for the Fourier Transform. The set of basis functions is obtained by dilating and shifting a prototype, called a mother wavelet, on different levels. Wavelet coefficients are the projection of the original function onto the basis.

Fig.2.1 shows a typical example from the Daubechies

wavelet family, which is frequently used in power transients analysis because of its property of compact support.

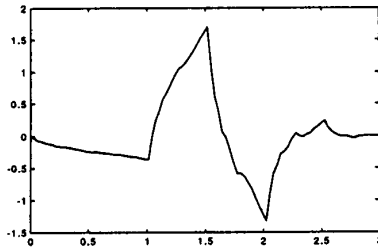


Figure 2.1. Daubechies 4 mother wavelet

The wavelet transform [14] can be thought of as the inner product of the original signal with the basis function  $\psi(t)$ :

$$W_{\psi} f(a, b) = \int f(t) \cdot \psi^* \left( \frac{t-b}{a} \right) dt = \langle f, \psi_{a,b} \rangle \quad (2.1)$$

Where

$$\psi_{a,b} = \frac{1}{\sqrt{a}} \psi \left( \frac{t-b}{a} \right) \quad (2.2)$$

In discrete cases, the selection of  $a=2^m$ ,  $b=k2^m$  ( $k, m \in \mathbb{Z}$ ) results in the dyadic wavelet transform.

The definition of the Wavelet Transform shows that the wavelet analysis is a measure of similarity between the basis functions (wavelets) and the original function. The calculated coefficients indicate how close the function is to the daughter wavelet at the particular scale. If the function has a major contribution at a particular scale, the coefficient computed at this point in the time-scale plane will be a relatively large number.

An invertible transform can be obtained by a simple dyadic translation and dilation scheme, which is based on functions  $\psi_{j,k}(t)=2^{j/2}\psi(2^j t-k)$ , where  $j, k \in \mathbb{Z}$ . That is, there exist functions  $\psi$  such that  $\{\psi_{j,k}, j, k \in \mathbb{Z}\}$  forms a complete orthonormal basis of  $L^2(\mathbb{R})$ , which is the space of the square integrable functions. For a function  $f \in L^2(\mathbb{R})$ , there is a wavelet expansion.

$$f = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \langle f, \psi_{j,k} \rangle \psi_{j,k} \quad (2.3)$$

Where  $\langle f, \psi_{j,k} \rangle = \int f(t) \psi_{j,k}^*(t) dt$ . That is at each scale  $J$ , there is a projection of  $f$  on some linear space. This space can be alternatively spanned by translates of an accordingly dilated and shifted version of a so-called scaling function  $\phi$ . Let  $\phi_{j,k}(t)=2^{j/2}\phi(2^j t-k)$ . Now the wavelet expansion of the function  $f$  is

$$f = \sum_k \langle f, \phi_{j,k} \rangle \phi_{j,k} + \sum_{j=J}^{\infty} \sum_k \langle f, \psi_{j,k} \rangle \psi_{j,k} \quad (2.4)$$

### 3. Reconstruction of Power Transients

Typical power transients include momentary interruptions, such as autoreclosure, voltage sags and swells, which are the results of power system faults, equipment failure and control malfunctions. Oscillatory transients occur when there is a sudden, non-power frequency change in the steady state condition of voltage, current, or both, that include both positive and negative polarity values. A typical example of the oscillatory transient is caused by capacitor switching.

Figure 3.1 shows an autoreclosure signal for a period of 0.2 seconds with 2 cycles of power outage. After the Wavelet Transform is carried out, the coefficients are used to reconstruct the original signal. In this case, only 2% of the total number of coefficients are used and a good approximation is obtained.

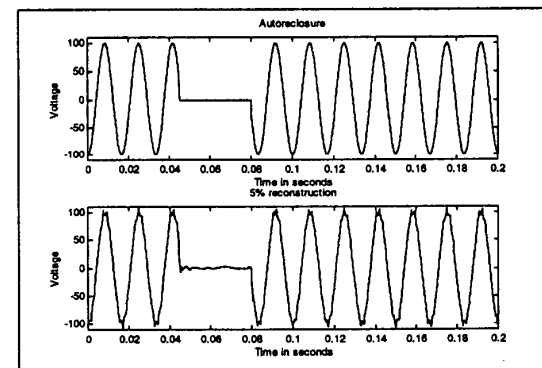


Figure 3.1 Autoreclosure and its 5% reconstruction with DB4 mother wavelet.

