$$R_{ThE} = \frac{(g_{\pi p} + g_{sp})g_{cp}}{g_E g_{\pi p} g_{cp} + g_E g_{sp} g_{cp} + g_{\pi p} g_{sp} g_{cp} + g_m g_{sp} g_{cp}} \qquad (3.12)$$

Substituting back in terms of the resistance notations, i.e., $g_E = 1/R_E$, $g_{\pi p} = 1/r_{\pi p}$, $g_o = 1/r_o$, and so on, one can get

$$R_{ThE} = \frac{(R_{sp} + r_{\pi p})R_E}{R_{sp} + r_{\pi p} + R_E + g_m r_{\pi p} R_E} \qquad (3.13)$$

Using $g_m\, r_{\pi p} = h_{fe}$, and simplifying, one arrives at $R_{ThE} = \frac{(R_{sp} + r_{\pi p})R_E/(1+h_{fe})}{(R_{sp} + r_{\pi p})/(1+h_{fe}) + R_E}$, i.e.,

$$R_{ThE} = R_E \,\|\, (\frac{R_{sp} + r_{\pi p}}{1+h_{fe}}) = R_E \,\|\, (\frac{r_\pi + r_x + R_1 \| R_2 \| R_{sig}}{1+h_{fe}}) .$$

****

The overall lower -3 dB frequency is calculated approximately by the formula $\omega_L = \omega_{L1} + \omega_{L3} + \omega_{LE}$. If out of the several poles of the low-frequency transfer function $F_L(s)$, one is very large compared to all other poles and zeros, the overall lower -3 dB frequency ω_L becomes $\cong$ dominant pole (i.e., largest of ω_{L1} or ω_{LE} or ω_{L3}).

If the numerical values of the various pole frequencies are known (by exact circuit analysis followed by numerical computation), the lower 3-dB frequency can be calculated approximately by a formula of the form $\omega_L = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2 + ...}$ where, ω_1, ω_2 , .. are the individual pole frequencies and the zero-frequencies are very small compared with the pole frequencies.

Example 3.4.1: Consider the following values in a BJT amplifier.

R_{sig} =50Ω, R_B=R_1||R_2 = 10 kΩ, r_π =2500, r_x =25Ω, h_{fe} =100 and R_E =1kΩ, R_C =1.5kΩ, R_L =3.3 kΩ, V_A =20 volts, $I_C \approx 1$ mA. Further, C_1 =1uF and C_E= 10uF and C_3 = 1uF. What is ω_L ?

According to above formulas, R_{Th1} = 2.05kΩ , R_{ThE} =25.25Ω and R_{Th3} =1.39kΩ+3.3kΩ = 4.69kΩ. Then ω_{L1} = 487.8 rad/s, ω_{LE} = 3.96E3 rad/sec and ω_{L3} = 213.2 rad/sec. Then , $\omega_L \approx \sqrt{\omega_{L1}^2 + \omega_{LE}^2 + \omega_{L3}^2} = 3995.62$ rad/s, which is pretty close to ω_{LE}.

For nodal admittance matrix (NAM) formulation we need to transform the voltage source with its internal resistance to its Norton equivalent, i.e., a signal current source $i_S = {}^{v_{sig}}\!/_{R_{sig}}$ in parallel with R_{sig} (the student is encouraged to complete this part to modify Fig.3.15(b)). The NAM by inspection will be:

$$\begin{bmatrix} g_{sig}+g_E+g_\pi+g_o+sC_\pi & -g_\pi-sC_\pi & -g_o \\ -g_\pi-sC_\pi & g_x+g_\pi+sC_\pi+sC_\mu & -sC_\mu \\ -g_o & -sC_\mu & g_o+g'_C+sC_\mu \end{bmatrix}\begin{bmatrix} v_E \\ v_{B'} \\ v_C \end{bmatrix}=\begin{bmatrix} i_S+g_m v_\pi \\ 0 \\ -g_m v_\pi \end{bmatrix} \quad (3.23)$$

