CHAPTER 3

Frequency Response of Basic BJT and MOSFET Amplifiers

(*Review materials in Appendices III and V*)

In this chapter you will learn about the general form of the frequency domain transfer function of an amplifier. You will learn to analyze the amplifier equivalent circuit and determine the critical frequencies that limit the response at low and high frequencies. You will learn some special techniques to determine these frequencies. BJT and MOSFET amplifiers will be considered. You will also learn the concepts that are pursued to design a wide band width amplifier. Following topics will be considered.

- Review of Bode plot technique.
- Ways to write the transfer (i.e., gain) functions to show frequency dependence.
- Band-width limiting at low frequencies (i.e., DC to f_L). Determination of lower band cut-off frequency for a single-stage amplifier – short circuit time constant technique.
- Band-width limiting at high frequencies for a single-stage amplifier. Determination of upper band cut-off frequency- several alternative techniques.
- Frequency response of a single device (BJT, MOSFET).
- Concepts related to wide-band amplifier design – BJT and MOSFET examples.

3.1 A short review on Bode plot technique

Example: Produce the Bode plots for the magnitude and phase of the transfer function $T(s) = \dfrac{10s}{(1+s/10^2)(1+s/10^5)}$, for frequencies between 1 rad/sec to 10^6 rad/sec.

We first observe that the function has zeros and poles in the numerical sequence 0 (zero), 10^2 (pole), and 10^5 (pole). Further at ω=1 rad/sec i.e., lot less than the first pole (at ω=10^2 rad/sec), $T(s) \cong 10s$. Hence the first portion of the plot will follow the asymptotic line rising at 6 dB/octave, or 20 dB/decade, in the neighborhood of ω=1 rad/sec. The magnitude of *T(s)* in decibels will be approximately 20 dB at ω= 1 rad/sec.

The second asymptotic line will commence at the pole of $\omega=10^2$ rad/sec, running at -6 dB/octave slope relative to the previous asymptote. Thus the overall asymptote will be a line of slope zero, i.e., a line parallel to the ω- axis.

The third asymptote will commence at the pole $\omega=10^5$ rad/sec, running at -6 dB/Octave slope relative to the previous asymptote. The overall asymptote will be a line dropping off at -6 dB/octave beginning from $\omega=10^5$ rad/sec.

Since we have covered all the poles and zeros, we need not work on sketching any further asymptotes. The three asymptotic lines are now sketched as shown in figure 3.1.

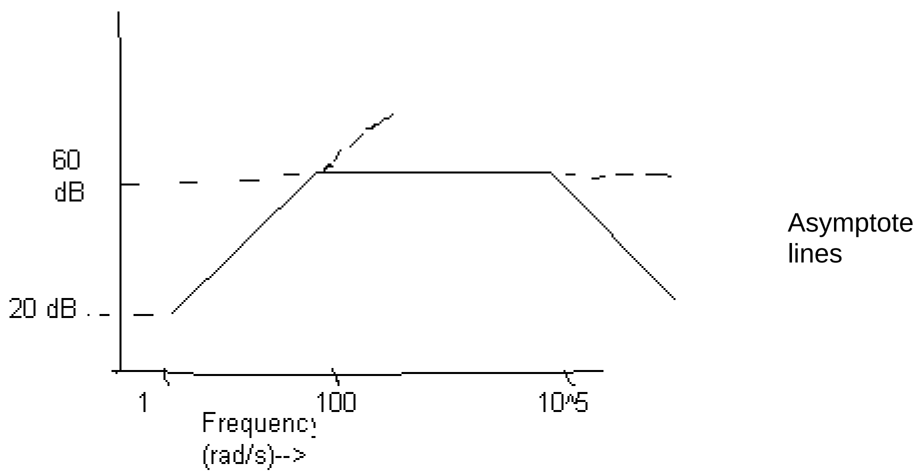


Figure 3.1: The asymptotic line plots for the *T(s)*.

The actual plot will follow the asymptotic lines being 3 dB below the first corner point (i.e.,at ω=100)i.e., 57 dB ,and 3 dB below the second corner point (i.e., ω=10^5), i.e. 57 dB. In between the two corner point the plot will approach the asymptotic line of constant value 60 dB. The magnitude plot is shown in figure 3.2.

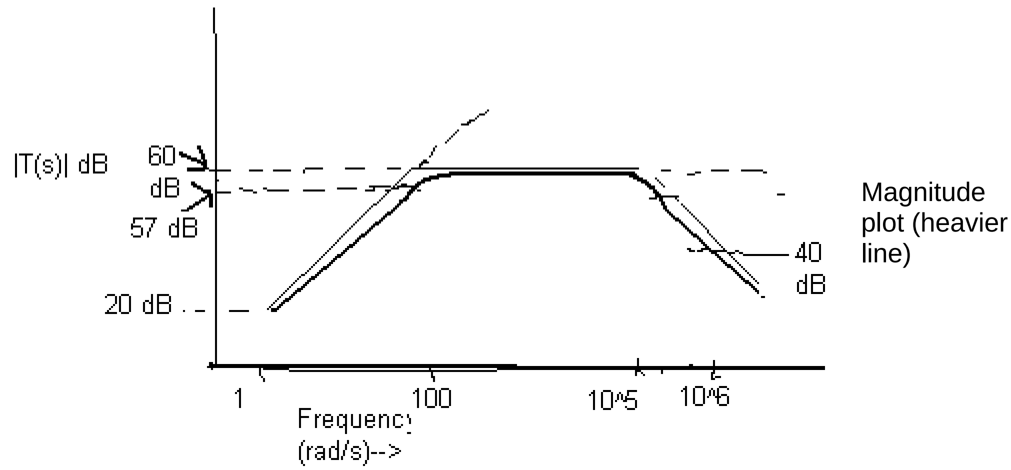


Figure 3.2:Bode magnitude plot for *T(s)*

For phase plot, we note that the 's' in the numerator will give a constant phase shift of +90^o degrees (since $s \to j\omega \to 0 + j\omega$, angle: $\tan^{-1}(\omega/0) \to \tan^{-1}(\infty) \to 90^o$), while the terms in the denominator will produce angles of $-\tan^{-1}(\omega/10^2)$, and $-\tan^{-1}(\omega/10^5)$ respectively. The total phase angle will then be:

$$\phi(\omega) = 90^o - \tan^{-1}(\omega/10^2) - \tan^{-1}(\omega/10^5) \qquad (3.1)$$

Thus at low frequency (<< 100 rad/sec), the phase angle will be close to 90^o. Near the pole frequency ω=100, a -45^o will be added due to the ploe at making the phase angle to be close to +45^o. The phase angle will progressively decrease, because of the first two terms in φ(ω). Near the second pole ω=10^5, the phase angle will approach

$\phi(\omega) = 90^o - \tan^{-1}(10^5/10^2) - \tan^{-1}(10^5/10^5) \cong 90^o - 90^o - 45^o$ i.e., -45^o degrees.

(The student in encouraged to draw the curve)

3.2 Simplified form of the gain function of an amplifier revealing the frequency response limitation

3.2.1 Gain function at low frequencies

Electronic amplifiers are limited in frequency response in that the response magnitude falls off from a constant mid-band value to lower values both at frequencies below and above an intermediate range (the mid-band) of frequencies. A typical frequency response curve of an amplifier system appears as in figure3.3.

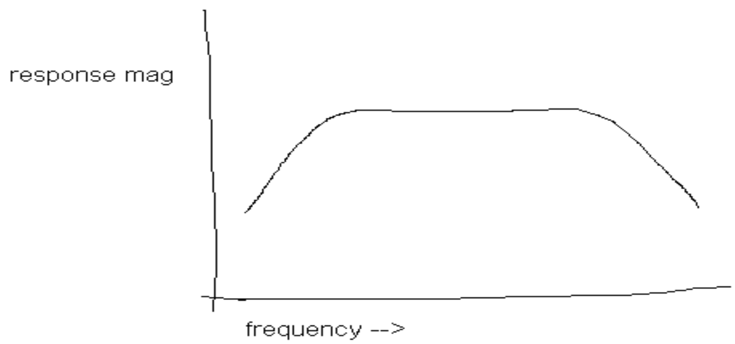


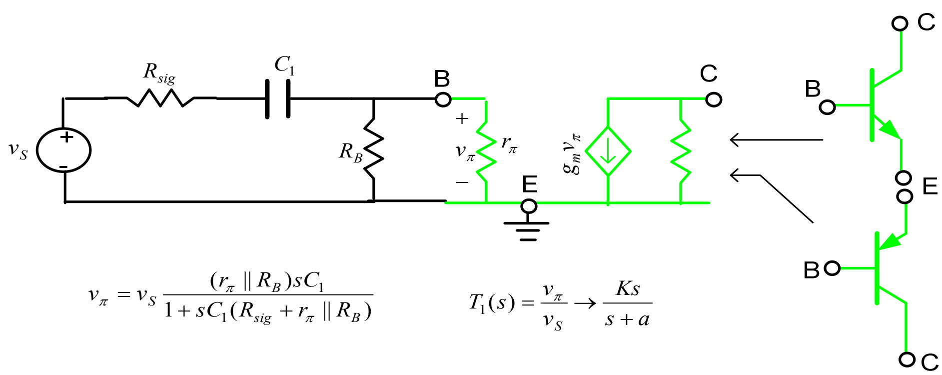
Figure 3.3: Typical frequency response function magnitude plot for an electronic amplifier

Using the concepts of Bode magnitude plot technique, we can approximate the low-frequency portion of the sketch above by an expression of the form $T_L(s) = \frac{Ks}{s+a}$, or $T_L(s) = \frac{K}{1+a/s}$. In this K and a are constants and s=$j\omega$, where ω is the (physical, i.e., measurable) angular frequency (in rad/sec). In either case, when the signal frequency is very much smaller than the pole frequency 'a', the response $T_L(s)$ takes the form Ks/a. This function increases progressively with the frequency $s = j\omega$, following the asymptotic line with a slope of +6 dB per octave. At the pole frequency 'a', the response will be 3 dB below the previous asymptotic line, and henceforth follow an asymptotic line of slope (-6+6=0) of zero dB/ octave. Thus $T_L(s)$ will remain constant with frequency, assuming the mid-band value. Note that $T_L(s)$ is a first order function in 's' (a single time-constant function).

The *frequency* at which the magnitude plot reaches *3 dB below the mid-band* (i.e., the flat portion of the magnitude response curve) gain value is known as the *-3 dB frequency* of the gain function. For the low-frequency segment (i.e., T_L(s)) of the magnitude plot this will be designated by f_L (or $\omega_L = 2\pi f_L$).

In a practical case the function $T_L(s)$ may have several poles and zeros at low frequencies. The pole which is *closest* to the flat mid-band value is known as the *low frequency dominant pole* of the system. Thus it is the pole of *highest* magnitude among all the poles and zeros at low frequencies. Numerically the *dominant* pole differs from the -3 dB frequency. But for simplicity, one can approximate dominant pole to be of same value as the -3dB frequency. The -3dB frequency at low frequencies is also sometimes referred to as the *lower* cut-off frequency of the amplifier system.

