Multilevel Selective Harmonic Elimination PWM Technique in Series-Connected Voltage Inverters

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Abstract – Selected harmonic elimination PWM (SHEPWM) method is systematically applied for the first time to multilevel series-connected voltage source PWM inverters. The method is implemented based on optimization techniques. The optimization starting point is obtained using a phase-shift harmonic suppression approach. Another less computationally demanding harmonic suppression technique, called a mirror surplus harmonic method, is proposed for five-level (double-cell) inverters. Theoretical results of both methods are verified by experiments and simulations for a double-cell inverter. Simulation results for a five-cell (11-level) inverter are also presented for the multilevel SHEPWM method.

I. INTRODUCTION

Medium/large motor drives, large UPS systems, and high power inverters in FACTS (flexible alternate current transmission systems) need switching elements which can bear high voltage/current. To overcome limits of semiconductor switches, several new techniques and topologies have been developed [1]-[4], such as multiple switching elements in one leg of an inverter [5], [6], series-connected inverters [7]-[15], parallel-connected inverters [16], [17], multilevel reactive power compensators [18]-[21], multiple rectifiers for unity power factor correction [22], optimization of motor performance indices (such as harmonic current, torque ripple, common mode voltage, and bearing currents) [23], [24] and neutral point clamped (NPC) inverters [25]-[27]. This paper focuses on series-connected voltage-source PWM inverters. In this area, present control techniques are based on the following methods: 1) sinusoidal PWM (SPWM) [8], [9]; 2) space vector PWM (SVPWM) [13]; 3) nonsinusoidal carrier PWM [7]; 4) mixed PWM [12], [14], [5] special structure of cell connections [11]; and 6) selected harmonic elimination PWM (SHEPWM) [1], [15]. The SHEPWM based methods can theoretically provide the highest quality output among all the PWM methods. The drawback of these methods is a heavy computational burden and a complicated hardware [31]. The SHEPWM method presented in [1] offers the same number of control variables as the number of inverter levels. Results given in [15] are only for a five-level inverter up to seven switching angles without taking into account that inverter cells should equally share the output power.

In this paper, a SHEPWM model of a multilevel series-connected voltage-source inverter is developed that can be used for arbitrary number of levels and switching angles. Simulation and experimental results for a 5-level (or double-cell) 22-angle single-phase inverter and a 5-level 20-angle three-phase inverter are presented. Simulation results for a 11-level (five-cell) 45-angle three-phase inverter are also given. A reduced-order SHEPWM method by mirror surplus harmonic shaping for 5-level inverters is proposed and experimentally verified. The paper is organized in the following manner: the mirror surplus harmonic method is presented in Section II followed by a description and results of the general multilevel SHEPWM in Section III. Conclusions are given in Section IV.

II. HARMONIC SUPPRESSION WITH MIRROR SURPLUS HARMONICS TECHNIQUE

In this section, a new concept of mirror surplus harmonics is introduced for selected harmonics elimination in double-cell series-connected PWM inverters. This concept allows for reducing the amount of computations in comparison with the multilevel SHEPWM. It will be shown that the obtained output harmonic spectrum is close to that of multilevel SHEPWM.

A. Double-Cell Series-Connected Inverter Harmonic Model

In 1973, the selected harmonic elimination method for PWM inverters was introduced [28] for single-cell (two- and three-level) inverters. This method is sometimes called a programmed PWM technique. Fig. 1 illustrates the general quarter-wave symmetric triple-level programmed PWM switching pattern. The square wave is chopped m times per half cycle. Owing to the symmetries in the PWM waveform, only odd harmonics exist. The Fourier coefficients of odd harmonics in triple-level programmed PWM inverters with odd switching angles are given by:

$$b_n = \frac{4E}{\pi R} \left[ \cos \alpha_1 - \cos \alpha_2 + \ldots \right.$$  

$$+ (-1)^{n-1} \cos \alpha_j + \ldots + \cos \alpha_m \right]$$  

(1)

where $n$ is the harmonic order.

Any $m$ harmonics can be eliminated by solving the $m$ equations obtained from setting (1) equal to zero [29]. Usu-
ally, the Newton iteration method is used to solve such systems of nonlinear equations [30]. The correct solution must satisfy the condition
\[ 0 < \alpha_1 < \alpha_1 < \ldots < \alpha_m < \frac{\pi}{2}. \] (2)

