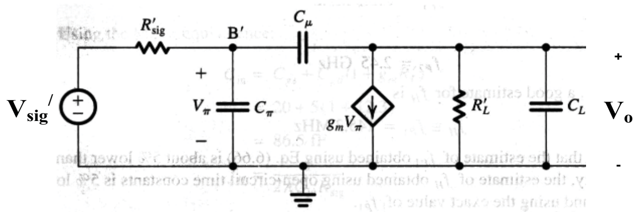
ELEC 312: ELECTRONICS – II : ASSIGNMENT-set 2

Department of Electrical and Computer Engineering

Winter 2013

1. A common-emitter amplifier that can be represented by the following equivalent circuit, has $C_\pi = 10$ pF, $C_\mu = 0.5$ pF, $C_L = 2$ pF, $g_m = 20$ mA/V, $\beta = 100$, $r_x = 200$ Ω, $R_L^{/} = 5$ kΩ and $R_{sig} = 1$ kΩ. Find (i) the mid band gain A_M, (ii) the frequency of the zero f_Z, and (iii) the approximate values of the pole frequencies f_{P1} and f_{P2}. Hence estimate the 3-dB frequency f_H. Note that R'_{sig} is the equivalent Thevenin resistance looking towards the signal source and includes the effects of R_{sig}, r_x and r_π. For approximate estimates, you may use OCTC method.

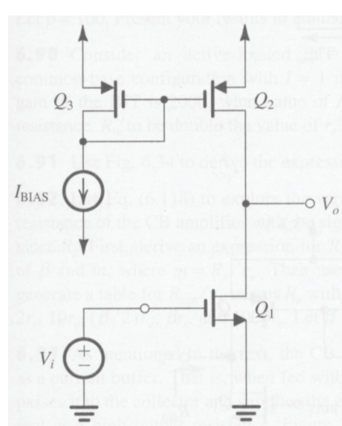


Hints:

(i) $A_M = -r_\pi(g_m R'_L)/(R_{sig} + r_x + r_\pi)$; (ii) $f_Z = g_m/(2\pi C_\mu)$ (iii) $f_{P1} = 1/[2\pi\{(C_\pi + C_\mu(1+ g_m R'_L)) R'_{sig} + (C_L + C_\mu)R'_L\}]$; $f_{P2} = [(C_\pi + C_\mu(1+ g_m R'_L))R'_{sig} + (C_L + C_\mu)R'_L]/[2\pi\{C_\pi(C_L + C_\mu) + C_L C_\mu)\} R'_{sig} R'_L]$; $f_{P1} << f_{P2}$ & $f_{P1} << f_Z$, hence $f_H \approx f_{P1}$

2. Analyze the high-frequency response of the CMOS amplifier shown below. The dc bias current is 100 μA. For Q_1, $\mu_n C_{ox} = 90$ μA/V^2, $V_A = 12.8$ V, W/L = 100 μm/1.6 μm, $C_{gs} = 0.2$ pF, $C_{gd} = 0.015$ pF. For Q_2, $C_{gd} = 0.015$ pF, $C_{gs} = 36$ fF and $|V_A| = 19.2$ V. Assume that the resistance of the input signal generator is negligibly small. Also, for simplicity assume that the signal voltage at the gate of Q_2 is zero. Find the low-frequency (i.e., at DC) gain, the frequency of the pole, and the frequency of the zero. You may use nodal analysis.

Note: fF=10^{-15} F, pF=10^{-12} F.



Hints:
DC gain = $-g_m(r_{01}// r_{02})$, where $g_m = \sqrt{[2\mu_n C_{ox} I_D W/L]}$, $r_0 = V_A/I_D$ and
Small-signal gain, $v_o/v_i = (g_m - sC_{gd1})/[1/r_{01} + 1/r_{02} + s(C_L + C_{gd1})]$ where $C_L = C_{gd2}$
$f_Z = g_m/(2\pi C_{gd1})$; $f_p = (1/2\pi)[(1/r_{01} + 1/r_{02})/(C_L + C_{gd1})]$

3. A CG amplifier is specified to have $C_{gs} = 2$ pF, $C_{gd} = 0.1$ pF, $C_L = 2$ pF, $g_m = 5$ mA/V, $\chi = 0.2$, $R_{sig} = 1$ kΩ and $R_L^{/} = 20$ kΩ. Neglecting the effects of r_o, find the low-frequency gain v_o/v_{sig}, the frequencies of the poles f_{P1} and f_{P2} and hence an estimate of the 3-dB frequency f_H. For a CG amplifier you can use $g_{mb} = \chi g_m$. Use ac equivalent circuit.