The frequency response limitation at low frequency occurs because of coupling and by-pass capacitors used in the amplifier circuit. For single-stage amplifiers, i.e., CE, CB..CS,CG amplifiers these capacitors come in series with the signal path (i.e., they form a loop in the signal path), and hence impedes the flow of signal coupled to the internal nodes (i.e., BE nodes of the BJT, GS nodes of the MOSFET) of the active device. The students can convince themselves by considering the simple illustration presented in figure 3.4.



$$v_\pi = v_S \frac{(r_\pi \parallel R_B) s C_1}{1 + s C_1 (R_{sig} + r_\pi \parallel R_B)} \qquad T_1(s) = \frac{v_\pi}{v_S} \rightarrow \frac{Ks}{s+a}$$

Figure 3.4: Illustrating the formation of s *zero* in the voltage transfer function because of a capacitor in the signal loop. The controlling voltage v_π for the VCCS has a *zero* because of the presence of C_1.

3.2.2 Gain function at high frequencies

A similar scenario exists for the response at high frequencies. By considering the graph in Fig.3.3 at frequencies beyond (i.e., higher than) the mid-band segment, we can propose the form of the response function as: $T_H(s) = \frac{K}{s+b}$. K and b are constants. Other alternative forms are: $T_H(s) = \frac{K_o b}{s+b}$, or $T_H(s) = \frac{K_o}{1+s/b}$. Note that in all cases, for frequencies << the pole frequency 'b', the response function assumes a constant value (i.e., the mid-band response). For $T_H(s)$, which is a *first-order* function, the frequency b becomes the *-3db frequency* for high frequency response, or the *upper* cut-off frequency. When there are several poles and zeros in the high frequency range, the pole with the *smallest* magnitude and hence closest to the mid-band response zone is referred to as the *high frequency dominant* pole. Again, numerically the high frequency dominant pole will be different from the upper cut-off frequency. But in most practical cases, the difference is small. In case the high frequency response has several poles and zeros, one can formulate the function as

$$T_H(s) = \frac{(1+s/\omega_{z1})(1+s/\omega_{z2})..}{(1+s/\omega_{p1})(1+s/\omega_{p2})..} \qquad (3.2)$$

In an integrated circuit scenario coupling or by-pass capacitors are absent. The frequency dependent gain function (i.e., *transfer function*) is produced because of the intrinsic capacitances (*parasitic* capacitances) of the devices. As a consequence the zeros occur at very high frequencies and only one of the poles fall in the signal frequency range of interest, with the other poles at substantially higher frequencies. Thus if ω_{p1} is the pole of smallest magnitude, the amplifier will have ω_{p1} as the dominant pole. In such case $T_H(s) \cong \frac{\omega_{p1}}{s+\omega_{p1}}$, and ω_{p1} will also be the -3 dB or upper cut-off frequency of the system. Otherwise, the -3 dB frequency ω_H can be calculated using the formula[1]

$$\omega_H \cong 1 \Big/ \left[\left(\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + ...\right) - 2\left(\frac{1}{\omega_{z1}^2} + \frac{1}{\omega_{z2}^2} + ..\right)\right]^{1/2} \qquad (3.3)$$

3.2.3 Simplified (first order) form of the amplifier gain function

[1] Sedra and Smith, "Microelectronic Circuits", 6th edn., ch.9, p.722, Oxford University Press, ©2010.

Considering the discussions in sections 3.2.1-2 we can formulate the simplified form of the amplifier gain function can then be considered as :

$$A(s)=A_M\, F_L(s)\, F_H(s) \qquad (3.4)$$

In (3.4), A_M is independent of frequency, F_L has a frequency dependence of the form $s/(s+w_L)$, while F_H has a frequency dependence of the form $w_H/(s+w_H)$. Thus for frequencies higher than w_L and for frequencies lower than w_H the gain is close to A_M. This is a constant gain and the frequency band w_H - w_L is called the mid-band frequencies. So in the mid-band frequencies the gain is constant i.e., A_M. At frequencies << w_L, $F_L(s)$ increases with frequency (re: Bode plot) by virtue of the '*s*' in the numerator, at 6dB/octave. As the frequency increases, the rate of increase slows down and the Bode plot merges with the constant value A_M shortly after $w=w_L$. At $w=w_L$ the response falls 3 dB below the initial asymptotic line of slope 6dB/octave. Similarly, as frequency increases past w_H, the response A(s) tends to fall off, passing through 3dB below A_M (in dB) at $w=w_H$ and then following the asymptotic line with slope *minus* 6dB/octave drawn at $w=w_H$. It is of interest to be able to find out these two critical frequencies for basic single stage amplifiers implemented using BJT or MOSFET.

3.3 Simplified high-frequency *ac* equivalent circuits for BJT and MOSFET devices

It can be noted that for amplifiers implemented in integrated circuit technology only the upper cut-off frequency w_H is of interest. To investigate this we must be familiar with the ac equivalent circuit of the transistor at high frequencies. The elements that affect the high frequency behavior are the parasitic capacitors that exist in a transistor. These arise because a transistor is made by laying down several semiconductor layers of different conductivity (i.e., p-type and n-type materials). At the junction of each pair of dissimilar layers, a capacitance is generated. We will consider the simplified high-frequency equivalent circuits for the BJT and MOSFET as shown in Figs.3.5-3.6. In these models each transistor is assigned with only two parasitic capacitance associated with its internal nodes. These arise out of the semiconductor junctions that are involved in building the transistor. For the BJT, the base material produces a small resistance r_x, which assumes importance for high (signal) frequency applications (signal processing). The models for N-type (i.e., NPN, NMOSFET) and P-type (i.e., PNP, PMOSFET) transistors are considered same. In more advanced models (used in industries) more number of parasitic capacitances and resistances are employed.

