Appendix-I

(Electronics-I and Network Analysis Background)

1. BJT and MOS circuit calculations

1. DC calculations

1.1 For a dc bias current I_E at the emitter of a BJT, the collector current is: $\alpha_{DC} I_E$.

1.2 For a dc bias current I_E at the emitter of a BJT, the base current is: $I_E/(1+\beta_{DC})$.

1.3 The relationships between α and β for DC calculations are: $\alpha_{DC} = \beta_{DC}/(1+\beta_{DC}); \beta_{DC} = \alpha_{DC}/(1-\alpha_{DC})$.

1.4 A resistance R_E connected between the emitter node and *ac* ground appears as a resistance $R_E(1+\beta_{DC})$ at the base node.

1.5 A resitance R_B connected between the base node and *ac* ground appears as a resistance $R_B/(1+\beta_{DC})$ at the emitter node.

1.6 Either of 1.4 or 1.5 can be used to carry out DC circuit calculations with reference to the base side and emitter side respectively.

1.7 For DC and large signal (i.e., in digital circuits) calculations, the EB junction behaves like a constant battery. If a value is not given, you can assume this to be 0.7 volts. For NPN, the voltage is V_{BE} while for PNP it is V_{EB}.

1.8 For an enhancement type MOSFET (E-MOS) with N-channel, the DC current formulae are:

$$V_{GS} < V_{THN},\ I_{DS} \cong 0 \text{ (sub-threshold region)}$$

$$V_{GS} > V_{THN}, \text{ and } V_{DS} < V_{GS} - V_{THN},\quad I_{DS} = \mu_n C_{ox} \frac{W}{L}[(V_{GS} - V_{THN})V_{DS} - \frac{V_{DS}^2}{2}] \text{ (linear region)}$$

$$V_{GS} > V_{THN}, \text{ and } V_{DS} \le V_{GS} - V_{THN},\quad I_{DS} = \mu_n C_{ox} \frac{W}{2L}[(V_{GS} - V_{THN})^2(1 + \lambda_n V_{DS})] \text{ (saturation region)}$$

1.9 For a P- channel E-MOS, the formulae become (note the changes in the symbols!):

$$V_{SG} < |V_{THP}|, I_{SD} \cong 0 \text{ (sub-threshold region)}$$

$$V_{SG} > |V_{THP}|, \text{ and } V_{SD} < V_{SG} - |V_{THP}|, \quad I_{SD} = \mu_p C_{ox} \frac{W}{L}[(V_{SG} - |V_{THP}|)V_{SD} - \frac{V_{SD}^2}{2}] \text{ (linear region)}$$

$$V_{SG} > |V_{THP}|, \text{ and } V_{SD} \le V_{SG} - |V_{THP}|, \quad I_{SD} = \mu_p C_{ox} \frac{W}{2L}[(V_{SG} - |V_{THP}|)^2 (1 + \lambda_p V_{SD})] \text{ (saturation region)}$$

2. <u>AC (i.e., small signal) calculations</u>

2.1 A DC current I_C at the collector produces an ac equivalent circuit with a transconductance $g_m = I_c / V_T$, where V_T is the thermal voltage kT/q. Unless given otherwise, assume V_T = 25 mV at room temperature (300^o Kelvin).

2.2 A DC current I_E at the emitter produces a resistance of $r_e = V_T / I_E$, in the ac equivalent circuit of the BJT. Unless given otherwise, assume $r_e = 25mV/I_E$ at room temperature (300^o Kelvin).

2.3 Since $I_C = \alpha_{ac} I_E$, it follows that $g_m = \alpha_{ac} / r_e$. For quick and approximate calculations you can assume that $g_m = 1/r_e$.

2.4 For ac components of the BJT in CE mode of operation, $i_c = h_{fe} i_b$, $i_e = (h_{fe}+1) i_b$, where h_{fe} is the ac current gain in CE mode of operation. Similarly, for CB mode of ac operation $\alpha_{ac} = h_{fb} = h_{fe}/(1+h_{fe})$.

2.5 For small (ac) signals (i.e., $\le V_T/10$), the BJT base-emitter junction behaves like a diode with an ac resistance of r_e at the <u>emitter node</u>. Looking <u>from base node</u> this transforms into a resistance of r_e (h_{fe} +1). This is the famous reflection rule for CE BJT amplifier.**

2.6 An extension of the reflection rule is : if R_E is the total resistance from the emitter lead to ground (ac voltage =0), the equivalent resistance by looking at the base node will be $R_E(h_{fe}+1)$. On the other hand if there is an ac equivalent resistance R_B connected between the base and ac ground, the resistance that appears across these two terminals is $R_B/(h_{fe}+1)$. This is the inverse reflection rule.