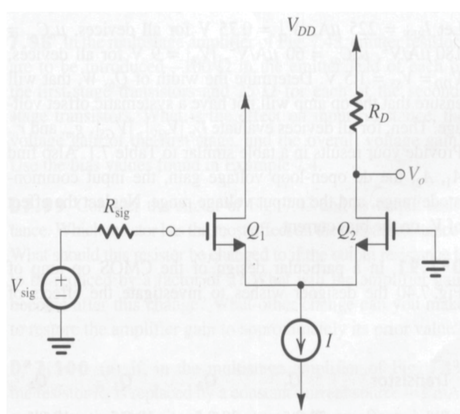
Hints:
From the small-signal equivalent circuit,
$v_o/v_i = [\{1/(g_m+g_{mb})\}/\{R_S+1/(g_m+g_{mb})\}](g_m+g_{mb})R'_L$; $f_{p1} = 1/[2\pi C_{gs}\{R_{sig}//(1/(g_m+g_{mb}))\}]$;
$f_{p2} = 1/[2\pi(C_{gd}+C_L)R'_L]$. $f_{p2} << f_{p1}$, f_{p2} is the dominant pole and $f_H \approx f_{p2}$

4. (a) Consider a CS amplifier having $C_{gd} = 0.2$ pF, $R_{sig} = R_L = 20$ kΩ, $g_m = 5$ mA/V, $C_{gs} = 2$ pF, C_L (including C_{db}) = 1 pF, and $r_o = 20$ kΩ. Find (i) the low-frequency gain A_M, and (ii) estimate f_H using open-circuit time constants.

   Hence determine the gain-bandwidth (GBW=mid-freq. gain *times* f_H).

Hints:
$A_M = g_m R'_L$; $f_H = 1/(2\pi\tau_H)$ where $\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R'_L$, $R_{gs} = R_{sig}$, $R_{gd} = R_{sig}(1+g_m R'_L) + R'_L$; GBW = $|A_M| f_H$

5. Consider the following circuit for the case: I = 200 μA and $V_{OV} = 0.25$ V, $R_{sig} = 200$ kΩ, $R_D = 50$ kΩ, $C_{gs} = C_{gd} = 1$ Pf (for both transistors). Find the dc (i.e., low-frequency) gain, the high-frequency poles, and an estimate of f_H. (hint: need to find g_m from I and V_{OV} data!).



Hints:
~~$V_{G1} = V_S.[(2/g_m)/((2/g_m)+R_S)]$, $I = V_{G1}/(2/g_m)$, $V_O = IR_D$ hence, $A_O = V_O/V_S = g_m R_D/(2+g_m R_S)$; $f_{P1} = 1/[2\pi R_S(C_{gs}/2+C_{gd})]$; $f_{P2} = 1/(2\pi R_D C_{gd})$~~ (See Later)

6.

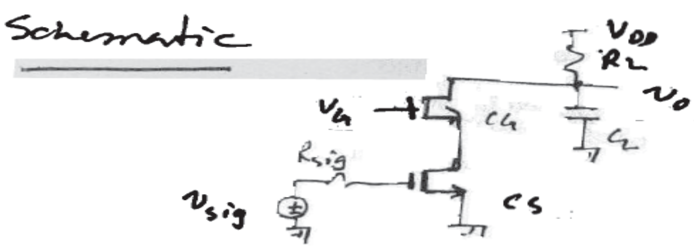
(a) Consider a CS stage having C_{gd} = 0.2 pF, R_{sig} = 20 kΩ, g_m =5 mA/V, C_{gs} = 2 pF, and r_o = 20 kΩ.

(b) A CG stage is connected in totem-pole configuration with the CS transistor in (a) to create a cascode amplifier. The ac parameters of this stage are identical with those of the CS stage. Regarding the body-effect in the CG stage assume χ = 0.2. Further R_L= 20 kΩ, and is shunted by a load capacitance C_L=1 pF. Show a schematic diagram of the system using NMOS transistors. Show the *ac* equivalent circuit.

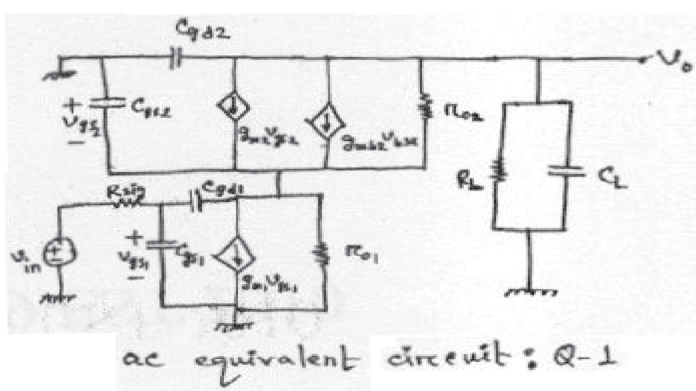
Find (i) the low-frequency gain AM, and (ii) estimate the gain-bandwidth of the system. You may use OCTC method to determine the dominant high frequency pole f_H of the system.