3.3.1 High frequency response characteristics of a BJT

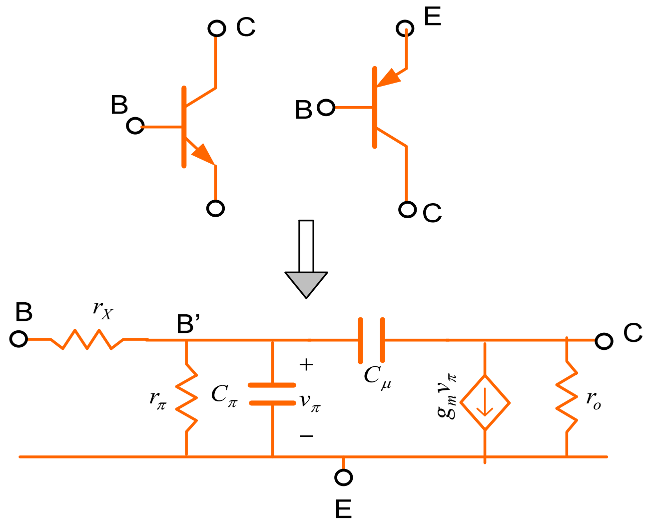


Figure 3.5: Simplified ac equivalent circuit for a BJT device for high signal frequency situation.

An important performance parameter of a BJT device is the small signal short circuit current gain of the device under CE mode of operation. Thus in Fig.3.5, if we insert an ac current source at terminal **B** and seek the ac short-circuit output current at node C, we can construct the CE ac equivalent circuit as in Fig.3.6. The short-circuit current gain i_o/i_i of the device can be derived from the KCL equations (returning terminal C to *ac* ground) at the nodes B and B'. Writing $g_i = 1/r_i$ in general, we get

$$i_i = g_x(v_B - v_\pi), \quad 0 = -g_x v_B + (g_x + g_\pi + sC_\pi + sC_\mu)v_\pi \tag{3.5}$$

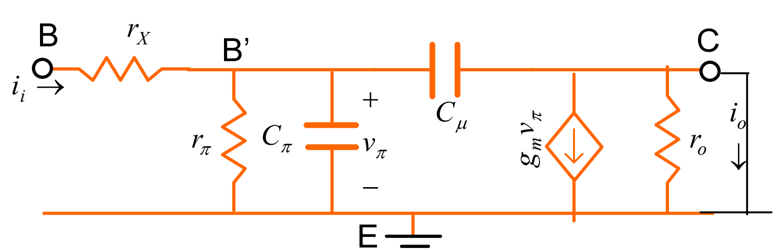


Figure 3.6: Configuring the BJT device for CS short-circuit current gain calculation.

Solving for v_π and noting that at C node (which is short circuited for *ac*) $i_o = -g_m v_\pi + sC_\mu$, we can finally derive the *short-circuit* current gain of the BJT under CE mode of operation as:

$$h_{fe}(s) = {}^{i_o}\!/_{i_i} = -\frac{-(g_m - sC_\mu)g_x}{g_x(g_\pi + s(C_\pi + C_\mu))} = -\frac{g_m - sC_\mu}{C_\pi + C_\mu}\frac{1}{s + \frac{1}{r_\pi(C_\pi + C_\mu)}} \qquad (3.6)$$

Eq.(3.6) represents a transfer function with a low-frequency (i.e., $\omega \approx 0$) value of $h_{fe}|_{\text{low-frequency}} = h_{fe}(s)|_{s=j\omega=0} = -g_m r_\pi = -\beta$, the familiar symbol for the current gain of a BJT in CE operation. Because C_μ is very small, the *zero* of $h_{fe}(j\omega)$ i.e., g_m/C_μ lies at very high frequencies. Using the symbol h_{fe} (0) for low-frequency ($\omega \approx 0$) value of h_{fe}, and for frequencies << $\omega_z = {}^{g_m}\!/_{C_\mu}$, the Bode magnitude plot of h_{fe} appears as in Fig. 3.7.

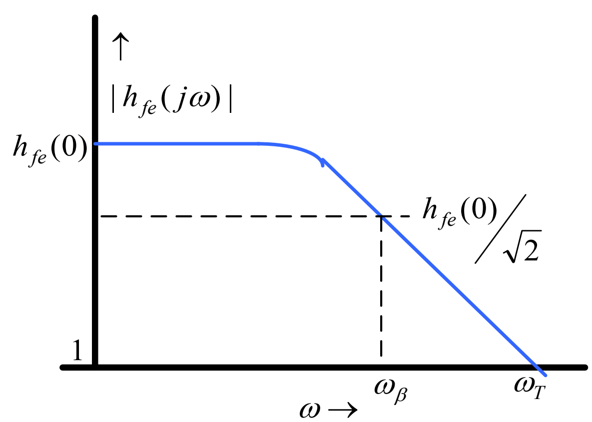


Figure 3.7: The Bode magnitude plot of | $h_{fe}(j\omega)$ |.