2.7 The rules as above arise because for same signal voltage across the base-emitter junction, there is a difference in the values of signal currents in the base with that at the emitter. The factor is (h_{fe} +1). Continuity of voltage and current through the EB junction is maintained if this scaling factor is used to scale up (or down) the respective resistances.**

2.8 The reflection rule applies equally well for impedances connected at the emitter or at the base terminal.**

2.9 Because of the reflection rule, the ac resistance of the intrinsic transistor, when looked from the base side becomes $(h_{fe}+1) r_e = r_\pi$.**

** The above cases (2.5-2.9) can be applied as a rule of thumb only if r_o is infinite (i.e., V_A = infinity, or unspecified). For finite value of r_o, the rule is applicable if one end of r_o is *grounded for ac signals.*

2.10 How to figure out if to use r_e or r_π in the equivalent circuit? If, while following the path of signal flow (from the signal source end), you encounter the E-terminal of the BJT before the B-terminal, you will use r_e . But if you meet with the B-terminal before the E-terminal, you will use r_π . Another clue is: for CE and CC amplifier you will use r_π and for CB amplifier you will use r_e .

2.11 V_A is the early voltage for the transistor, the output resistance of the intrinsic device (BJT or MOS) is r_o (r_{ds} for MOS) = V_A / I_{DC} , where I_{DC} is the DC bias current value at the collector (drain for MOS) of the transistor.

2.12 An E- MOS transistor, when operating in saturation region, has the ac transconductance g_m : $\sqrt{2\mu C_{ox}(W/L)I_{DC}}$

2.13 An E- MOS transistor, operating in the linear region, has an output resistance of: $[\mu C_{ox}(W/L)(V_{GS} - V_{THN})]^{-1}$; for PMOS use V_{SG}, and $|V_{THP}|$.

The informations above will be useful to draw the ac equivalent circuit for a transistor and to perform quick and simple hand calculations regarding certain charcteristics of the transistor amplifier.

Appendix-II: Linear Network Analysis Fundamentals

The time domain integro-differential equations for linear networks (i.e., networks containing linear circuit elements like R,L and C, linear controlled sources such as VCVS,VCCS,CCCS and CCVS) can be arranged in a matrix form:

$$w(p)\ x(t) = f(t)$$

where, w(p) is an impedance/admittance matrix operator containing integro-differential elements (x(t) is the current/voltage variables (vectors) in the edges of the network graph and f(t) are the source voltage/current variables (vectors). The p-operator implies *p = d /dt* and *1/p =   dt*. On taking Laplace transform of both sides, one could derive

$$W(s)\ X(s) = F(s) + h(s)$$

where h(s) contain the contributions due to *initial values.*On inserting s = jω one can get the *Frequency Domain* characterization of the system. The above sequence of operation, is, however, rather lengthy and impractical. A more efficient technique is to characterize each network component (R,L and C) in s-domain including the contribution of initial conditions and formulate the network equations as was done before. Impedance elements so expressed are *transformed impedances* and the network becomes a *transformed network.*

2.1 Transformed Impedances

The transformed impedances are impedance elements referred to a transform (i.e., Lapalce Transform) domain. Characterizations for transformed basic network elements are discussed below. For ideal voltage or current sources the transformed quantities are simply the Lapalce transforms (e.g., $V_g(s)$, $I_g(s)$). For the i-v relation across a resistor, one can write either $V_R(s)=I_R(s)R$ or the dual $I_R(s)=V_R(s)/R$. Thus there are two characterizations (viz., an I-mode and a V-mode) for each element. The particular choice depends upon which of I(s) and V(s) is the independent variable. In *loop analysis, the I(s) becomes independent variable* and the voltage and impedance around the loop are recorded as part of the systematic procedure to obtain the matrix formulation (for example, by inspection). In this case the series equivalent model of the transformed impedance has to be used.The cases of interest are shown in Figures 1.1(a)-(b) and 1.2 (a)-(b) for the inductance and capacitance respectively.

For an inductance, the v - i relations

$$v_L(t) = L\frac{di_L}{dt}, \text{ and } i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0\_),$$

lead to the s - domain equations :

$$V_L(s) = sLi_L(s) - Li_L(0\_) \text{ and,}$$

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0\_)}{s}.$$ These lead to the two equivalent networks shown on right. Figure(a) is the series representation and Figure(b) is the shunt representation.