<u>Hints:</u>

<u>For the cascade amplifier:</u>



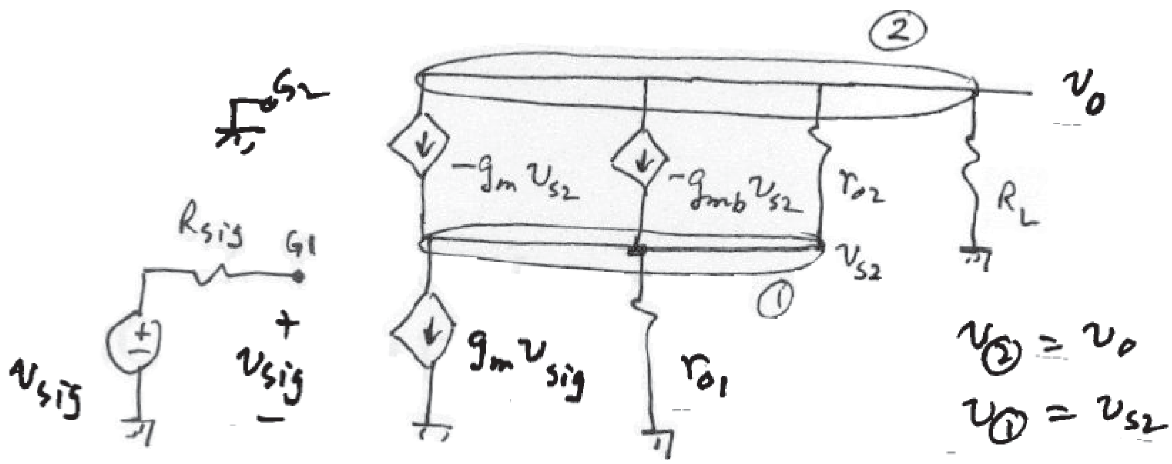
<u>AC Equivalent circuit:</u>



$V_{gs2} = V_g - V_{s2} = 0 - V_{s2}$

$V_{bs2} = V_b - V_{s2} = 0 - V_{s2}$

For low frequency gain, ignore all C_{gs} and C_{gd}



Consider the 2- node system and derive V_o/V_{sig}

$$\begin{bmatrix} g_{01} + g_{02} & -g_{02} \\ -g_{02} & g_{02} + g_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -g_m v_{sig} - (g_m + g_{mb})v_1 \\ (g_m + g_{mb})v_1 \end{bmatrix}$$

$$\frac{v_0}{v_{sig}} = -\frac{g_m\,(g_m + g_{mb} + g_0)}{(g_m + g_{mb})g_L + g_0(g_o + 2g_L)}$$

Here $g_{01} = g_{02} = g_0 = \frac{1}{r_0}$

Using the values: $\frac{v_0}{v_{sig}}$ **≈ - 98.37 v/v**

For Dominant role calculation, note:

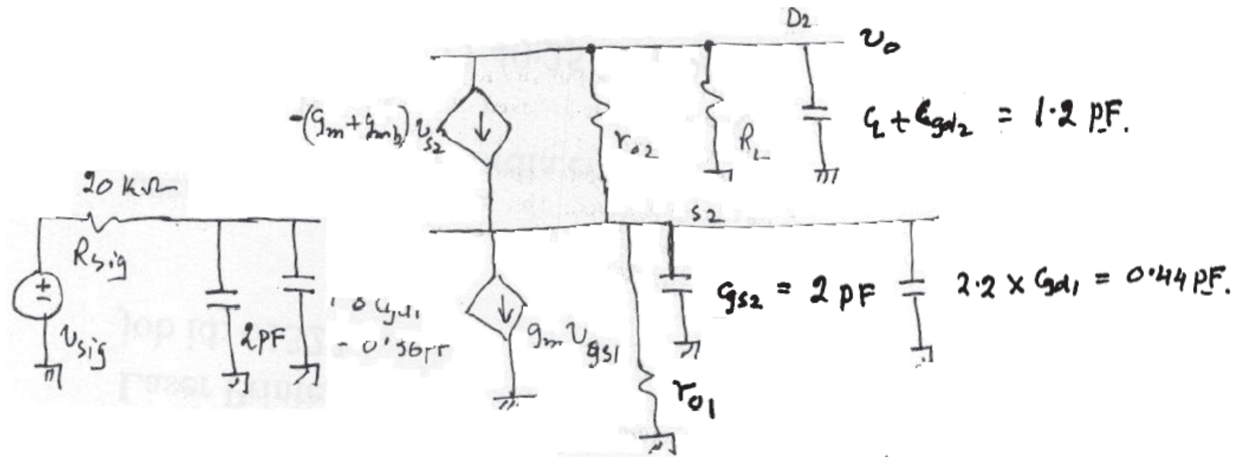
For C_{gd1} , the Miller effect amplifications are :

i) At input $(1 + K_1)\, C_{gd1}$, $K_1 = \frac{g_m}{g_m + g_{mb}} = \frac{1}{1.2} = 0.8$

ii) At input $(1 + \frac{1}{k_1})\, C_{gd1} = (1 + 1.2)\, C_{gd1}$

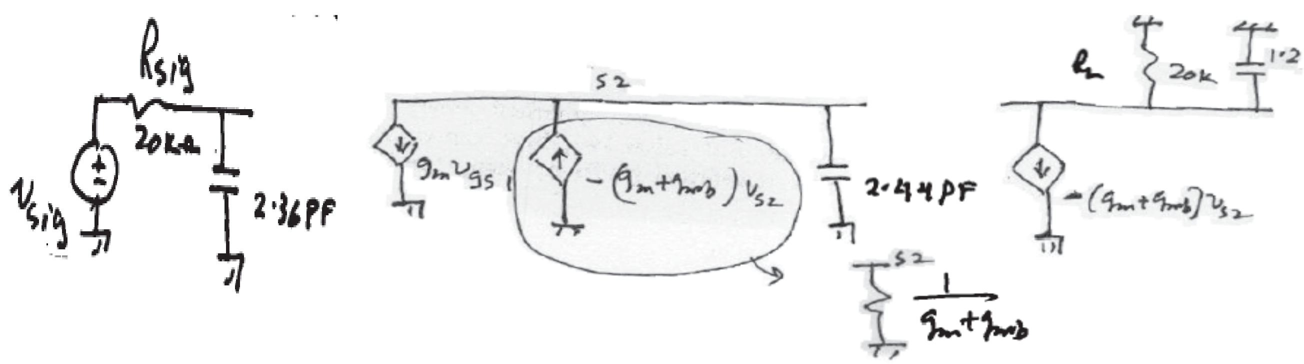
C_{gd2} does not have miller effect

So the AC equivalent circuit is



Ignoring r_{01} and r_{02} as was done in the class lecture.