But $v_\pi = v_{B'} - v_E$. Hence (3.23) reduces to

$$\begin{bmatrix} g_{sig}+g_E+g_\pi+g_o+sC_\pi+g_m & -g_\pi-sC_\pi-g_m & -g_o \\ -g_\pi-sC_\pi & g_x+g_\pi+sC_\pi+sC_\mu & -sC_\mu \\ -g_m-g_o & g_m-sC_\mu & g_o+g'_C+sC_\mu \end{bmatrix}\begin{bmatrix} v_E \\ v_{B'} \\ v_C \end{bmatrix}=\begin{bmatrix} i_S \\ 0 \\ 0 \end{bmatrix} \quad (3.24)$$

In order to compare the performance of the CB and CE BJT amplifiers, it will be convenient to assume r_X as negligible. Then node *B'* will be at *zero* ac potential and (3.24) will approximate to (by discarding the *row* and *column* associated with the ***B'*** node)

$$\begin{bmatrix} g_{sig}+g_E+g_\pi+g_o+sC_\pi+g_m & -g_o \\ -g_m-g_o & g_o+g'_C+sC_\mu \end{bmatrix}\begin{bmatrix} v_E \\ v_C \end{bmatrix}=\begin{bmatrix} i_S \\ 0 \end{bmatrix} \quad (3.25)$$

The VTF is given by

$v_o/v_{sig} = v_C/v_{sig} =$

$$\frac{g_{sig}(g_m+g_o)}{s^2C_\pi C_\mu + s[C_\pi(g_o+g'_C)+C_\mu(g_\pi+g_E+g_{sig}+g_o+g_m)]+g_o(g_{sig}+g_\pi+g_E+g'_C)+g'_C(g_{sig}+g_E+g_o+g_\pi+g_m)}$$

Using the VTF we can deduce

- The mid-band gain (i.e., $s \rightarrow j\omega$, $\omega \rightarrow 0$) for CB mode of operation is:

$$A_M\,|_{CB} = \frac{g_{sig}(g_m+g_o)}{g_o(g_{sig}+g_\pi+g_E+g'_C)+g'_C(g_{sig}+g_E+g_o+g_\pi+g_m)} \quad (3.26)$$

- The dominant high frequency pole for CB mode of operation is

$$\omega_{HD}|_{CB} = \frac{g_o(g_{sig}+g_\pi+g_E+g'_C)+g'_C(g_{sig}+g_E+g_o+g_\pi+g_m)}{C_\pi(g_o+g'_C)+C_\mu(g_\pi+g_E+g_{sig}+g_o+g_m)} \quad (3.27)$$

Example 3.5.3.3: Determine the mid-band gains and dominant high frequency poles of the BJT amplifier operated as CE (see Fig.3.9a) and CB (see Fig.3.15a) modes of

Example 3.5.3.3: Determine the mid-band gains and dominant high frequency poles of the BJT amplifier operated as CE (see Fig.3.9a) and CB (see Fig.3.15a) modes of operation. The devices, the DC biasing conditions, and the circuit components are assumed identical for both the configurations.

[Given R_{sig} =50Ω, R_1=82 kΩ, R_2=47 kΩ, R_E=270 Ω, R_C =2.7 kΩ, R_L=4.7 kΩ

g_m =40 m mhos, r_o=50 kΩ, r_X =10Ω ,$h_{fe}(0)$=h_{FE} = 49

C_π =1.2 pF, C_μ =0.1 pF]

On substitution into the expressions (3.26) and (3.27), we get: A_M =20.99 v/v,

$\omega_{HD}|_{\mathrm{CB}}=5.3117\times10^9$ rad/sec.

Note I*:* We observed that the CB-BJT amplifier has somewhat *lower voltage gain* (i.e., 20.99) compared with that of CE-BJT (i.e., 63.12 in magnitude) amplifier, but has a *higher high-frequency bandwidth* (~ dominant high-frequency pole=4.5796×10^9 rad/sec) compared with that of CE-BJT (i.e., 1.6158×10^9 rad/sec) amplifier.