It is observed that at the frequency $\omega_\beta = {}^{1}\!/_{r_\pi(C_\pi + C_\mu)}$, $|h_{fe}|$ drops to ${}^{h_{fe}(0)}\!/_{\sqrt{2}}$, i.e., -3db below $h_{fe}(0)$. This frequency is known as the β cut-off frequency for the BJT under CE mode of operation.

At frequencies much higher than ω_β, $h_{fe}(j\omega)$ changes as (see eq.(3.6)) $\approx -\frac{g_m}{j\omega(C_\pi + C_\mu)}$. This reaches a magnitude of *unity* (i.e. =1), at a frequency

$$\omega_T = g_m / (C_\pi + C_\mu) \qquad (3.7)$$

This is known as the *transition frequency* of the BJT for operation as CE amplifier. The *transition frequency* $\omega_T = 2\pi f_T$ is a very *important* parameter of the BJT for high-frequency applications. For a given BJT, the high-frequency operational limit of the device can be increased by increasing ω_T via an increase in g_m , the *ac* transconductance of the BJT. This, however, implies an increase in the DC bias current (since $g_m = I/V_T$) and hence an increase in the DC power consumption of the system. Recalling the relation $g_m r_\pi = \beta+1$, we can deduce that

$$\omega_T = (1+\beta)\omega_\beta = (1+h_{fe}(0))\omega_\beta \qquad (3.8)$$

In real BJT devices $C_\pi >> C_\mu$, and $C_\pi + C_\mu >> C_\mu$. Hence, the *zero* frequency $\omega_z = g_m / C_\mu$ will be >> the *transition* frequency ω_T. Since $|h_{fe}(j\omega)|$ becomes <1 beyond ω_T, the *zero* frequency bears no practical interest.

3.3.2 High frequency response characteristics of a MOSFET

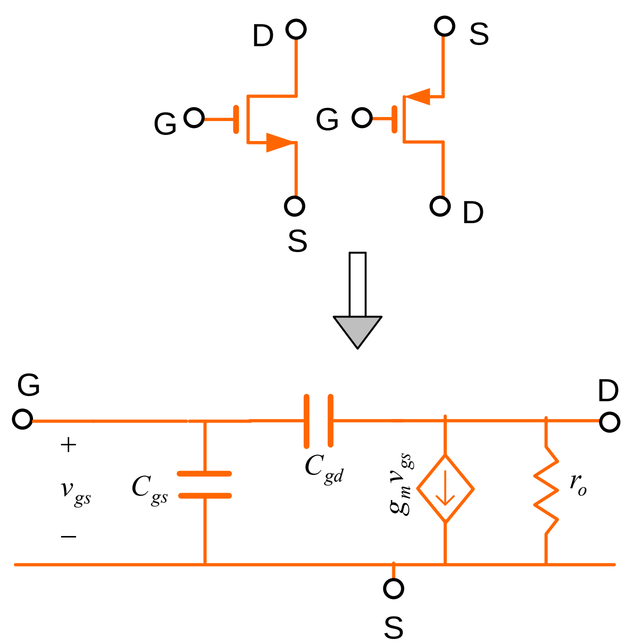


Figure 3.8: Simplified ac equivalent circuit for a MOSFET device for high signal frequency situation.

A simplified ac equivalent circuit for the MOSFET is shown in figure 3.8. The body terminal (B) for the MOSFET , and the associated parasitic capacitances as well as the body transconductance (g_{mb}) have not been shown. By following a procedure similar to that of a BJT, it can be shown that the *short circuit current gain* of the MOSFET configured as a CS amplifier is given by

$i_o/i_i = -\dfrac{g_m - sC_{gd}}{s(C_{gs} + C_{gd})}$ which can be approximated as $\dfrac{i_o}{i_i} = -\dfrac{g_m}{s(C_{gs} + C_{gd})}$ for frequencies well below the *zero* frequency g_m/sC_{gd}.

Under the above assumption the frequency at which the magnitude of the current gain becomes *unity* i.e., the *transition frequency*, becomes:

$$\omega_T = \frac{g_m}{(C_{gs} + C_{gd})} \qquad (3.9)$$

The *transition frequency* of a MOSFET is a very important parameter for high frequency operation. This can be increased via an increase in g_m with the attendant increase in the DC bias current and hence increase in DC power dissipation.

3.4 Calculation of ω_L – the lower cut-off frequency (*Short Circuit Time Constant method*)

Figure 3.9(a) depicts a typical CE-BJT amplifier with coupling capacitors C_1, C_3, and the by-pass capacitor C_E. Each of these capacitors fall in the *signal path* for the operation of the amplifier and hence influences the voltage gain function in terms of introducing several poles and zeros in the gain transfer function.

A simplified method to determine *the poles* is to consider *only one of the capacitors* effective at a time and assume that the other capacitors behave approximately as *short circuits*. Because only one capacitor is present in the system, it is easy to determine the *time constant* parameter of the *associated ac equivalent* circuit. Hence the method is known as *short circuit time constant* method (SCTC). Figures 3.9(b)-(d) show the three *ac* equivalent circuits under the assumption of only one of C_1, C_2, or C_E present in the circuit. The location(s) to be used for the calculation of the equivalent *Thevenin* resistance for each of the capacitors (C_1, C_E, C_3) are shown in blue lines on the diagrams.

The internal capacitances of the BJT offer very high impedance at low frequencies and hence they are considered as open circuits (so these are not shown).