CE-CB Cascade



The time constants are:

$\tau_1 = 2.36 \times 10^{-12} \times 20 \times 10^{3} = 4.72 \times 10^{-8}$ sec

$\tau_2 = \frac{2.44 \times 10^{-12}}{1.2 \times 5 \times 10^{-3}} = 4.07 \times 10^{-10}$ sec

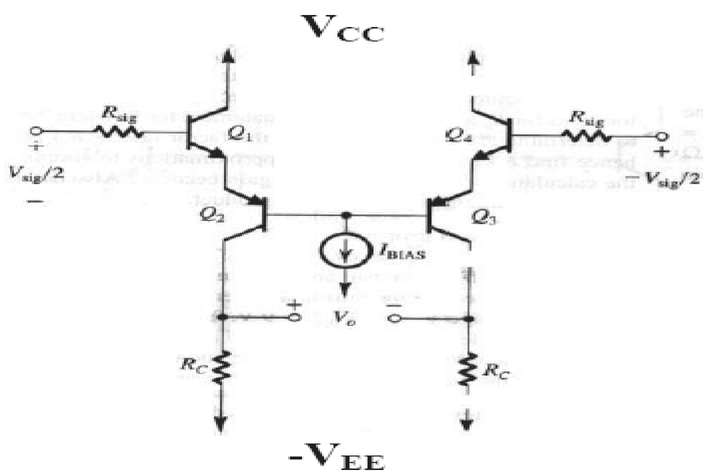
$\tau_3 = 1.2 \times 10^{-12} \times 20 \times 10^{3}$ sec

τ_1 , τ_3 are close enough ,so dominant time constant principle may not apply

We will take $\tau_H = \tau_1 + \tau_2 + \tau_3 = f_H = \frac{1}{2\pi} \frac{1}{7.161 \times 10^{-8}} = \mathbf{2.22}\ \boldsymbol{MHZ}$

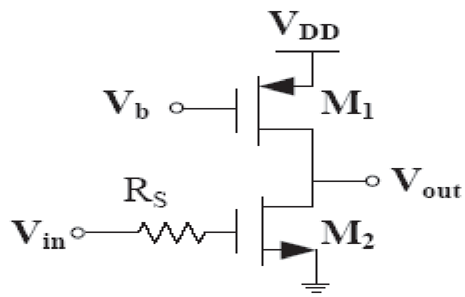
GBW = $|-98.37| \times 2.2 \times 10^6 =$ ***218.6 MHZ***

7. For the following circuit, let the bias be such that each transistor is operating at 100-μA collector current. Let the BJTs have h_{fe} = 200, f_T = 600 MHz, and C_μ = 0.2 pF, and neglect r_o and r_x. Also, $R_{sig} = RC$ = 50 kΩ.
Show the *ac* equivalent circuit.
Find (i) the low-frequency gain, (ii) the high-frequency poles, and (iii) an estimate of the dominant high frequency pole f_H of the system. Now find the GBW (gain-bandwidth) of the system. You may use half-circuit technique.



Hints:
~~(i)$A_o = \alpha R_C/[(R_{sig}/(\beta+1))+2r_e]$; where $\beta = h_{fe}$ (ii) $f_{P1} = 1/[2\pi(R_{sig}||2r_\pi)(C_\pi/2+C_\mu)]$; $f_{P2} = 1/[2\pi R_C C_\mu]$; where $2\pi f_T = g_m/(C_\pi + C_\mu)$, calculate C_π from here. (iii)$f_H = \sqrt{[(1/f_{P1})^2+(1/f_{P2})^2]}$; (iv)GBW = $|A_o| f_H$~~ (See Later)

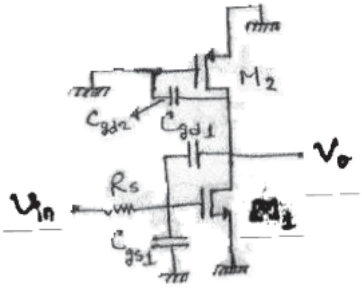
8.- In the following circuit assume both transistors operate in saturation and $\lambda \neq 0$. For each transistor you can assume the parasitic capacitances as C_{gsi}, C_{gdi}, (i=1,2).

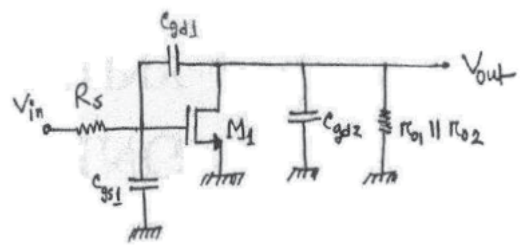


Draw the *ac* equivalent circuit, analyze and derive the expression for the dominant pole frequency.