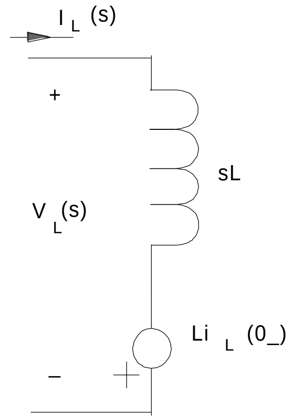


Figure 1.1(a)

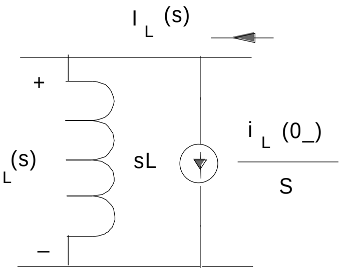


Figure 1.1(b)

Similarly, the capacitance current $i_c = C\frac{dv_c}{dt}$, and

$$v_c = \frac{1}{C}\int_0^t i_c dt + v_c(0\_),$$ lead to the s - domain equations :

$$I_c(s) = sCV_c(s) - Cv_c(0\_), \text{ and } V_c(s) = \frac{I_c(s)}{sC} + \frac{v_c(0\_)}{s}.$$

These lead to the two network representations shown to the right. Figure (a) is the series representation, while Figure (b) is the shunt representation.

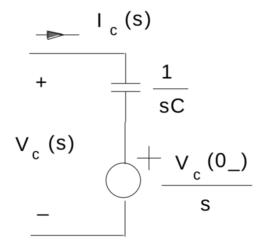


Figure 1.2(a)

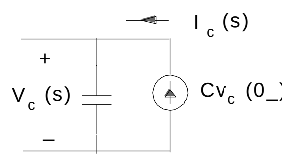


Figure 1.2(b)

In *nodal analysis*, where the node voltage becomes independent variable, the shunt architecture are to be used for the transformed impedances.

A network containing transformed impedances is referred to as transformed network.

Appendix-III: Network analysis technique

3.1 Nodal analysis

In *nodal analysis, a voltage source in series with an element should be transformed into a current source with a shunt element* by employing source transformation (i.e., Thevenin to Norton equivalent). This will reduce the number of nodes and also make *the analysis more homogeneous in that we have to deal with only node voltages and current sources (independent, dependent)* which are the principal variables in nodal analysis. It may be recalled that the nodal system of equations is represented by the matrix equation Y(s)V(s) = J(s), where Y(s) is the admittance matrix, V(s) is the node voltage vector (i.e., a column matrix) and J(s) is the current source vector. If a voltage source feeds more than one unique impedance in a parallel connetion, E-shift technique [1]is to be used before embarking on the source transformation operation. The given network has to be converted to a network with transformed impedances with shunt model representation for inductances and capacitances.

Steps to setup the matrix equation for Nodal Analysis

Step 1: **Identification of the sources:** identify all dependent and independent sources. These are to be included initially as elements of the vector J(s).

Step 2: **Set up the matrix elements**

(a) y_{ii} elements are *sum* of admittances (conductances) meeting at the node. This is the *self admittance*.

(b) y_{ij} elements are *negative* of the admittances running between the node pair (i,j). This is *trans admittance.*

The above two sets are to be included in the Y(s) part of the matrix equation Y(s)V(s) = J(s).

(c) j_{nk} element is the sum of current sources meeting at k^{th} node, taken *positive if towards and negative when away* from the node. These are to be included in the J(s) part of the matrix equation Y(s)V(s) = J(s)

Step 3: **In** J(s) **decode the dependent I-sources** in terms of the node (voltage) variables i.e., elements of V(s).

Step 4: **Transpose** the quantities obtained in step 3 to the other side and allocate them to appropriate locations in the Y(s) matrix.

The above four steps complete the setting up of the node system of matrix equation in the form Y(s)V(s) = J(s).

3.2 Loop analysis

[1]Linear Networks and Systems, 2nd edition, vol.1, by Dr. Wai-Kai Chen.

The *first thing is to draw the equivalent circuit* with transformed impedances using series model versions for inductors and capacitances. Further, all current sources are to be converted to equivalent voltage sources with series impedances using source transformation (i.e., Norton to Thevenin). If a current source exists with no unique impedance in parallel, I-shift technique is to be used before applying source transformation. It may be recalled that the loop system matrix equation has the form Z(s)I(s) = E(s), where Z(s) is the impedance matrix, I(s) is the loop current vector and E(s) is the loop voltage vector.