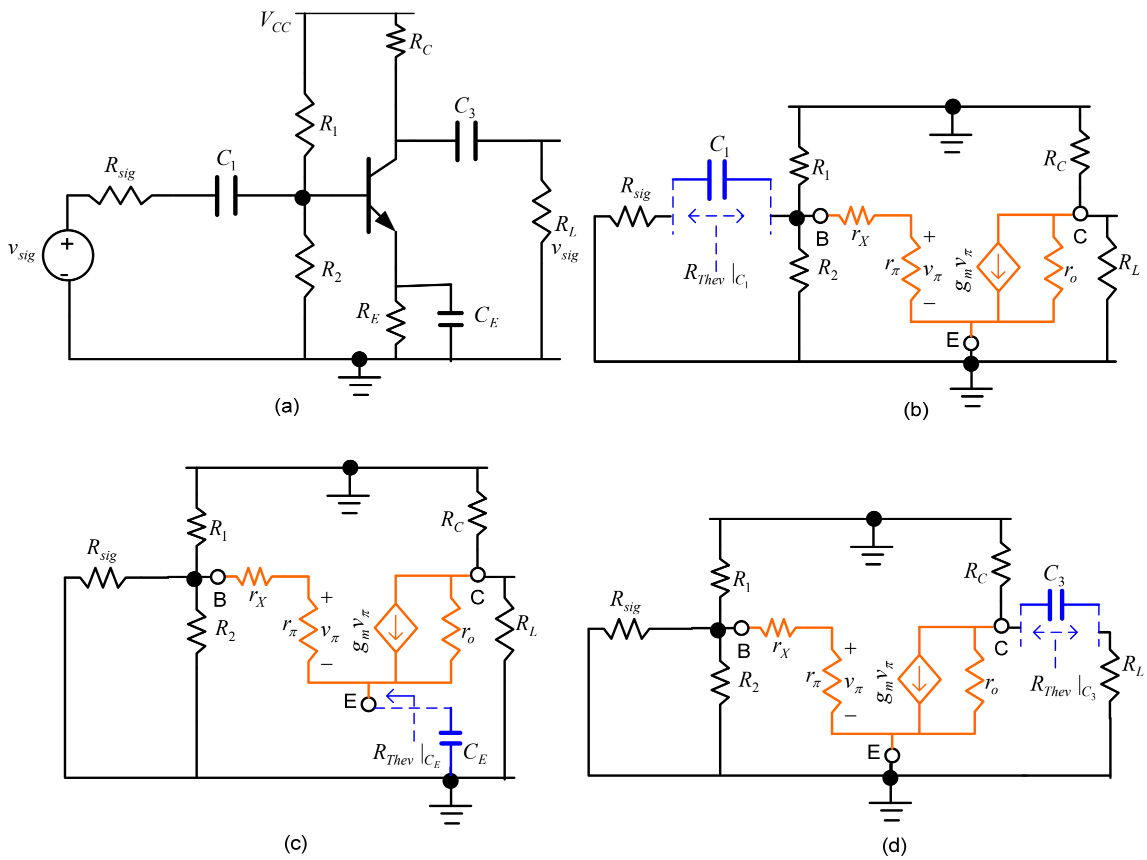


Figure 3.9: (a) Schematic of a CE amplifier with four resistor biasing; (b) the *ac* equivalent circuit with C_E ,C_3 as *short circuits*; (c) the *ac* equivalent circuit with C_1 ,C_3 as *short circuits*, and (d) the *ac* equivalent circuit with C_E ,C_1 as *short circuits*.

Analysis of the equivalent circuit in Fig.3.9(b) is straightforward. By inspection, the *Thevenin* resistance associated with C_1 is $R_{Th1} = R_{sig} + R_1 \parallel R_2 \parallel (r_x + r_\pi)$, where the notation $\parallel$ implies *in parallel with*. The associated time-constant is C_1R_{Th1}, and the corresponding *pole-frequency* is $\omega_{L1} = 1/(C_1R_{Th1})$. Similarly, the *Thevenin* resistance for C_3 is $R_{Th3} = R_C \parallel r_o + R_L$ (see Fig.3.9(d)). The corresponding *pole-frequency* is $\omega_{L3} = 1/(C_3R_{Th3})$. The calculation of the *Thevenin* resistance associated with C_E can be simplified considerably by assuming r_o as *inifinity*. Then by inspection (see Fig.3.9(c)), $R_{ThE} = R_E \parallel (\dfrac{r_\pi + r_x + R_1 \parallel R_2 \parallel R_{sig}}{1 + h_{fe}})$. The corresponding *pole-frequency* is ω_{LE} = $1/(C_ER_{ThE})$.

A more adventurous student may discard the assumption of r_o → *infinity* and proceed to set up a 3 by 3 nodal admittance matrix (NAM) (see *Appendix III*) by using the substitutions

$r_{\pi p} = r_\pi + r_x$, $R_{sp} = R_{sig} \| R_1 \| R_2$, $R_{cp} = R_c \| R_L$, and by inserting a *dummy* current source i_x at the node labeled as E in Fig. 3.9(c). The NAM will appear as

$$\begin{bmatrix} g_E + g_{\pi p} + g_o & -g_{\pi p} & -g_o \\ -g_{\pi p} & g_{\pi p} + g_{sp} & 0 \\ -g_o & 0 & g_o + g_{cp} \end{bmatrix} \begin{bmatrix} V_E \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_x + g_m v_\pi \\ 0 \\ -g_m v_\pi \end{bmatrix} \qquad (3.10)$$

