Course		Number	Section
Electronics I I		ELEC 312	F
Examination	Date	Time	# of pages
Mid-Term Test	October 21, 2012	75 minutes	3
Instructor(s)			
Dr. R. Raut			

Materials allowed: [X] No [ ] Yes (Please specify)

Calculators allowed: [ ] No [X] Yes

NO formula sheet is allowed.
ONLY ENCS approved calculator is allowed.

Special Instructions:

Write your ID# on the front page of your answer book
Attempt all questions.
Show all steps clearly in neat and legible handwriting.
Students are required to return question paper together with exam booklet(s).

****** **Some useful formulae** ******

BJT:

$r_e = \alpha / g_m \quad g_m = I_C / V_T \quad r_\pi = h_{fe} / g_m \quad h_{fe} = i_c / i_b \quad r_o = V_A / I_C$

$\alpha = \dfrac{h_{fe}}{h_{fe}+1} \qquad V_T = \dfrac{kT}{q} \approx 25mV$ at room temperature

DIODE: $I = I_s \, exp(v_{BE}/V_T)$

MOSFET: $I_D = \mu C_{ox} \dfrac{W}{2L}(V_{GS} - V_{TH})^2$,(ignoring channel modulation effect, and assuming that the MOSFET is working in the saturation region)

$$g_m = \sqrt{\frac{2\mu C_{ox} W I_D}{L}}$$

Q.1: Consider the amplifier in Figure 1, built with MOS transistors. The transistor M3 functions as an active load for the transistors M1 and M2. M1 is a common source stage while M2 is a common gate stage.

(a) Draw the small signal ac equivalent circuit for the system. Ignore the *body effect* for M2.
(b) Produce the nodal admittance matrix for the system with a goal to find the small signal voltage gain v_o/v_i. You <u>*do not* have to calculate</u> the voltage gain.

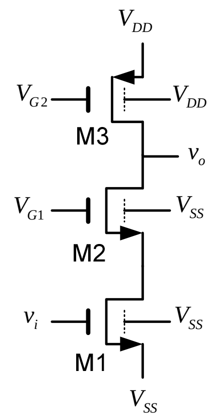


Figure 1:

Q.2: Figures 2(a)-(b) show the *schematics* of a basic and an improved current mirror respectively using MOSFET devices.

(a) Draw the pertinent *ac equivalent* circuits for the two circuits.

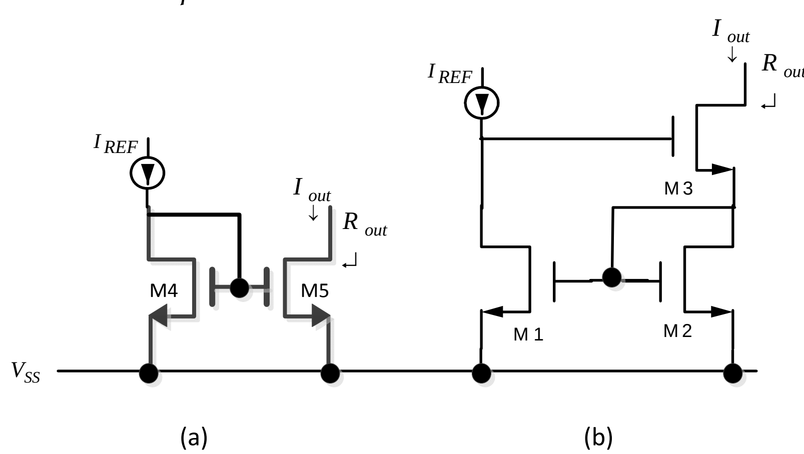


Figure 2:

Given (for all transistors), μC_{ox} = 100 μA/V^2, W/L=10, V_{SS} =-5 V, V_{TH} =0.8 V, V_{DD} = 5 V, V_{GS}-V_{TH} = 1 V, and V_A=20 V. Approximate analysis gives the output resistance of the mirror in Fig.2(b) as $R_{out} = g_m r_o^2$, where r_o is the output resistance of each MOSFET device.