Note II*:* It is known that the CE-BJT amplifier as moderate to high (kΩ to tens of kΩ) input resistance, as well as moderately high (kΩ) output resistance. In comparison, a CB-BJT amplifier has low (Ω to tens of Ω) input resistance while a moderately high (kΩ) output resistance.

Note III: A CB-BJT is preferred over a CE-BJT amplifier for most *radio-frequency* (MHz and above) applications because it has a *higher* high-frequency bandwidth (for identical device and biasing conditions), and affords to provide better impedance matching at the input with the *radio-frequency* (RF) source (R_{sig} in the range of 50Ω to 75Ω).

Example 3.5.3.4: Derivation of the voltage gain transfer function (VTF) *of a* CS-MOSFET *amplifier with active load*

Consider figures 3.16(a)-(b) which depict respectively the schematic of a CS-MOSFET amplifier and the associated high frequency equivalent circuit.

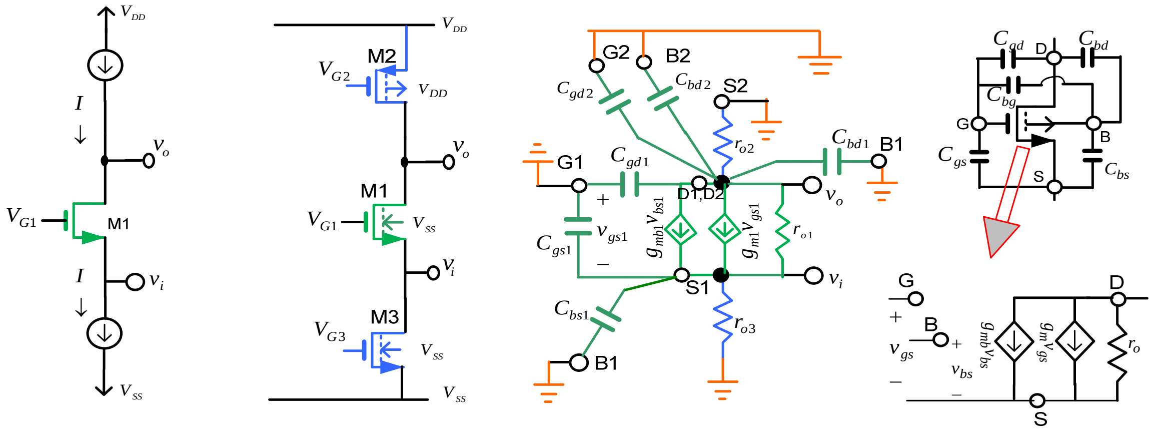

Figure 3.17: (a) Schematic of the CG-MOSFET amplifier with ideal load and bias source, (b) practical biasing and active load arrangement, (c) high-frequency equivalent circuit model for the CG-MOSFET stage, (d) more complete high-frequency model for a MOSFET.

transconductance g_{mb} of the MOSFET, since the topology is such (i.e., the amplifying device is in a *totem pole* connection) that the source and body terminals of the amplifying transistor M1 cannot be connected together!

It may be noted that despite the complicated look of Fig.3.17(c), there are only two signal nodes in the equivalent circuit. Thus the nodal admittance matrix will appear as:

$$\begin{bmatrix} g_{o3}+g_{o1}+s(C_{gs1}+C_{bs1}+C_{gd3}+C_{bd3}) & -g_{o1} \\ -g_{o1} & g_{o1}+g_{o2}+s(C_{bd1}+C_{gd1}+C_{bd2}+C_{gd2}) \end{bmatrix}\begin{bmatrix} v_i \\ v_o \end{bmatrix}=\begin{bmatrix} i_x+g_{m1}v_{gs1}+g_{mb1}v_{bs1} \\ -g_{m1}v_{gs1}-g_{mb1}v_{bs1} \end{bmatrix}$$