Steps to setup the matrix equation for Loop Analysis

Step 1 **(Identification of sources):** look up all voltage sources (independentn and dependent) to be initially included as elements of the E(s) vector.

Step 2 **(Identify the loop impedance matrix operator elements and loop voltage source vector)**

(a) *self loop impedance* z_{ii} is the sum of all impedances in the loop i.

(b) *mutual loop impedance* z_{ij} is the impedance shared by loop i and loop j. If currents in loops i and j are in the same direction z_{ij} is taken with a positive sign. On the other hand, if the currents in loops i and j are in opposite directions, it is taken with a negative sign.

(c) *loop source vector* e_I is the algebraic sum of all the voltage sources in loop i. The components are taken with a positive sign if a *potential rise* occurs in the direction of the loop current. If a *potential drop* takes place in the direction of the loop current, the voltage element is taken with a negative sign. We tus have the preliminary form Z(s)I(s) = E(s).

Step 3 **In** E(s) found above and **express the dependent sources** in terms of the loop current variables (i.e.,elements of I(s)).

Step 4 **Transpose the dependent components** of E(s) on the other side and allocate the associated coefficiens to proper location of the Z(s) matrix.

The above four steps will produce the final matrix equation form Z(s)I(s) = E(s) for loop analysis.

3.3 Network Functions

If we study the relationships developed in connection with nodal and loop analysis, we can discover a general format, i.e., W(s) X(s) = F(s) + h(s), where W(s) can be either an admittance or an impedance matrix, X(s) can be nodal voltage vector or loop current vector, F(s) the vector of independent sources and h(s) the vector of initial conditions. Using this equation ,one can easily arrive at: $X(s) = W^{-1}(s)F(s) + W^{-1}(s)h(s)$. The FIRST part of the solution on the RHS is the complete solution if initial values were zero (so h(s)=0). This is called **zero (initial) -state response.** The SECOND part of the solution on the RHS is the complete solution if the forcing functions were zero (so F(s)= 0). This is known as **zero -input or natural response.**

*A network function is defined with regard to zero-state response in a network when there is **only one independant voltage/current (driving function) forcing function** in the network.*. It is the ratio of the *Lapalce- transform of the zero-state response at a given point (node) in a network to the Lapalce- transform of the input (or another zero-state response) effective at another location of the network.*

Depending upon the location of the pair of points there are two nomenclatures. If *the pair of points are same,* we talk about **driving point impedance or driving point admittance.** If the *pair of points are separate,* we can derive (i) **transfer function in voltage, (ii) transfer function in current, (iii) transfer impedance and (iv) transfer admittance**

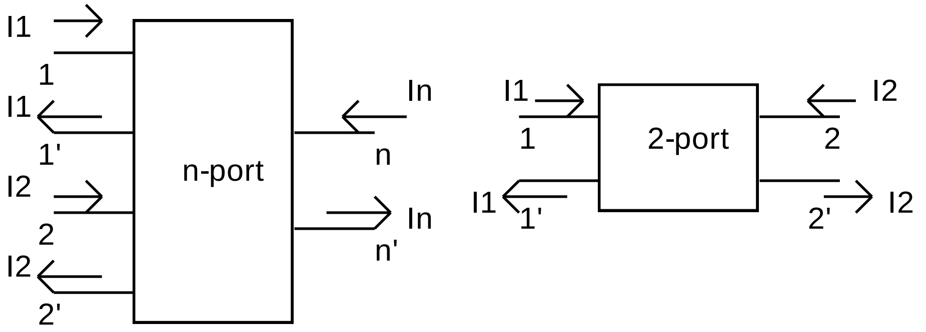
3.4 Characteristics of Network functions

It is well-known that Laplace transform of time-domain integro-differential equations are algebraic functions in the variable s, which is regraded as *complex frequency* i.e., $s = \sigma + j\omega$. When we set σ =0, the resulting algebraic function (i.e., $s=j\omega$.) represents a frequency domain function. The network functions are thus rational algebraic functions of s . The parameter σ relates to damping (or growth) of the time domain response while ω gives the frequency of the time-domain waveform.

When $\sigma > 0$, the system becomes unstable (time domain response grows) and when $\sigma << \omega$, the system exhibits frequency selectivity. For a self oscillatory network $\sigma <= 0$. For certain values of s, the network function ->0. These values (of s) are the 'zeros' of the network function. Similarly, for some values of s, the network function -> ∞. These values (of s) are called the 'poles' of the network function**.**

3.5 Two-port Networks

A **node pair**, such that current entering one node is exactly equal to the current exiting out of the other node constitute a **one-port**. If there is another node pair with the same property, we have another one-port. If the *node pairs belong to the same system, we have a two-port network system.* As an illustration, consider the following diagram. The concept of an n-port can also be developed similarly.