One measure of the accuracy of the reconstruction, commonly used in the signal processing is

$$error = \frac{\text{norm}(x - z)}{\text{norm}(x)} * 100\% \quad (3.1)$$

that is, the norm of the difference between the original signal  $x$  and reconstructed signal  $z$ , divided by the norm

of the original signal  $x$ . The effective reconstruction requires the error as small as possible.

In the autoreclosure case, the relative error of reconstruction is 4.47%. The reconstructed signal retains approximately 95% of the energy in the original signal with 5% of the coefficients.

The result above is obtained via Wavelet Transform using the DB4 mother wavelet. If a higher order of the Daubechies mother wavelet is used, the reconstruction will be more effective. Figure 3.2 gives the reconstruction of an autoreclosure using the DB10 mother wavelet. The relative error according to equation (3.1) is 1.21%. That means that only 5% of the coefficients retain 98.8% of the energy in the original signal. The effectiveness comes from the increase of the order of Daubechies mother wavelet. The selection of the optimal mother wavelet is a major concern in the practical implementations.

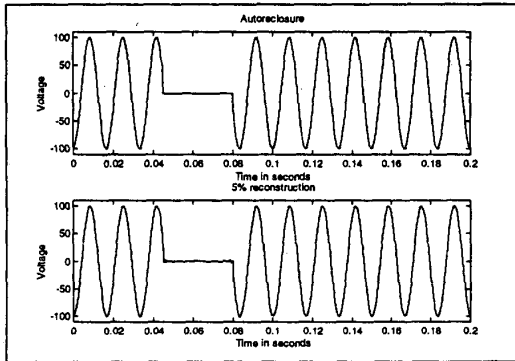


Figure 3.2 Autoreclosure and its 5% reconstruction with DB10 mother wavelet.

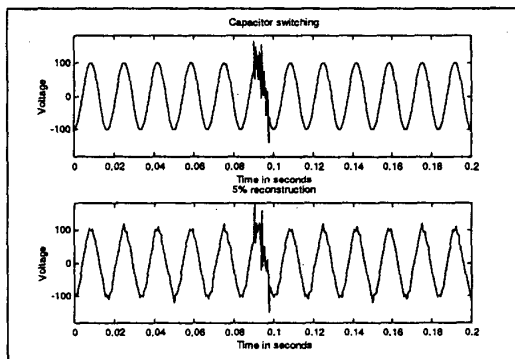


Figure 3.3 Capacitor switching and 5% reconstruction with DB4 mother wavelet

Figures 3.3~3.5 show the reconstruction of a capacitor switching, voltage sag and voltage swell with 5% of the wavelet coefficients. The related error with different Daubechies mother wavelet is given in Table 1. It is clear that the use of high order mother wavelet reduces the reconstruction error.

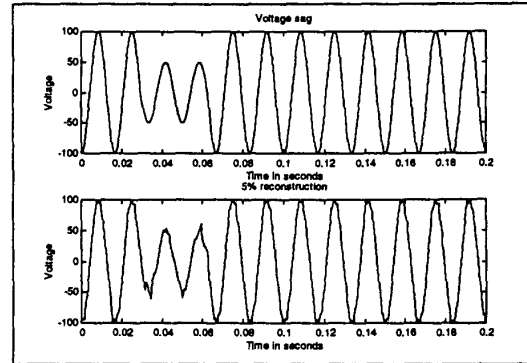


Figure 3.4 Voltage sag and its 5% reconstruction with DB4 mother wavelet

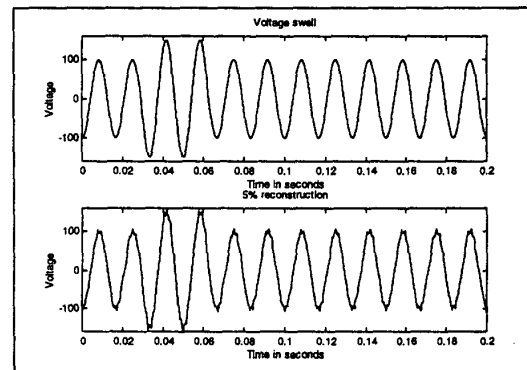


Figure 3.5 Voltage swell and its 5% reconstruction with DB4 mother wavelet

Table 1 Reconstruction error of four types of power transients with different Daubechies mother wavelet and 5% of the total coefficients (%)

	autoreclosure	switching	sag	swell
DB4	4.47	9.24	5.09	5.50
DB6	2.15	5.33	2.05	2.06
DB8	1.28	2.88	0.98	0.86
DB10	1.21	1.98	0.61	0.64

An other example of a power disturbance is the waveform distortion, which is a steady state deviation from an ideal sine wave of power frequency principally characterized by the spectral content of the deviation. A

example given in this paper is voltage notching, which is a periodic voltage disturbance caused by the normal operation of power electronics devices when current is commutated from one phase to another.