(b) Compare the numerical values of the output resistances of the mirrors in Figs. 2(a) and 2(b)

Q.3: The BJT differential amplifier in Figure 3 is supplied with a differential ac signal $v_D = v_1 - v_2$. The differential output signal is given by the expression

$$v_{o1} - v_{o2} = R_c I\left[\frac{\exp(-v_D / V_T)}{1+\exp(-v_D / V_T)} - \frac{\exp(v_D / V_T)}{1+\exp(v_D / V_T)}\right]$$

Where, V_T is the thermal voltage (~ 25 mV). The bias current I is arranged to be 5 mA

Determine the voltage gain $(v_{o1}-v_{02})/v_D$, when (i) v_D=10 mV and (ii) v_D= 1 mV. How do these compare with the theoretical small signal voltage gain value of $|g_m R_C|$, where g_m is the trans-conductance of each BJT device, and R_C is the resistance at the collector.

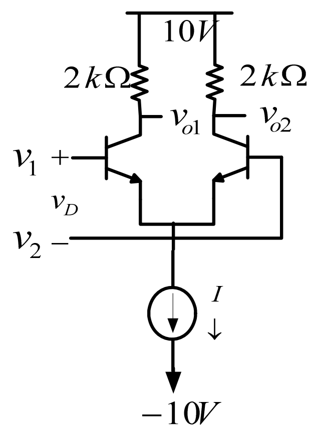


Figure 3:

Q.4. Figure 4 illustrates an implementation of a differential amplifier with active load using complementary BJT devices. The bias current I_{EE} is 2 mA. The early voltages are :

V_{AN} (for NPN BJT) =25 V, V_{AP} (for PNP BJT)=50 V.

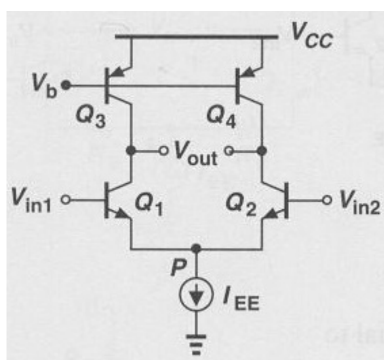


Figure 4:

(a) Draw the ac equivalent circuit for the differential amplifier. Assume that the source I_{EE} has an *ac* resistance of R_I.

(b) What will be the differential voltage gain $V_{out}/(V_{in1} - V_{in2})$ when V_{in1} and V_{in2} are *balanced differential* signals?

ELEC 312/2-F, Fall 2012, MT test soln.

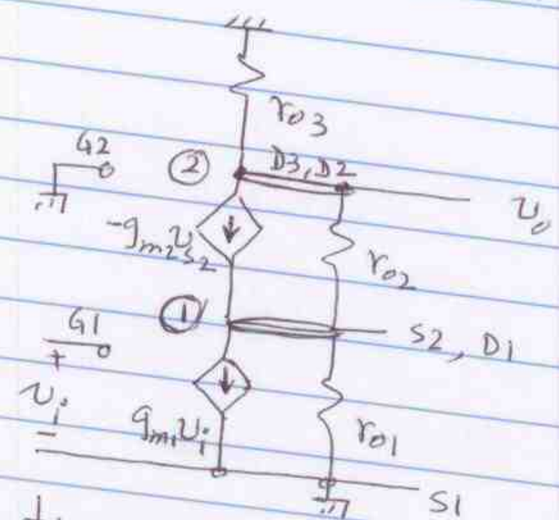
4/6, 1/7

Q1.

(a) M3 has source and gate at DC (= 0 ac). So it acts as a current-source active load = r_{o3}.