Let us consider a double-cell series-connected PWM inverter shown in Fig. 2. Each cell of the inverter switches \( m \) times per quarter-cycle and produces a three-level \{-1,0,1\} PWM waveform. This results in a five-level \{-2,-1,0,1,2\} inverter output. Theoretically, \( 2m - 2 \) odd harmonics can be eliminated from the inverter’s spectrum while keeping the fundamental components of both cells equal to each other. Even harmonics are not present due to the PWM waveform symmetry. The switching angles must be obtained from the following system of \( 2m \) nonlinear transcendental equations
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos \alpha_i = \frac{\pi}{4} M
\]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos \beta_i = \frac{\pi}{4} M
\]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos 3\alpha_i + \sum_{i=1}^{m} (-1)^{i+1} \cos 3\beta_i = 0
\]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos (4m - 3)\alpha_i + \sum_{i=1}^{m} (-1)^{i+1} \cos (4m - 3)\beta_i = 0
\] (3)
where \( \alpha_i \)'s are switching angles of the first cell, \( \beta_i \)'s are switching angles of the other cell, and \( M \) is the modulation index. The proposed in this paper model (3) explicitly requires an even fundamental power sharing among cells. Convergence of numerical procedures used to solve (3) depends greatly on starting values of switching angles and requires a lot of computational power.

B. Reduced Order Model of a Double-Cell Series-Connected Inverter

Elimination of low order harmonics from only one cell, which will be called a general SHEPWM method, can be obtained by solving a system of \( m \) equations [28]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos \alpha_i = \frac{\pi}{4} M
\]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos \beta_i = \frac{\pi}{4} M
\]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos 3\alpha_i = 0
\]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos 3\beta_i = 0
\]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos (4m - 3)\alpha_i = 0
\]
\[
\sum_{i=1}^{m} (-1)^{i+1} \cos (4m - 3)\beta_i = 0
\] (4)

The first significant surplus harmonic from this cell has an amplitude \( A_{2m+1} \). If we want to eliminate \( A_{2m+1} \) from the output spectrum of the inverter, the other cell must produce the \( 2m+1 \) harmonic of an amplitude \(-A_{2m+1}\). To preserve elimination of \( 2m-1 \) low-order odd harmonics and to set the amplitude of the \( 2m+1 \) harmonic to \(-A_{2m+1}\), the number of switching angles in the second cell must be increased by one to \( m+1 \). The switching angles of the second cell fulfill the following system of \( m+1 \) equations
\[
\sum_{i=1}^{m+1} (-1)^{i+1} \cos \alpha_i = \frac{\pi}{4} M
\]
\[
\sum_{i=1}^{m+1} (-1)^{i+1} \cos \beta_i = \frac{\pi}{4} M
\]
\[
\sum_{i=1}^{m+1} (-1)^{i+1} \cos 3\alpha_i = 0
\]
\[
\sum_{i=1}^{m+1} (-1)^{i+1} \cos 3\beta_i = 0
\]
\[
\sum_{i=1}^{m+1} (-1)^{i+1} \cos (4m - 3)\alpha_i = 0
\]
\[
\sum_{i=1}^{m+1} (-1)^{i+1} \cos (4m - 3)\beta_i = 0
\]
\[
\sum_{i=1}^{m+1} (-1)^{i+1} \cos (2m+1)\beta_i = \frac{(2m+1)\pi}{4} A_{2m+1}. \] (5)

An unexpected benefit of such a \( 2m+1 \) harmonic cancellation is that the whole first cluster of significant harmonics from the second cell becomes nearly a mirror image of the first cluster of significant harmonics from the first cell. Thus, the solution of (4) and (5) approximates very closely the solution of (3). Such a cancellation feature will be called an optimal PWM technique. This feature has been checked for several values of \( m \) and for wide range of the modulation index \( M \). An example for \( m = 10 \) and \( M = 1.0 \) is given below. Since systems of equations of the...
(4) type do not have analytical solutions, it is difficult to find a theoretical explanation for the proposed method. It will be a subject of a future research. Nonetheless, the proposed approach is a practical way of finding an approximate solution to (3) and, hence, harmonic suppression in double-cell series-connected inverters. For three and higher number of cells, the direct SHEPWM described in Section III should be used.

C. Simulation Example

Frequency spectra of waveforms obtained with the mirror surplus harmonic method for single-phase and three-phase systems are shown in Figs. 3(a) and 4(a), respectively. The difference between single- and three-phase calculations is that for a three-phase system triplen harmonics need not to be included in the set of selected eliminated harmonics. It can be clearly seen that first clusters of significant harmonics of two cells in Figs. 3(a) and 4(a) (second and third spectrum from the top) almost cancel each other in the output spectrum. In fact, the output spectra in Figs. 3(a) and 4(a) are very close to what can be expected if the solution of (3) is used for generation of switching patterns.

D. Experimental Results

The calculated switching patterns have been implemented using TMS320C30 DSP unit driving IPM PM200S3000 power modules. The resulting inverter was loaded by a free-running induction motor. Fig. 7(a) and Fig. 8(a) show measured voltage harmonic amplitudes of both cells and the inverter output for modulation index $M = 1.0$ in this scaled-down experimental system. Time-domain waveforms of inverter outputs are also presented in Fig. 7(a) and Fig. 8(a). The experimental results are in a full agreement with theoretical predictions.