Hints:

Simplified circuit models for small signal analysis are given below:





~~Analyzing the small signal equivalent circuit we get,~~

$\cancel{\omega_{P1} = 1/[\{1+g_{m1}(r_{01}||r_{02})\}C_{gd1}R_S+R_SC_{gs1}+(r_{01}||r_{02})(C_{gd1}+C_{gd2})]}$

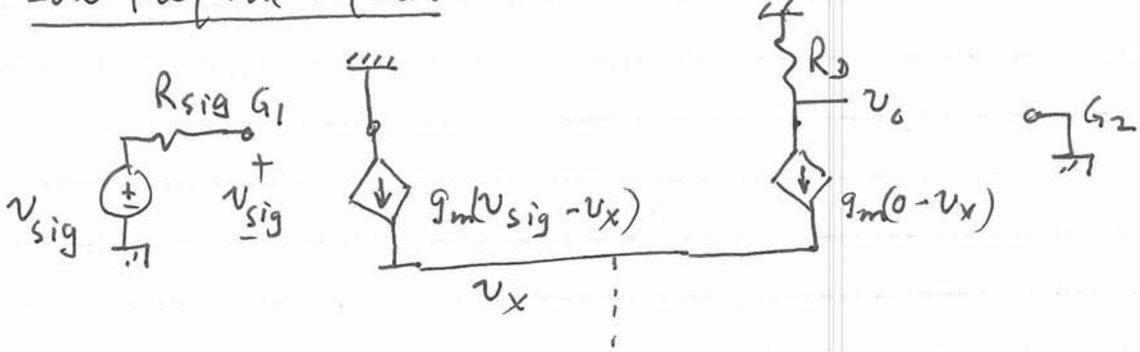
$\cancel{\omega_{P2}=[\{1+g_{m1}(r_{01}||r_{02})\}C_{gd1}R_S+R_SC_{gs1}+(r_{01}||r_{02})(C_{gd1}+C_{gd2})]/[R_S(r_{01}||r_{02})(C_{gs1}C_{gd1}+C_{gd2}C_{gd1}+C_{gs1}C_{gd2})]}$ (See Later)

5. $I = 200\mu A = \mu_n C_{ox} \frac{W}{2L} V_{OV}^2$ (square-law)

With $V_{OV} = 0.25$ $\mu_n C_{ox} \frac{W}{2L} = 3.2 \times 10^{-3}$

$$g_m = \mu_n C_{ox} \frac{W}{L} \cdot V_{OV} = 6.4 \times 10^{-3} \times 0.25 = 1.6 \times 10^{-3}$$

Low freq. ac equiv:

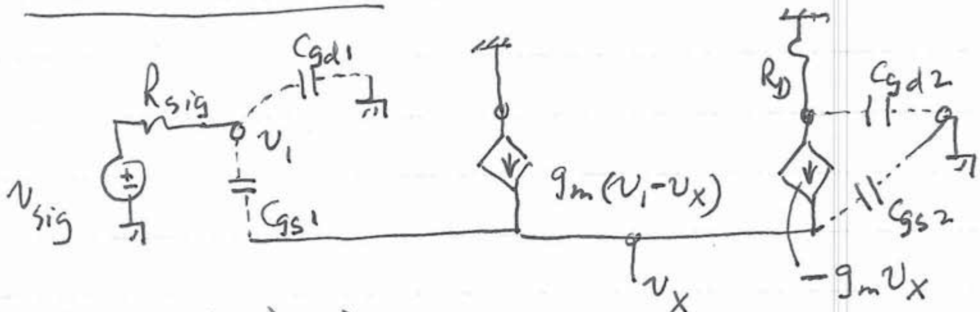


$v_o = g_m v_x R_D$, but $-g_m(v_{sig} - v_x) - g_m(-v_x) = 0$

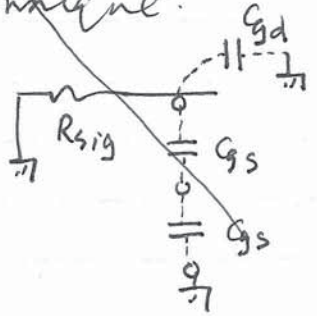
gives $v_x = v_{sig}/2$

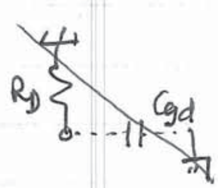
$v_o = g_m R_D \frac{v_{sig}}{2}$; $v_o / v_{sig} \Big|_{\text{Low freq.}} = \frac{g_m R_D}{2}$

H.F. equiv. cct:



By OCTC technique:





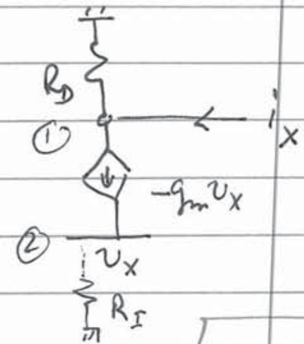
I-sources $\to \infty$ resistance

5 (Cont.)