Writing $C_1=C_{gs1}+C_{bs1}+C_{gd3}+C_{bd3}$, $C_2=C_{gd1}+C_{bd1}+C_{gd2}+C_{bd2}$, and observing that the gate and body terminals of the MOSFETs are at *zero* ac potentials so that $v_{gs1}=-v_{s1}=-v_i$, $v_{bs1}=-v_{s1}=-v_i$, we can re-write the admittance matrix equation as

$$\begin{bmatrix} g_{o3}+g_{o1}+sC_1 & -g_{o1} \\ -g_{o1} & g_{o1}+g_{o2}+sC_2 \end{bmatrix}\begin{bmatrix} v_i \\ v_o \end{bmatrix}=\begin{bmatrix} i_x-g_{m1}v_i-g_{mb1}v_i \\ g_{m1}v_i+g_{mb1}v_i \end{bmatrix} \quad (3.30)$$

Re-arranging, we get

$$\begin{bmatrix} g_{o3}+g_{o1}+g_{m1}+g_{mb1}+sC_1 & -g_{o1} \\ -g_{m1}-g_{mb1}-g_{o1} & g_{o1}+g_{o2}+sC_2 \end{bmatrix}\begin{bmatrix} v_i \\ v_o \end{bmatrix}=\begin{bmatrix} i_x \\ 0 \end{bmatrix} \quad (3.31)$$

The VTF is given by:

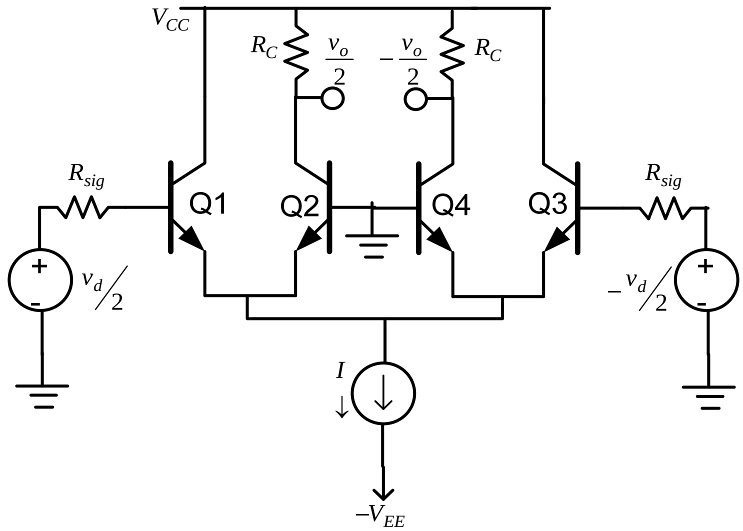

Figure 3.26: A CC-CB cascade doublet wide band differential amplifier.

The amplifier in Fig.3.26 will provide wide band operation with good voltage gain. But there are four transistors which need be supplied with DC bias current. The DC power consumption will be high. A *totem-pole* configuration where two transistors are stacked in one column will result in a lower DC power consumption. This is possible by using complementary (i.e., NPN and PNP) transistors to form the CC-CB cascade. Figure 3.27 presents the configuration.

3.6.3.3: CC-CB cascade doublet differential amplifier with complementary transistors

In the configuration of Fig.3.27 DC bias current is to be supplied only to two columns of transistors. So compared with Fig.3.26, the complementary CC-CB cascade doublet differential amplifier (Fig.3.27) will consume less DC power.