III. MULTILEVEL SELECTIVE HARMONIC ELIMINATION PWM TECHNIQUE

The multilevel SHEPWM technique has a theoretical potential to achieve the highest output power quality at low switching frequencies in comparison to other methods. However, because of its mathematical complexity, no significant results have been reported so far. One of the main challenges is to obtain a good starting point when solution of a nonlinear system of equations of the (3) type is attempted. This paper presents a concept of obtaining the starting point by means of SHEPWM with a phase shift. Then, a nonconstraint optimization [32] is used to calculate the final solution of the multilevel SHEPWM problem.

A. Specific Harmonic Elimination with Phase Shift

Phase shift is an effective and simple method to decrease harmonic content in multilevel converters. The most common application of phase shifting in PWM inverters is in carrier based modulation schemes [8], [9]. In this paper, phase shift together with three-level SHEPWM is used to obtain a starting point for multilevel SHEPWM calculations.

Let us consider a three-cell PWM inverter as an example. Assume that the cells have identical voltage spectra with low order harmonics eliminated by the classical SHEPWM. To preserve high amplitude of the fundamental, the phase-shift angle $\beta$ among the three cells should be small. Using (1), the harmonic content of the reference cell $b_1\text{m}$, the leading cell $b_2\text{m}$, and the lagging cell $b_3\text{m}$ can be described as

$$b_1\text{m} = \frac{4E}{n\pi}[\cos n\alpha_1 - \cos n\alpha_2 + \ldots + (-1)^{j-1}\cos n\alpha_j + \ldots + \cos n\alpha_m]$$

$$b_2\text{m} = \frac{4E}{n\pi}[\cos (n\alpha_1 - \beta) - \cos (n\alpha_2 - \beta) + \ldots + (-1)^{j-1}\cos n(\alpha_j - \beta) + \ldots + \cos n(\alpha_m - \beta)]$$

$$b_3\text{m} = \frac{4E}{n\pi}[\cos (n\alpha_1 + \beta) - \cos (n\alpha_2 + \beta) + \ldots + (-1)^{j-1}\cos n(\alpha_j + \beta) + \ldots + \cos n(\alpha_m + \beta)].$$

(6)

Adding the cell voltages in (6), the multilevel inverter output harmonics are given by

$$V_n = (1 + 2\cos n\beta)b_1\text{m}$$

(7)

where $V_n$ is the $n$-th harmonic of the inverter output voltage.

The phase shift angle $\beta$ may be selected by the following heuristic approach. If the number of switching angles in a quarter-period is $m$, the first significant harmonic crest for each cell consists typically of $2m + 1$, $2m + 3$, $2m + 5$, and $2m + 7$ harmonics. One of these harmonics can be eliminated by the phase shift, others will be suppressed. By selecting the $2m + 3$ harmonic for elimination, the phase shift angle $\beta$ can be obtained as:

$$\beta = \frac{2\pi}{3(2m + 3)}.$$  

(8)

Fig. 5 shows the phase diagram of the fundamental and $2m + 3$ harmonic. Fig. 6 presents an example of the surplus harmonic suppression with phase shift in a three-cell inverter with $m = 9$. The resulting $\beta$ equals 5.35 degrees.
Fig. 3. Calculated frequency spectra for single-phase harmonic elimination (a) Mirror surplus harmonic method: (from top to bottom) general 11 switching angles, optimal 11 switching angles, general 10 switching angles, and inverter output versus harmonic order. \( M = 1.0 \). (b) Multilevel selective harmonic elimination method: (from top to bottom) general 11 switching angles, two optimal 11 switching angles, and inverter output versus harmonic order. \( M = 1.0 \).

Fig. 4. Calculated frequency spectra for three-phase harmonic elimination (triplen harmonics not shown) (a) Mirror surplus harmonic method: (from top to bottom) general 11 switching angles, optimal 11 switching angles, general 9 switching angles, and inverter output versus harmonic order. \( M = 1.0 \). (b) Multilevel selective harmonic elimination method: (from top to bottom) general 10 switching angles, two optimal 10 switching angles, and one phase output versus harmonic order. \( M = 1.0 \).
B. Starting Point of Multilevel SHEPWM

The phase shift technique for surplus harmonic suppression presented in the previous subsection can be easily generalized for an $K$-cell ($K = 2, 3, \ldots$) system. The obtained switching angles may be used as starting points to calculate a solution for a nonlinear system of equations of the (3) type. Next subsections present an application of this idea to double-cell and five-cell inverters.