M2 has gate at DC. So the $g_{m2} v_{gs2}$ source becomes $g_{m2}(0 - v_{s2}) = -g_{m2} v_{s2}$

The ac equivalent circuit is:



soln

(b) This is a two node system ~~with~~ v_i is input node. Since v_i does not have any component attached, we will exclude it in formulating the nodal admittance matrix (NAM). By inspection (and letting $g = \frac{1}{r}$)

$$\begin{array}{cc} \begin{bmatrix} g_{o1} + g_{o2} & -g_{o2} \\ -g_{o2} & g_{o2} + g_{o3} \end{bmatrix} \\ \quad ① \qquad\qquad ② \end{array} \begin{bmatrix} v_{①} \\ v_{②} \end{bmatrix} = \begin{bmatrix} -g_{m1}v_i - g_{m2}v_{s2} \\ g_{m2}v_{s2} \end{bmatrix}$$

But $v_{s2} = v_{①}$. Substituting and moving on left side (i.e. $g_{m2}v_{s2} = g_{m2}v_{①} \rightarrow -g_{m2}v_{①}$ on left)

$$\begin{bmatrix} g_{m2} + g_{o1} + g_{o2} & -g_{o2} \\ -g_{m2} - g_{o2} & g_{o2} + g_{o3} \end{bmatrix} \begin{bmatrix} v_{①} \\ v_{②} \end{bmatrix} = \begin{bmatrix} -g_{m1}v_i \\ 0 \end{bmatrix}$$

soln.

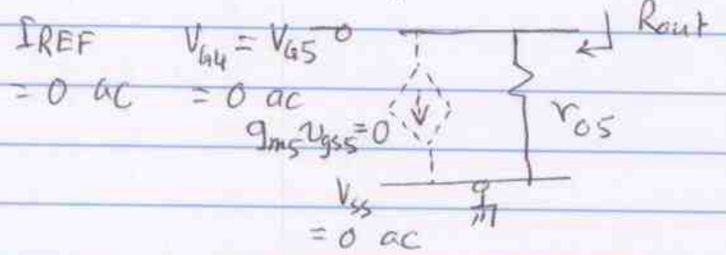
Q2:

For Fig. 2(a), it is a basic current mirror made from M4, M5. R_{out} for M5 is simply r_o of M5.

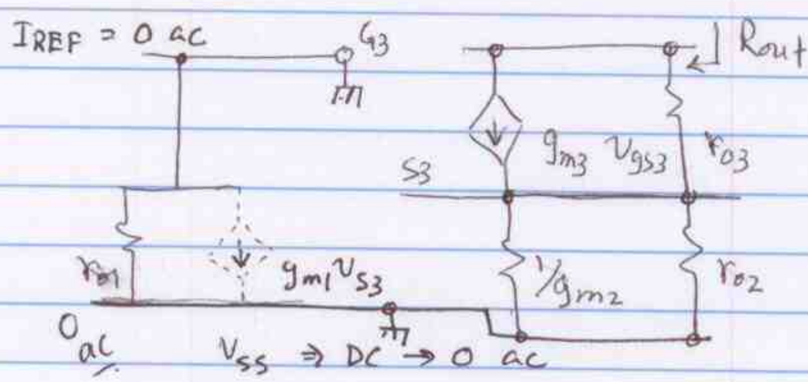
$$r_o = \frac{V_A}{I_{REF}}$$, so I_{REF} need be found out.

For Q2. (a)

AC equiv. circuit of Fig 2(a) will be



For (b), remembering I_{REF} is a DC value i.e. zero 'ac' the ac equivalent circuit will be:



$v_{gs3} = 0 - v_{s3} = -v_{s3}$

M2 → gate-drain connected diode

In Q 2(b) we can use $g_m = g_{m3} = g_{m2} = g_{m1}$ etc.

$$I_{REF} = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_{TH})^2 = 100 \times \frac{10}{2} \times 1^2 = 500\,\mu A$$

So r_o for all the MOSFET $= V_A / I_{REF} = \frac{20}{.5}$ kΩ

$r_o = 40$ kΩ

$$g_m = \sqrt{\frac{2 \mu C_{ox} W}{L} I_D}$$ Where $I_D = I_{out} = I_{REF}$

Q2 (b) (cont.) We take $I_{out} = I_{REF}$ since no specific data are given to make any difference. The data set given implies all transistors are identical.