- *Analysis for gain bandwidth*

A simplified analysis for the band width of the differential amplifier can be carried out on the assumption that the NPN and the PNP transistors are matched pairs (it is *seldom true* in practice) and then working on one half circuit of the system. Thus considering the pair

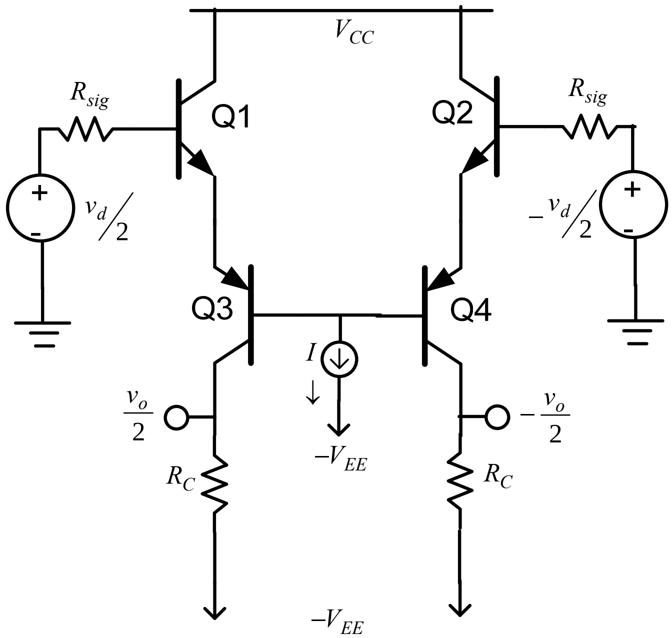

Figure 3.27: Schematic of complementary CC-CB cascode doublet differential amplifier

Q1,Q3 we can construct the high frequency equivalent circuit as shown in figure 3.28. We have ignored[2] r_x and r_o to further simplify the analysis.

We can now rearrange the direction of one of the controlled current sources by reversing the direction of the controlling voltage v_π. This leads to figure 3.29(a). Figure 3.29(b) reveals further simplification by combining the two identical parallel C_π , r_π circuits which occur in series connection. The current $g_m v_\pi$ coming *toward* and *leaving* from the node marked E1,E2 can be lifted off from this junction and has been shown as a single current $g_m v_\pi$ meeting the node C2.

[2] A *rule –of- thumb* regarding such simplifications is: a resistance in *series* connection can be neglected as *short circuit* if it is *small* compared with other resistances. Similarly, a resistance in *shunt* connection can be neglected *as an open circuit* if it is *high* compared with other resistances.

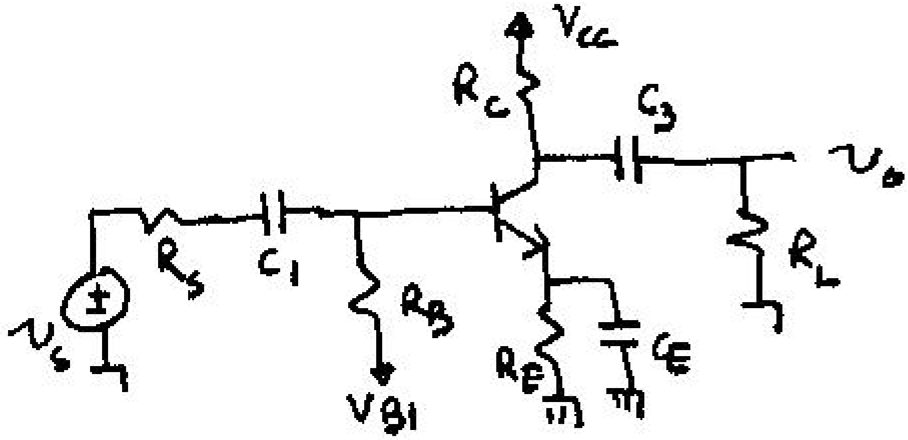

Figure for Problem# 3.7.2

3.7.3: In a BJT, CE amplifier the network parameters are: R_S= 100 ohms, R_B=1 k ohms, r_x= 50 ohms, I_C= 1 mA, h_{fe}= 99, C_π=1.2 pF, C_μ=0.1 pF, V_A=50 V, R_C=1.5 k ohms. Find,

