Q.1: (Sol/hints)

Consider Early effect for both M1 and M2 (for best accuracy)

```
> VA:=25.;
```

$$VA := 25.$$

```
> Vo:=3.;
```

$$Vo := 3.$$

```
> Kn:=100.;
```

$$Kn := 100.$$

```
> L:=1.;
```

$$L := 1.$$

```
> W1:=2.;
```

$$W1 := 2.$$

```
> W2:=4.;
```

$$W2 := 4.$$

```
> Vth:=0.6;
```

$$Vth := .6$$

```
> Io:=50.;
```

$$Io := 50.$$

```
> y1:=Io-Kn*(W2/(2*L))*Vov^2*(1+Vo/VA);
```

$$y1 := 50. - 224.0000000\, Vov^2$$

```
> solve(y1,Vov);
```

$$-.4724555913,\ .4724555913$$

```
> Vov1:=.4724555913;
```

$$Vov1 := .4724555913$$

```
> VG2:=Vth+Vov1;
```

$$VG2 := 1.072455591$$

```
> VGS1:=VG2;
```

$$VGS1 := 1.072455591$$

```
> VDS1:=VGS1;
```

$$VDS1 := 1.072455591$$

```
> x1:=1+VDS1/VA;
```

$$x1 := 1.042898224$$

```
> x2:=1+Vo/VA;
```

$$x2 := 1.120000000$$

```
> IREF:=(x1/x2)*(Io/2);
```

$$IREF := 23.27897822$$

```
> R:=1E6*(5-VG2)/IREF;
```

$$R := 168716.3574$$

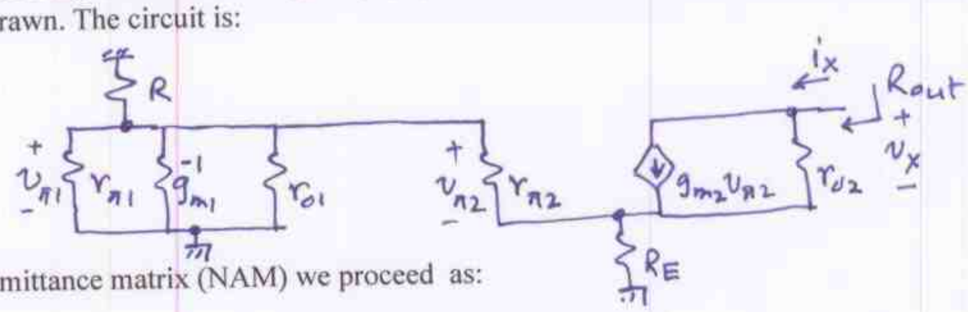
R=168.72 k ohms

Q.2: (Sol/hint)

We need to add a series resistance with the internal resistance of the transistor to divide the 50mV into 3mV +47mV.
For each BJT, 're' =25/.5 =50 ohms. If RE is added to each emitter leg,
(RE+re)/re =25/3. This gives RE

Show the schematic with RE included in the emitter leg. An alternative could be to add (beta+1)*RE in series with the base of each BJT.
The voltage gain will be the gain of a single CE amplifier with emitter resistance RE. It is approximately =-alpha*RC/(re+RE), where alpha =beta/(beta+1). Note the minus sign!

Q.3: (Sol/hint)
The Rout to be investigated is a 'small signal' (ac) quantity. So the ac equivalent circuit for the BJTs will have to be drawn. The circuit is:



For the nodal admittance matrix (NAM) we proceed as:

$$\begin{bmatrix} g_{o2} & -g_{o2} & 0 \\ -g_{o2} & g_E+g_{o2}+g_{\pi2} & -g_{\pi2} \\ 0 & -g_{\pi2} & g_{\pi2}+g_{o1}+g_{\pi1}+g_R \end{bmatrix}\begin{bmatrix}V_x\\V_2\\V_3\end{bmatrix} = \begin{bmatrix} i_x - g_{m2}v_{\pi2} \\ g_{m2}v_{\pi2} \\ -g_{m1}v_{\pi1}\end{bmatrix}$$

Noting $v_{\pi2}=V_3-V_2$, $v_{\pi1}=V_3$, we get the NAM

$$\begin{bmatrix} g_{o2} & -g_{m2}-g_{o2} & g_{m2} \\ -g_{o2} & g_E+g_{o2}+g_{\pi2}+g_{m2} & -g_{\pi2}-g_{m2} \\ 0 & -g_{\pi2} & g_{\pi2}+g_{o1}+g_{\pi1}+g_R+g_{m1} \end{bmatrix}\begin{bmatrix}V_x\\V_2\\V_3\end{bmatrix} = \begin{bmatrix} i_x \\ 0 \\ 0\end{bmatrix}$$

```
>
```