So $g_m = \sqrt{\underset{\mu C_{ox}}{2 \times 100} \times \underset{\frac{W}{L}}{10} \times \underset{I_D = I_{REF} = I_{out}}{500}}$ ← given formula

$= 1000\ \mu ℧ = 1$ milli mho

So R_{out} for Fig 2(b) circuit is $= 1 \times 10^{-3} \times 40 \times 10^{3} \times 40 \times 10^{3}$ (by given formula)

So $R_{out}\big|_{2b} = 1600\ k\Omega = 1.6\ M\Omega$

Comparison: $\left.\begin{array}{l} R_{out}\big|_{2a} = r_o = 40\ k\Omega \\ R_{out}\big|_{2b} = 1.6\ M\Omega \end{array}\right\}$

4/7 ~~3/7~~ ~~5/6~~

Q3: $g_m R_c = \dfrac{I_c}{V_T} \cdot R_c = \dfrac{I/2}{V_T} R_c = \dfrac{2.5mA}{25mV} \times 2000$

$g_m R_c = 200$ V/V → Small signal gain.

(i) For $v_D = 10$ mV

$$\exp(v_D/V_T) = \exp(10/25) = 1.4918$$

$$\exp(-v_D/V_T) = 0.6703$$

Then $\dfrac{\exp(-v_D/V_T)}{1+\exp(-v_D/V_T)} = \dfrac{0.6703}{1+.6703} = 0.401$

$$\frac{\exp(v_D/V_T)}{1+\exp(v_D/V_T)} = \frac{1.4918}{2.4918} = 0.5987$$

Then $v_{o1} - v_{o2} = 2000 \times 5\times10^{-3}\left[0.401 - 0.5987\right]$

$$= -1.977$$

$$\text{Gain} = -\frac{1.977}{10\,mV} = -197.7 \text{ V/V}$$

(ii) For $v_D = 1$ mV

$$\exp(v_D/V_T) = \exp(1/25) = 1.04$$

$$\exp(-v_D/V_T) = 0.96$$

$$\frac{\exp(-v_D/V_T)}{1+\exp(-v_D/V_T)} = \frac{0.96}{1.96} = 0.489$$

$$\frac{\exp(v_D/V_T)}{1+\exp(v_D/V_T)} = \frac{1.04}{2.04} = 0.509$$

~~5/7~~ 5/6

Q3: Case $v_D = 1\,mV$
(cont.)
$$v_{o1} - v_{o2} = 2000 \times 5 \times 10^{-3} \left[0.489 - .509 \right]$$
$$= -0.2\,V$$
$$\text{Gain} = -\frac{0.2}{1\,mV} = -200\ V/V.$$

For $v_D = 1\,mV$ which is $\ll V_T$, the gain $\rightarrow -200$ exactly matches with that given by the formula $|g_m R_C|$ in magnitude.