In the above $g_E = 1/R_E$,, $g_{\pi p}=1/r_{\pi p}$, $g_o = 1/r_o$, and so on, have been used. With the further assumption (it is very good if r_x is << r_π) of $v_\pi = V_B - V_E$, the matrix equation (3.10), becomes, after rearrangement (i.e., changing sides):

$$\begin{bmatrix} g_E + g_{\pi p} + g_o + g_m & -g_{\pi p} - g_m & -g_o \\ -g_{\pi p} & g_{\pi p} + g_{sp} & 0 \\ -g_o - g_m & 0 + g_m & g_o + g_{cp} \end{bmatrix} \begin{bmatrix} V_E \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_x \\ 0 \\ 0 \end{bmatrix} \qquad (3.11)$$

Then R_{ThE} is given by V_E/i_x. The result is (using *Maple* program code):

$$R_{ThE} = \frac{(g_{\pi p} + g_{sp})(g_o + g_{cp})}{g_E g_{\pi p} g_o + g_E g_{\pi p} g_{cp} + g_E g_{sp} g_o + g_E g_{sp} g_{cp} + g_{\pi p} g_{sp} g_o + g_{\pi p} g_{sp} g_{cp} + g_o g_{\pi p} g_{cp} + g_o g_{sp} g_{cp} + g_m g_{sp} g_{cp}}$$

Now introducing the assumption $g_o \rightarrow 0$ (i.e., $r_o \rightarrow$ *infinity*), one will get

$$R_{ThE} = \frac{(g_{\pi p} + g_{sp}) g_{cp}}{g_E g_{\pi p} g_{cp} + g_E g_{sp} g_{cp} + g_{\pi p} g_{sp} g_{cp} + g_m g_{sp} g_{cp}} \qquad (3.12)$$

Substituting back in terms of the resistance notations, i.e., $g_E = 1/R_E$,, $g_{\pi p}=1/r_{\pi p}$, $g_o = 1/r_o$, and so on, one can get

$$R_{ThE} = \frac{(R_{sp} + r_{\pi p}) R_E}{R_{sp} + r_{\pi p} + R_E + g_m r_{\pi p} R_E} \qquad (3.13)$$

Using $g_m\, r_{\pi p} = h_{fe}$, and simplifying, one arrives at $R_{ThE} = \dfrac{(R_{sp} + r_{\pi p}) R_E /(1 + h_{fe})}{(R_{sp} + r_{\pi p})/(1 + h_{fe}) + R_E}$, i.e.,

$$R_{ThE} = R_E \| (\frac{R_{sp} + r_{\pi p}}{1 + h_{fe}}) = R_E \| (\frac{r_\pi + r_x + R_1 \| R_2 \| R_{sig}}{1 + h_{fe}}).$$

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The overall lower -3 dB frequency is calculated approximately by the formula $\omega_L = \omega_{L1} + \omega_{L3} + \omega_{LE}$. If out of the several poles of the low-frequency transfer function $F_L(s)$, one is very large compared to all other poles and zeros, the overall lower -3 dB frequency ω_L becomes $\cong$ dominant pole (i.e., largest of ω_{L1} or ω_{LE} or ω_{L3}).

If the numerical values of the various pole frequencies are known (by exact circuit analysis followed by numerical computation), the lower 3-dB frequency can be calculated approximately by a formula of the form $\omega_L = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2 + ...}$ where, ω_1, ω_2 , .. are the individual pole frequencies and the zero-frequencies are very small compared with the pole frequencies.

Example 3.4.1: Consider the following values in a BJT amplifier.
R_{sig} =50Ω, R_B=$R_1||R_2$ = 10 kΩ, r_π =2500, r_x =25Ω, h_{fe} =100 and R_E =1kΩ, R_C =1.5kΩ, R_L =3.3 kΩ, V_A =20 volts, $I_C \approx 1$ mA. Further, C_1 =1uF and C_E= 10uF and C_3 = 1uF. What is ω_L ?

According to above formulas, R_{Th1} = 2.05kΩ , R_{ThE} =25.25Ω and R_{Th3} =1.39kΩ+3.3kΩ = 4.69kΩ. Then ω_{L1} = 487.8 rad/s, ω_{LE} = 3.96E3 rad/sec and ω_{L3} = 213.2 rad/sec. Then , $\omega_L = \sqrt{487.8^2 + 3.96E3^2 + 213.2^2} = 3.9956E3$, which is pretty close to ω_{LE}.

Example 3.4.2: What if , ω_L= 1800 rad/sec is to be designed? We can assume, for example, ω_{L1} = $0.8\omega_L$, , ω_{LE}= ω_{L2}= $0.1\omega_L$ and ω_{L3}=$0.1\omega_L$. Then, design the values of the capacitors C_1, C_E and C_3. The student can try other relative allocations too.

3.5: Calculation of ω_H – the higher cut-off frequency

Several alternative methods exist in the literature. The following are presented.

3.5.1 <u>Open circuit time-constant (OCTC) method</u>

This is similar to the case as with low frequency response. For high frequency operation, we are interested in the capacitor which will have lower reactance value since this capacitance will start to degrade the high frequency response sooner than the other. Thus, we can consider one capacitor at a time and assume that the other capacitors are too small and have reasonably high reactance values (for a C, the reactance is ∝ 1/C) so that they could be considered as open circuits. We then calculate the associated time constant. Thus the method is named as *open circuit time constant* (OCTC) method. We shall illustrate the method using the case of a CE BJT amplifier.

Consider figure 3.10(a) which shows the *ac* equivalent circuit of the CE-BJT amplifier of Fig.3.9(a). The *high frequency* equivalent circuit for the BJT has been included. The coupling and by-pass capacitors are assumed as *short circuits* for *high frequency* situation.