By OCTC technique:

1) For C_{gd1} : $\tau_{gd_1} = C_{gd1} \cdot R_{sig}$

2) For C_{gd2} : $\tau_{gd_2} = C_{gd_2} \cdot R_D$



By NAM

$$\begin{pmatrix} g_D & 0 \\ 0 & g_I \end{pmatrix}\begin{pmatrix} v_1 \\ v_x \end{pmatrix} = \begin{bmatrix} i_x + g_m v_x \\ -g_m v_x \end{bmatrix}$$

$$\begin{bmatrix} g_D & -g_m \\ 0 & g_I + g_m \end{bmatrix}\begin{bmatrix} v_1 \\ v_x \end{bmatrix} = \begin{bmatrix} i_x \\ 0 \end{bmatrix}$$

$$\Delta = g_D g_I + g_D g_m$$

$$v_1 = \frac{1}{g_D g_m} \cdot \begin{vmatrix} i_x & -g_m \\ 0 & g_I + g_m \end{vmatrix} = \frac{g_m i_x}{g_D g_m} = \frac{i_x}{g_D}.$$

if $g_I \to 0$

$$\frac{v_1}{i_x} = R_D \quad R_{Thev}.$$

By basic circ.

$$-i_x + \frac{v_1}{R_D} - g_m v_x = 0$$

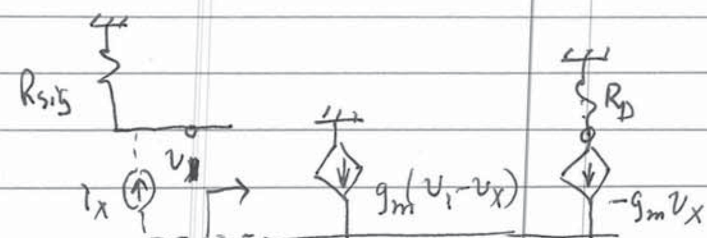
$$\frac{v_x}{R_I} + g_m v_x = 0$$

So $v_x = 0$;

Then $i_x = v_1 / R_D$; $\frac{v_1}{i_x} = R_D$.

R_{Thev}.

3) For C_{gs1}

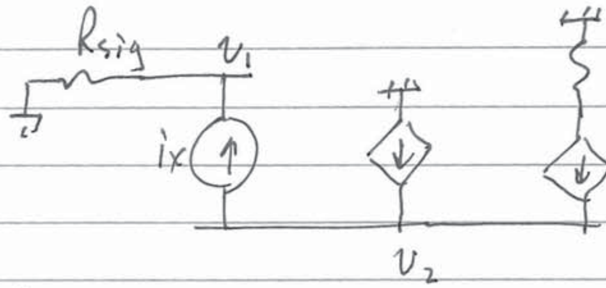


$$\begin{bmatrix} g_{sig} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g_D \end{bmatrix}\begin{bmatrix} v_1 \\ v_x \\ v_0 \end{bmatrix} = \begin{bmatrix} i_x \\ -i_x + g_m v_1 - g_m v_x - g_m v_x \\ + g_m v_x \end{bmatrix}$$

v_x

$$\begin{bmatrix} g_{sig} & 0 & 0 \\ -g_m & 2g_m & 0 \\ 0 & -g_m & g_D \end{bmatrix}\begin{bmatrix} v_1 \\ v_x \\ v_0 \end{bmatrix} = \begin{bmatrix} i_x \\ -i_x \\ 0 \end{bmatrix}$$

For C_{gs1} by basic ckt anal.:



At v_1 : $\dfrac{v_1}{R_{sig}} - i_x = 0$; $v_1 = i_x R_{sig}$

At v_2 : $+i_x - g_m(v_1 - v_2) - g_m(0 - v_2) = 0$

$$i_x - g_m v_1 + g_m v_2 + g_m v_2 = 0$$

$$\frac{v_1}{R_{sig}} - g_m v_1 + 2g_m v_2 = 0.$$

$$v_1\left(\frac{1}{R_{sig}} - g_m\right) + v_2 . 2g_m = 0.$$

$$v_2 = -v_1\left[\frac{1}{2R_{sig}g_m} - \frac{1}{2}\right] = -\frac{v_1}{2}\left[\frac{1}{R_{sig}g_m} - 1\right]$$

$$v_1 - v_2 = v_1 + \frac{v_1}{2}\left(\frac{1}{R_{sig}g_m} - 1\right)$$

$$= v_1 + \frac{v_1}{2}\left(\frac{1}{R_{sig}g_m} - 1\right)$$

$$\frac{v_1 - v_2}{i_x} = \frac{v_1 + \frac{v_1}{2}\left(\frac{1}{R_{sig}g_m} - 1\right)}{\frac{v_1}{R_{sig}}} = R_{sig}\left[1 + \frac{1}{2R_{sig}g_m} - \frac{1}{2}\right]$$

$$= R_{sig}\left(\frac{1}{2} + \frac{1}{2R_{sig}g_m}\right)$$

$$= R_{sig} + \frac{R_{sig}}{2}\left(\frac{1}{R_{sig}g_m} - 1\right)$$

$$= R_{sig} + \frac{1}{2g_m} - \frac{R_{sig}}{2} = \frac{R_{sig}}{2} + \frac{R_m}{2} = \frac{1}{2}(R_{sig} + R_m)$$

5 (cont.)