(a) The time constants associated with the capacitors using open-circuit time constant method.
(b) What is the approximate upper cut-off frequency?
(c) In the equivalent circuit of the amplifier use Miller's theorem assuming a gain of $-g_mR_C$ between the internal collector and base terminals of the BJT, and re-draw the equivalent circuit.
(d) Determine the pole frequencies in the equivalent circuit derived in step (c) above.
(e) What will be the approximate upper cut-off frequency using the results in (d)?
(f) Use the full transfer function determination method to the equivalent circuit of the CE amplifier and determine the pole frequencies using exact solution of *D(s)=0*, where the voltage gain function is : *N(s)/D(s).*
(g) Determine the 'dominant' pole from the *D(s)* derived in step (f) above.
(h) What are your estimates about the upper cut-off frequencies if you use the results in (f) and (g)?
(i) Tabulate the upper cut-off frequency values obtained in steps (b), (e), (f), and (g).

3.7.4: In a MOSFET amplifier, you are given the following: R_S=100 ohms, C_{gs}=0.1 pF, C_{gd}=20 fF, g_m =50 μ mho, I_{DC}=50 μA, V_A=20 V, and R_L= 5 k ohms. The MOSFET amplifier is configured to operate as CS amplifier. Find the dominant high-frequency pole of the amplifier using:

(a) Miller's theorem
(b) Full nodal analysis
(c) Open-circuit time constant method

3.7.5: Consider a basic MOSFET current mirror circuit. Find an expression for the high frequency current transfer function $i_o(s)/i_{in}(s)$. Use the high frequency ac equivalent circuit model for the transistors.

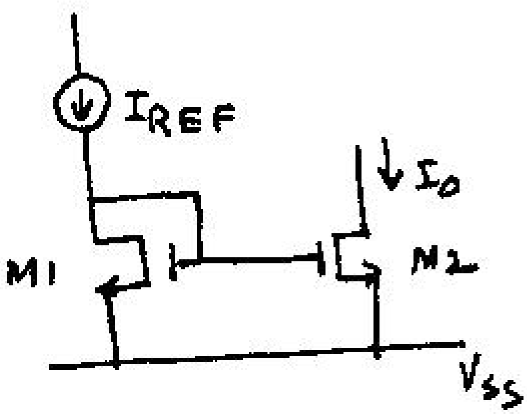

Figure for Problem # 3.7.5

3.7.6: For the BJT amplifier shown below, determine the high frequency voltage gain transfer function in the form: $A(s) = A_M \frac{\omega_H}{s+\omega_H}$. Given C_μ =0.5 pF, f_T=600 MHz, h_{fe}=49.

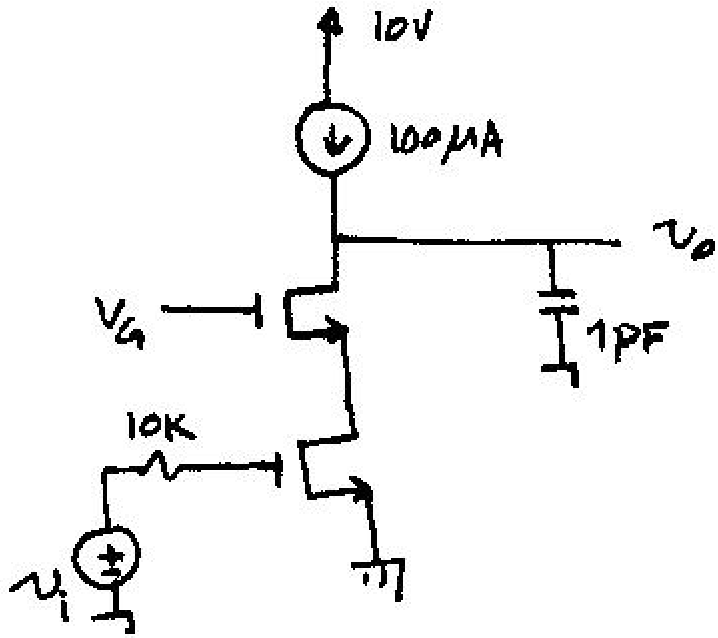

Figure: P 3.7.8