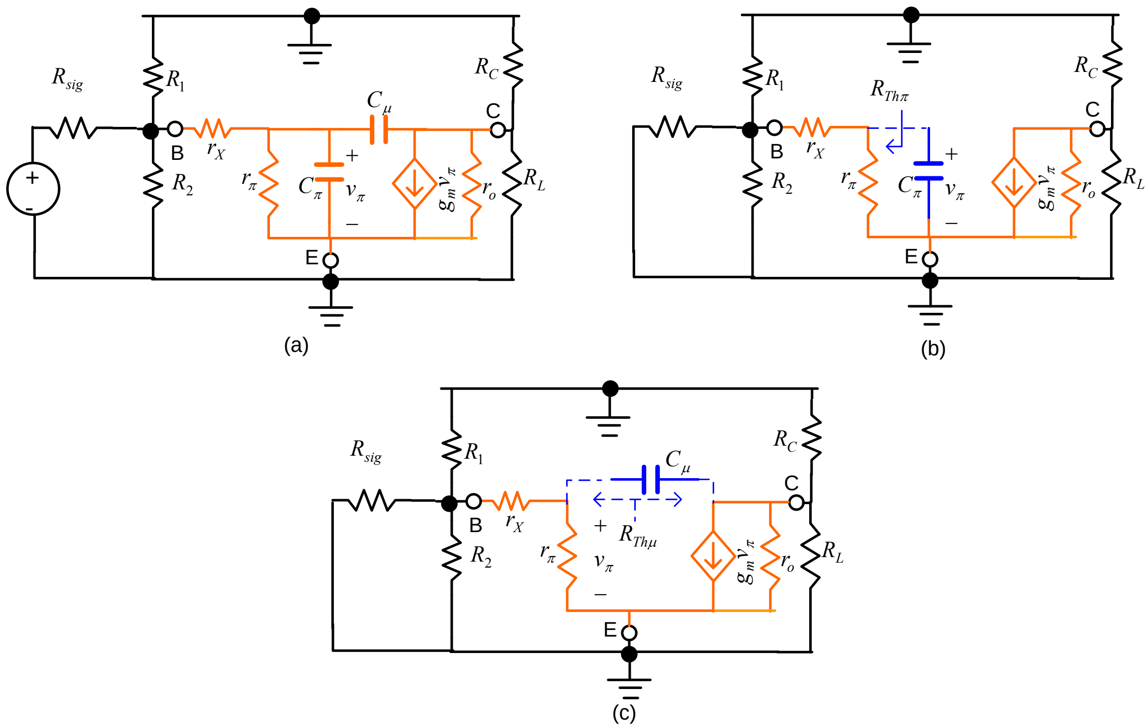

Figure 3.10: (a) high frequency equivalent circuit of the amplifier in Fig.3.9(a); (b) the equivalent circuit with C_μ open; (c) the equivalent circuit with C_π open.

Case 1: Cμ open

The ac equivalent circuit to determine the *Thevenin* equivalent resistance $R_{Th\pi}$ across C_π is shown in Fig.3.10(b). By simple inspection $R_{Th\pi} = r_\pi \| (r_x + R_{sig} \| R_1 \| R_2)$

The high frequency pole due to this situation is $\omega_{H1} = 1/(C_\pi R_{Th\pi})$.

Case 2: C_π open

We now need to determine the Thevenin equivalent resistance $R_{Th\mu}$ across C_μ. The associated equivalent circuit is shown in Fig.3.10(c). We can use a dummy signal current source i_x and carry out few steps of basic circuit analysis (see Fig.3.11).

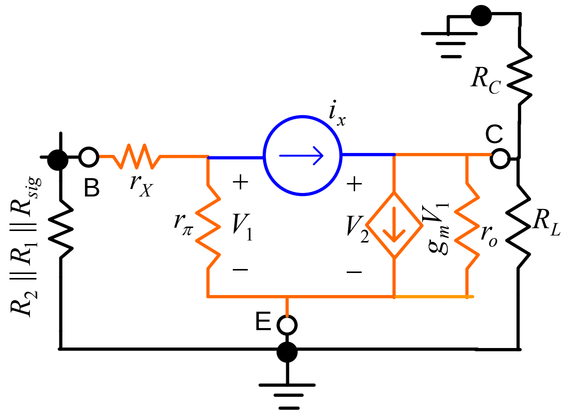

Figure 3.11: Equivalent circuit for calculating $R_{Th\mu}$

KCL at V_1 node gives: $V_1 G_S' + i_x = 0, \quad G_S' = \dfrac{1}{r_\pi \| (r_x + R_{sig} \| R_1 \| R_2)} = 1/R_S'$ (3.13a)

KCL at V_2 node gives: $-i_x + g_m V_1 + V_2 G_L' = 0, \quad G_L' = \dfrac{1}{r_o \| R_C \| R_L} = 1/R_L'$ (3.13b)

Solving (3.13(a),(b)) for V_1 and V_2 we can find $R_{Th\mu} = (V_2 - V_1)/i_x = R_s' + (1 + g_m R_s')R_L'$

The high frequency pole for C_μ is $\omega_{H2} = 1/\ C_\mu R_{Th\mu}$.

When the two pole frequencies are comparable in values, the upper -3 dB frequency is given approximately by $\omega_H = 1/(C_\pi R_{Th\pi} + C_\mu R_{Th\mu}) = 1/(\tau_1 + \tau_2)$ (3.14)

If, however, the two values are widely apart (say, by a factor of 5 or more), the upper -3 dB frequency will be called as the *dominant high frequency pole* and will be equal to the lesser of ω_{H1} and ω_{H2}.