The Rout is =Vx/ix. Solve by Krammer's rule:

$$V_x = \frac{\begin{vmatrix} i_x & -g_{m2}-g_{o2} & g_{m2} \\ 0 & g_E+g_{o2}+g_{\pi2}+g_{m2} & -g_{\pi2}-g_{m2} \\ 0 & -g_{\pi2} & g_{\pi2}+g_{o1}+g_{\pi1}+g_R+g_{m1} \end{vmatrix}}{\begin{vmatrix} g_{o2} & -g_{m2}-g_{o2} & g_{m2} \\ -g_{o2} & g_E+g_{o2}+g_{\pi2}+g_{m2} & -g_{\pi2}-g_{m2} \\ 0 & -g_{\pi2} & g_{\pi2}+g_{o1}+g_{\pi1}+g_R+g_{m1} \end{vmatrix}}$$

```
> x1:=gE+go2+gp2+gm2;
```

$$x1 := gE + go2 + gp2 + gm2$$

```
> x2:=gp2+go1+gp1+gR+gm1;
```

$$x2 := gp2 + go1 + gp1 + gR + gm1$$

```
> x3:=-gp2-gm2;
```

$$x3 := -gp2 - gm2$$

```
> x4:=-gp2;
```

$$x4 := -gp2$$

```
> z1:=x1*x2-x3*x4;
```

$$z1 := (gE + go2 + gp2 + gm2)\,(gp2 + go1 + gp1 + gR + gm1) + (-gp2 - gm2)\,gp2$$

```
> simplify(z1);
```

$$gE\,gp2 + gE\,go1 + gE\,gp1 + gE\,gR + gE\,gm1 + go2\,gp2 + go2\,go1 + go2\,gp1 + go2\,gR + go2\,gm1 + gp2\,go1 + gp2\,gp1 + gp2\,gR + gp2\,gm1 + gm2\,go1 + gm2\,gp1 + gm2\,gR + gm2\,gm1$$

```
> x5:=-(gm2+go2)*x2+gp2*gm2;
```

$$x5 := -(gm2 + go2)\,(gp2 + go1 + gp1 + gR + gm1) + gm2\,gp2$$

```
> simplify(x5);
```

$$-gm2\,go1 - gm2\,gp1 - gm2\,gR - gm2\,gm1 - go2\,gp2 - go2\,go1 - go2\,gp1 - go2\,gR - go2\,gm1$$

```
> x6:=-gm2*go1-gm2*gp1-gm2*gR-gm2*gm1-go2*gp2-go2*go1-go2*gp1-go2*gR
  -go2*gm1;
```

$$x6 := -gm2\,go1 - gm2\,gp1 - gm2\,gR - gm2\,gm1 - go2\,gp2 - go2\,go1 - go2\,gp1 - go2\,gR - go2\,gm1$$

```
> z2:=go2*(z1+x6);
```

$$z2 := go2\,\big((gE + go2 + gp2 + gm2)\,(gp2 + go1 + gp1 + gR + gm1) + (-gp2 - gm2)\,gp2 - gm2\,go1 - gm2\,gp1 - gm2\,gR - gm2\,gm1 - go2\,gp2 - go2\,go1 - go2\,gp1 - go2\,gR - go2\,gm1\big)$$

```
> simplify(z2);
```

$$go2\,(gE\,gp2 + gE\,go1 + gE\,gp1 + gE\,gR + gE\,gm1 + gp2\,go1 + gp2\,gp1 + gp2\,gR + gp2\,gm1)$$

```
> z3:=go2*(gE*gp2+gE*go1+gE*gp1+gE*gR+gE*gm1+gp2*go1+gp2*gp1+gp2*gR+
  gp2*gm1);
```

$$z3 := go2\,(gE\,gp2 + gE\,go1 + gE\,gp1 + gE\,gR + gE\,gm1 + gp2\,go1 + gp2\,gp1 + gp2\,gR + gp2\,gm1)$$

Rout=Vx/ix. Thus:

```
> Rout:=z1/z3;
```

$$Rout := \frac{(gE + go2 + gp2 + gm2)\,(gp2 + go1 + gp1 + gR + gm1) + (-gp2 - gm2)\,gp2}{go2\,(gE\,gp2 + gE\,go1 + gE\,gp1 + gE\,gR + gE\,gm1 + gp2\,go1 + gp2\,gp1 + gp2\,gR + gp2\,gm1)}$$

```
>
```

Q.4 (Sol/hint)

We can use the half-circuit concept by dividing Rs into two halves with an 'ac' ground at the center point. Thus each half will be like a CS MOS amplifier with a resistance Rs/2 at the source terminal.

From the result of a CS amplifier with Rx (=Rs/2) at the source end, the voltage gain is:

$$G = \frac{g_m R_D}{1 + g_m (R_s/2)}$$

Thus $A = g_m R_D$. This is the answer to part (a) of the question.

For part(b) of the question, we solve: $g_m R_D / 2 = \dfrac{g_m R_D}{1 + g_m R_S / 2}$

```
> yx:=gm*RD/2-(gm*RD)/(1+gm*Rs/2);
```

$$yx := \frac{1}{2} gm\ RD - \frac{gm\ RD}{1 + \frac{1}{2} gm\ Rs}$$

```
> solve(yx,Rs);
```

$$2 \frac{1}{gm}$$

Rs=2/gm is the solution.