$$\Delta = g_{sig} \cdot 2g_m \cdot g_D$$

$$v_1 \quad \frac{1}{\Delta} \cdot \begin{vmatrix} i_x & 0 & 0 \\ -i_x & 2g_m & 0 \\ 0 & -g_m & g_D \end{vmatrix} = \frac{1}{\Delta} \cdot i_x \cdot 2g_m g_D$$

$$v_x = \frac{1}{\Delta} \begin{vmatrix} g_{sig} & i_x & 0 \\ -g_m & -i_x & 0 \\ 0 & 0 & g_D \end{vmatrix} = \frac{1}{\Delta}\left[i_x \cdot g_m g_D - g_{sig} i_x g_D \right] = \frac{1}{\Delta} i_x g_D (g_m - g_{sig})$$

$$v_1 - v_x = \frac{1}{\Delta} \cdot \cancel{i_x g_m g_D} = i_x \frac{\cancel{g_m g_D}}{g_{sig} \cdot 2\cancel{g_m} \cdot \cancel{g_D}} = i_x \frac{1}{2g_{sig}}$$

$$= \frac{1}{\Delta} \cdot i_x (2g_m g_D - g_m g_D + g_D g_{sig})$$

$$= \frac{1}{\Delta} i_x \cdot (g_m g_D + g_D g_{sig})$$

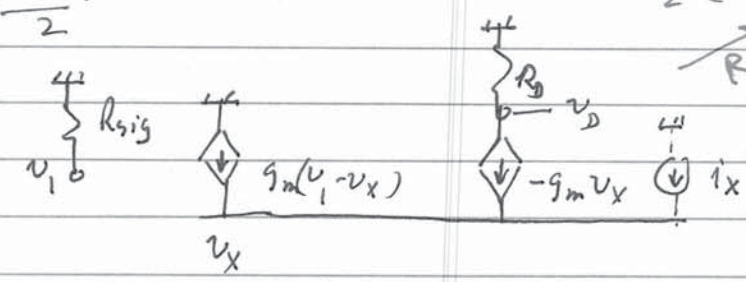
$$\frac{v_1 - v_x}{i_x} = \frac{g_D \cdot (g_m + g_{sig})}{g_D \cdot 2 g_m g_{sig}} = \frac{1}{2}\left(\frac{1}{g_{sig}} + \frac{1}{g_m}\right)$$

$$= \frac{1}{2}(R_{sig} + R_m)$$

R_{Thev}

3) $\tau_{gs1} = \frac{C_{gs1} \cdot R_{sig}}{2}$

4) For C_{gs2}



$$\begin{pmatrix} g_{sig} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g_D \end{pmatrix} \begin{pmatrix} v_1 \\ v_x \\ v_D \end{pmatrix} = \begin{bmatrix} 0 \\ i_x + g_m v_1 - g_m v_x - g_m v_x \\ g_m v_x \end{bmatrix}$$

$$\begin{bmatrix} g_{sig} & 0 & 0 \\ -g_m & 2g_m & 0 \\ 0 & -g_m & g_D \end{bmatrix} \begin{bmatrix} v_1 \\ v_x \\ v_D \end{bmatrix} = \begin{bmatrix} 0 \\ i_x \\ 0 \end{bmatrix}$$

5 (cont.)

Δ = same as before $= g_{sig} \cdot 2g_m \cdot g_D$

$$v_x = \frac{1}{\Delta} \cdot \begin{vmatrix} g_{sig} & 0 & 0 \\ -g_m & i_x & 0 \\ 0 & 0 & g_D \end{vmatrix} = \frac{1}{\Delta} \cdot g_{sig} \cdot i_x \, g_D = \frac{\cancel{g_{sig}} \cdot i_x \cdot \cancel{g_D}}{\cancel{g_{sig}} \cdot 2g_m \cdot \cancel{g_D}}$$

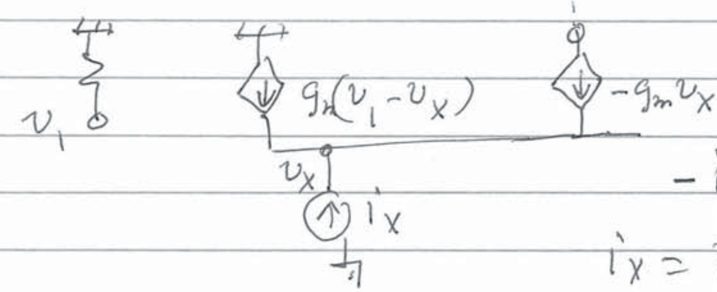
$$v_x = \frac{i_x}{2g_m} \; ; \; \underbrace{\frac{v_x}{i_x}}_{R_{Thev}} = \frac{1}{2g_m} = \frac{1}{2} R_m \; ; \; R_m = 1/g_m$$

Then:

$$\tau_{gs2} = C_{gs2} \cdot \underbrace{\frac{1}{2g_m}}_{R_{Thev}} = C_{gs2} \frac{R_m}{2}$$

Calculate after substitutions.

By basic ckt anal.?

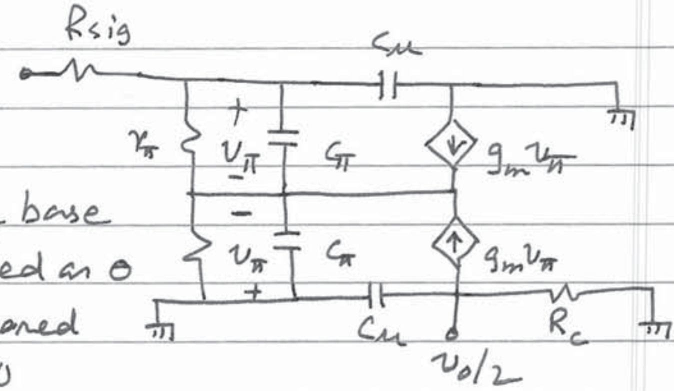


$\because v_1 = 0$

$$-i_x + g_m v_x + g_m v_x = 0$$

$$i_x = 2g_m v_x \; ; \; \underbrace{\frac{v_x}{i_x}}_{R_{Thev}} = (2g_m)^{-1} = \frac{1}{2} R_m$$