C. Multilevel Selective Harmonic Elimination

Consider a two-cell series-connected PWM inverter. With $m$ switching angles per quarter wave for each cell, there are $2m$ variables and, consequently, $2m - 1$ harmonics can be eliminated. The amplitude of the $n$-th voltage harmonic at the inverter output is given by

$$V_n = \frac{4E}{n \pi} \left[ \cos n \alpha_1 - \cos n \alpha_2 + \ldots + (-1)^{j-1} \cos n \alpha_j + \ldots + \cos n \alpha_m \right] + \frac{4E}{n \pi} \left[ \cos n \beta_1 - \cos n \beta_2 + \ldots + (-1)^{j-1} \cos n \beta_j + \ldots + \cos n \beta_m \right].$$

Setting the fundamental to a desired value dictated by the modulation index $M$ and equating selected harmonics to zero, results in a system of nonlinear equations which is very difficult to solve numerically. To alleviate computational problems, a nonconstrained optimization approach [32] has been proposed. The target function of this new optimization scheme can be written as

$$F = K_1 (V_1 - M)^2 + K_2 V_2^2 + \ldots + K_{2m-1} V_{2m-1}^2$$

where $K_1, K_2, \ldots, K_{2m-1}$ are penalty factors. The optimization starting point is obtained by the phase shift method described above.

D. Simulation and Experimental Results

Simulation results of multilevel SHEPWM in a double-cell inverter are given in Figs. 3(b) and 4(b) for single-phase and three-phase systems, respectively. It can be observed in the second and third spectra from top that low order harmonics are present. However, these harmonics are out of phase and of equal amplitudes. Consequently, they do not appear at the converter output. Experimental results for the double-cell single-phase and three-phase systems are presented in Fig. 7(b) and Fig. 8(b). The experimental setup was the same as that used for the mirror surplus harmonics method (see Section II).

The multilevel SHEPWM was also verified for a five-cell series-connected PWM inverter with nine switching angles per quarter wave per cell. The simulation results at a modulation index $M = 1.0$ are presented in Figs. 9 and 10. It can be seen in Fig. 9 that the low order harmonics are suppressed by more than 45 dB up to 137th harmonic. Fig. 10 shows that the cells share equally the fundamental component of the output power.

IV. Conclusions

A multilevel selected harmonic elimination PWM method is proposed. The computational difficulties of multilevel SHEPWM methods are overcome by development of an inverter model for nonconstraint optimization. The optimization starting point is obtained using a phase-shift surplus harmonic suppression technique. Simulation and experimental results are presented for a double-cell series-connected voltage source PWM inverter in single-phase and three-phase configuration. Simulation results for a three-phase five-cell inverter are also given. The multilevel SHEPWM method is capable of providing a very high quality output waveforms. It requires, however, a large amount of precalculations and big memory for storage of results.

A new reduced-order method of mirror harmonic suppression in a double-cell series-connected PWM inverter is also suggested. Instead of using a difficult-to-solve system of $2m$ nonlinear equations, the two inverter cells are considered separately. $m - 1$ low-order harmonics in the first cell are eliminated with a standard SHEPWM harmonic elimination scheme. An additional switching angle is allowed in the second cell to shape its frequency spectrum in such a way that it mirrors the spectrum of the first cell. The results obtained from solution of two systems of equations, $m$ and $m + 1$ order, closely approximate the solution of a system of $2m$ equation. Hence, the difficulty and amount of calculations are greatly reduced. Experimental tests, conducted for an inverter with $m = 10$ switching angles per quarter-wave in the first cell, show harmonic suppression that is comparable with that for a multilevel SHEPWM.

References

Fig. 7. Experimental single-phase spectra and waveforms. (a) Mirror surplus harmonic method: [from top to bottom] voltage spectra for optimal 11 switching angles, general 10 switching angles, and inverter output versus harmonic order as well as inverter voltage [V] and current [mA]. M = 1.0. (b) Multilevel selective harmonic elimination PWM method: [from top to bottom] voltage spectra for two optimal 11 switching angles and inverter output versus harmonic order as well as inverter voltage [V] and current [mA]. M = 1.0.


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Fig. 8. Experimental three-phase spectra and waveforms. (a) Mirror surplus harmonic method: (from top to bottom) voltage spectra for optimal 11 switching angles, general 9 switching angles, and inverter output versus harmonic order as well as inverter line voltage (V) and load current (mA). $M = 1.0$. (b) Multilevel selective harmonic elimination PWM method: (from top to bottom) voltage spectra for two optimal 10 switching angles and inverter output versus harmonic order as well as inverter line voltage (V) and load current (mA). $M = 1.0$.

Fig. 9. Simulation results of five cell series-connected inverter
Fig. 10. Voltage spectra and waveforms for each cell in five-cell series-connected inverter.

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