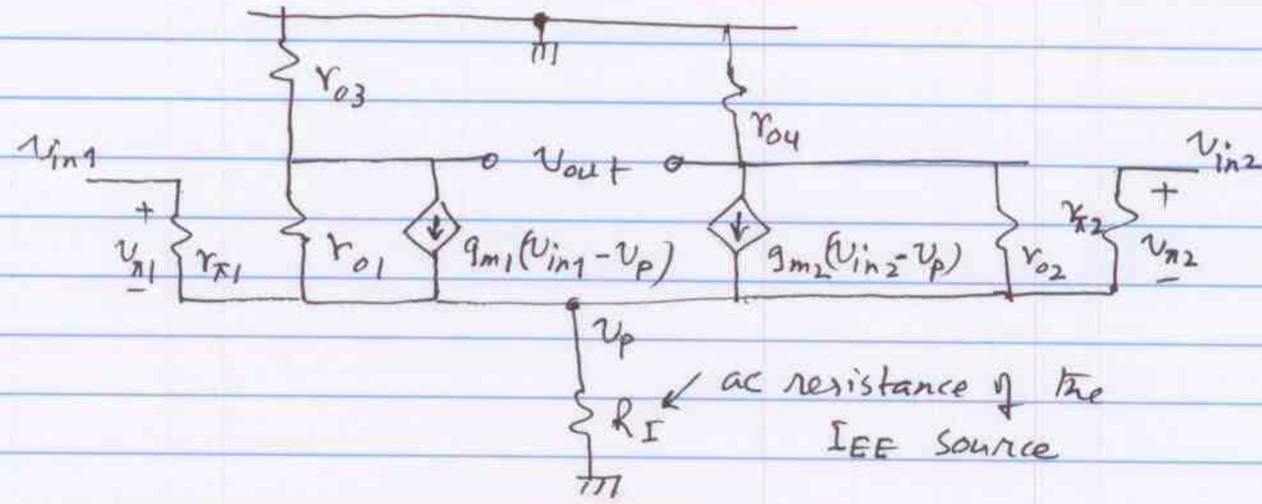
-197.7

For $v_D = 10\,mV$ which is $< V_T$, the gain is very close to the theoretical value $|g_m R_C|$? $+200$

Q4/

Q_3, Q_4 have emitters connected to DC (V_{cc}) and bases connected to DC (V_b). These are functioning like current source active loads.

(a) The ac equivalent circuit is:

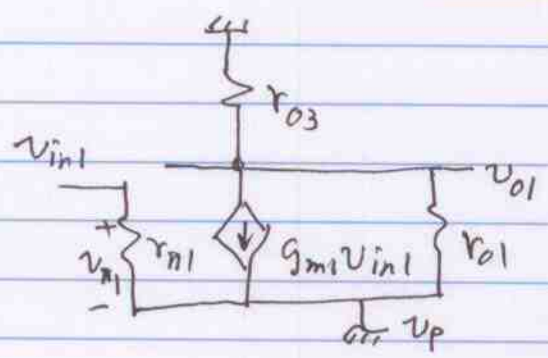


(b) When v_{in1}, v_{in2} are balanced differential signals, $v_p = 0$ (see lecture note derivation)

Each half of the circuit behaves as a CE-BJT amplifier.

Then $v_{o1} = -g_{m1}(v_{in1}) \cdot r_{o1} \| r_{o3}$

Similarly, $v_{o2} = -g_{m2}(v_{in2}) r_{o2} \| r_{o4}$



Assuming the BJTs are matched by pairs (i.e. $Q_3 \equiv Q_4$, $Q_1 \equiv Q_2$)

$v_{out} = v_{o1} - v_{o2}$

$= -g_{mn} \cdot r_{on} \| r_{op} \, v_{in1} + g_{mn} \, r_{on} \| r_{op} \, v_{in2}$

Q4 (Contd.)

Note: $r_{o1} = r_{o2} = r_{on}$
$r_{o3} = r_{o4} = r_{op}$
$g_{m1} = g_{m2} = g_{mn}$

$$v_{out} = v_{o1} - v_{o2}$$

$$= -g_{mn}\, r_{on} \parallel r_{op}\, (v_{in1} - v_{in2})$$

So $$\frac{v_{out}}{v_{in1} - v_{in2}} = -g_{mn} \cdot r_{on} \parallel r_{op}$$

In the above

$r_{o1} = r_{o2} = r_{on}$; $r_{o3} = r_{o4} = r_{op}$,

$g_{m1} = g_{m2} = g_{mn}$

From the given data: $I_{C1} = I_{C2} = I_{EE}/2 = 1mA$

$$r_{op} = \frac{V_{AP}}{I_C} = \frac{50V}{1mA} = 50\,k\Omega$$

$$r_{on} = \frac{V_{AN}}{I_C} = \frac{25V}{1mA} = 25\,k\Omega$$

$$g_{mn} = \frac{1mA}{25mV} = 0.04 \text{ mho}$$

Voltage gain $= -.04 \times 25k\Omega \parallel 50k\Omega = -666.67$ V/V