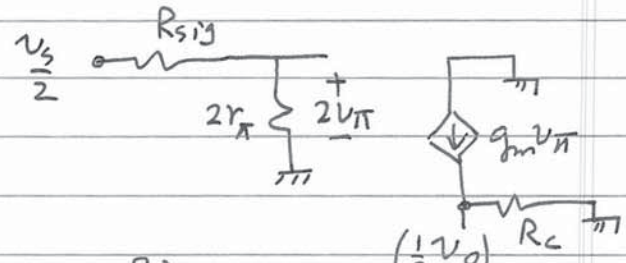
7/ Half circuit (see class lecture note)



• r_x of base ignored as 0

• r_o ignored as ∞

→ **Low Freq gain:**

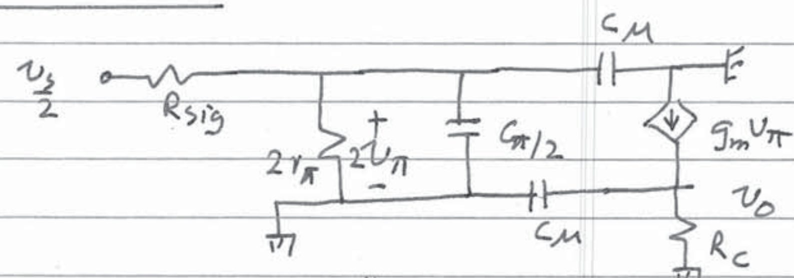


$$2v_\pi = \frac{2r_\pi}{R_{sig}+2r_\pi}\cdot\frac{v_s}{2} \quad ; \quad v_\pi = \frac{r_\pi}{R_{sig}+2r_\pi}\cdot\frac{v_s}{2}$$

$$\frac{v_o}{2} = \frac{g_m r_\pi R_c}{R_{sig}+2r_\pi}\cdot\frac{v_s}{2} \quad ; \quad \frac{v_o}{v_s} = \frac{R_c(h_{fe}+1)}{R_{sig}+2r_\pi} = A_M$$

subst. for r_π, R_{sig} ..etc.

HF. equiv. circ.



$$\tau_1 = \left(C_\mu + \frac{C_\pi}{2}\right)\cdot\left(2r_\pi \,\|\, R_{sig}\right)$$

$$\tau_2 = C_\mu R_c$$

$$\omega_T = 2\pi \times 600 \times 10^6 = \frac{g_m}{C_\pi + C_\mu}$$

$$0.004 \leftarrow g_m = \frac{I_c}{V_T} = \frac{0.1\,mA}{25\,mV} \; ; \; \text{find } C_\pi \quad \because C_\mu = 0.2\,pF.$$

$C_\pi \to 0.86$ pF.

7

$$\omega_H = \frac{1}{\tau_1 + \tau_2}$$

$$GBW = A_M \, \omega_H \quad \text{rad/sec.}$$

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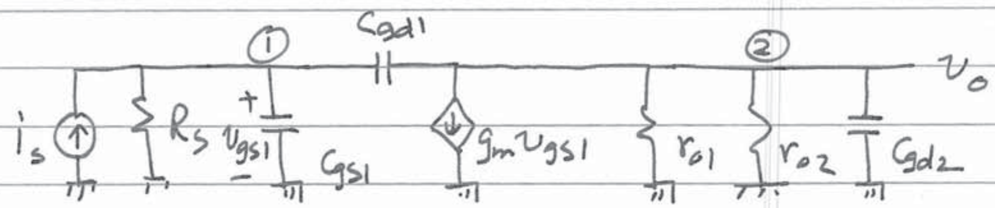
8 AC equivalent circuit for nodal analysis:



; $i_s' = \frac{v_{in}}{R_s}$

$$\begin{bmatrix} g_s + s(C_{gs1} + C_{gd1}) & -sC_{gd1} \\ -sC_{gd1} & g_{o1} + g_{o2} + sC_{gd1} + sC_{gd2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_s' \\ -g_m v_{gs1} \end{bmatrix}$$

$\because v_{gs1} = v_1$

$$\begin{bmatrix} g_s + s(C_{gs1} + C_{gd1}) & -sC_{gd1} \\ g_m - sC_{gd1} & g_o + g_{o2} + s(C_{gd1} + C_{gd2}) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

Δ of the matrix:

$$D(s) = \{g_s + s(C_{gs1} + C_{gd1})\}\{g_{o1} + g_{o2} + s(C_{gd1} + C_{gd2})\} + sC_{gd1}(g_m - sC_{gd1})$$

Expanding:

$$D(s) = As^2 + Bs + C$$

$$A = C_{gs1}C_{gd1} + C_{gd1}C_{gd2} + C_{gs1}C_{gd2}$$

$$B = g_s(C_{gd1} + C_{gd2}) + C_{gs1}(g_{o1} + g_{o2}) + g_m C_{gd1} + C_{gd1}(g_{o1} + g_{o2})$$

$$C = g_s(g_{o1} + g_{o2})$$

Dominant pole: C/B