In figure 3.6, a voltage notch is given. Since notching occurs continuously, it can be characterized by its harmonic spectrum. When wavelet reconstruction is used, more coefficients are needed to achieve the same accuracy than for the case of short term power transients. For a 20% reconstruction, the error is 4.2% with the DB4 mother wavelet. For DB10, the error is reduced to 3.1%.

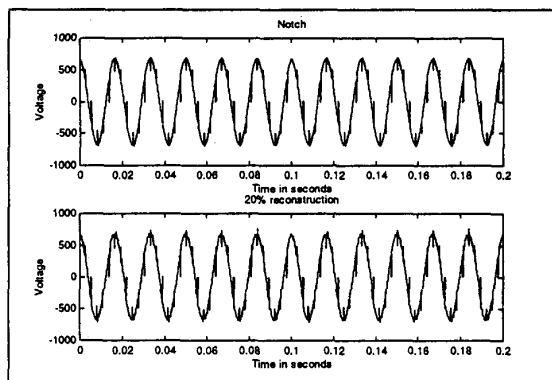


Figure 3.6 Notching and its 20% reconstruction with DB4 mother wavelet

#### 4. Conclusion

This paper shows that wavelets can be used to reconstruct non-stationary power transients as well as periodic disturbances. With the appropriate selection of the mother wavelet, the reconstruction accuracy can be improved. The modeling can form the basis for disturbance classification and protection.

#### 5. Reference

[1] P.Riberio, R.Ceilo and M.J. Smoty "Future Analysis Tools for Power Quality", Proc. PQA 93/PECON IV Conf., USA, Nov16-19, 1993, pp.7-3.1 to 7-3.10.  
 [2] P.F. Ribeiro, Wavelet transform: an advanced tool for analyzing non-stationary harmonic distortions in power systems, Proceedings of the IEEE International Conference on Harmonics in Power systems, Bologna, Italy, September 1994

[3] Robertson, D.C., Wavelet and electromagnetic power system transients, IEEE/PES 1995 Summer Meeting, 95 SM391-3 PWRD.  
 [4] Santoso, S., Power quality assessment via wavelet transform analysis, IEEE/PES 1995 Summer Meeting, 95 SM 371-5 PWRD  
 [5] Tunaboylu, N.S., Collins, E.R., The wavelet Transform Approach to Detect and Quantify Voltage Sags, Proc. ICHQP, October 1996, pp619-624  
 [6] Angrisani, L., Daponte, P., A New Wavelet Transform Based Procedure For Electrical Power Quality Analysis, Proc. ICHQP, October 1996, pp608-614  
 [7] Pillay, P., Bhattacharjee, A., Application of Wavelets to Model Short-time Power System Disturbances, 96 WM 284-0 PWRD, 1996  
 [8] P. Pillay, P. Ribeiro, Q. Pan, Power Quality Modeling Using Wavelets, Proc. ICHQP, October, 1996  
 [9] S. Santoso et al., "Power Quality Disturbance Identification using Wavelet Transforms and Artificial Neural Networks", Proc. ICHQP, October 1996, pp615-618.  
 [10] S. Santoso et al, "Power Quality Disturbance Waveform Recognition Using Wavelet-based Neural Classifier", PE-599-PWRD-0-01-1997, IEEE PES Winter Meeting 1997  
 [11] L. Angrisani, P. Daponte, "A Measurement Method Based on the Wavelet Transform for Power Quality Analysis", PE-056-PWRD-0-1-1998, IEEE PES Winter Meeting 1998.  
 [12] F. H. Magnago, A. Abur, "Fault Location Using Wavelets" PE-303-PWRD-0-12-1997, IEEE PES Winter Meeting 1998.  
 [13] S. Pandey, L. Satish, "Multiresolution Signal Decomposition: A New Tool For Detection In Power Transformers During Impulse Tests", PE-128-PWRD-0-11-1997  
 [14] Daubechies, I., Ten Lectures on Wavelets, Kluwer Academic Publishers, 1993