It is possible that the nodes 1',2' ... n' are one and the same node (i.e., ground). In 2-port network theory we consider only the external node pairs. The system is considered as a black box with no independent current/voltage source(s) residing inside the black box. Also all initial conditions are assumed to be zero (or taken care of by proper analysis prior to characterization as a 2-port network). The electrical characteristics of the two port is entirely defined in terms of the voltage/current source(s) effective at the external terminals of the (black box) network. Thus *the two port network theory can be very conveniently applied to fairly large sized networks.* There are several ways in which one can select a pair of independent voltage/current variables from the set V1, I1, V2 and I2. Thus we come across (i) *Short circuit* admittance parameters characterization, (ii) *Open circuit* impedance parameters .. , (iii) Hybrid parameters , and (iv) g-parameters.

3.5.1 Admittance Parameters

The characterization is best described by the matrix equation

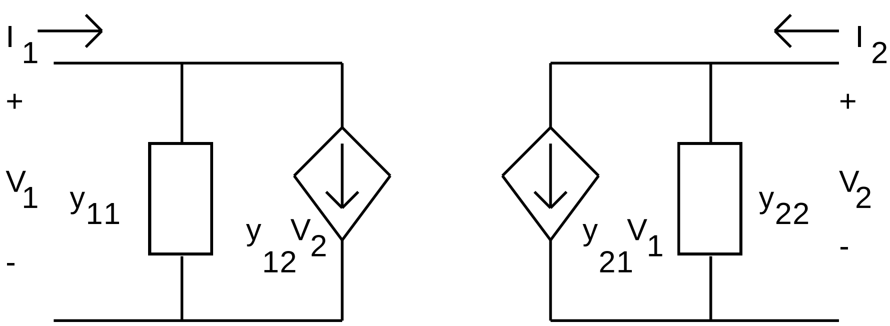
$$\mathbf{[Y][V] = [I]}$$

The above corresponds to the set of linear equations

$y_{11}V_1 + y_{12}V_2 = I_1$; $y_{21}V_1 + y_{22}V_2 = I_2$. The defining equations for the parameters are:

$y_{11} = [I_1 / V_1]_{V2=0}$, $y_{12} = [I_1 / V_2]_{v1=0}$, and so on.

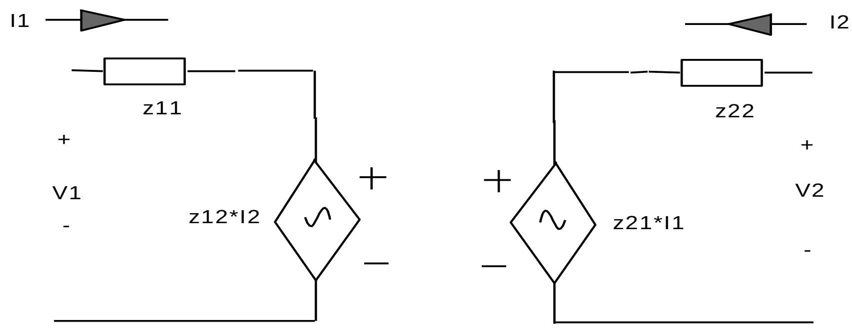
The electrical *network model (i.e., equivalent to the network inside the black box)* for the above set of equations is:



3.5.2 Impedance Parameters

Here the matrix relation is as **[Z] [I] = [V].** The set of equations are:

$z_{11} I_1 + z_{12} I_2 = V_1$; $z_{21} I_1 + z_{22} I_2 = V_2$. The network model is:

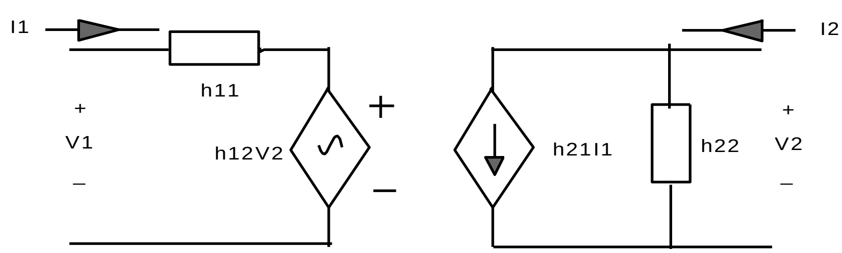


One can use the above relations to derive $z_{11} = (V_1 / I_1)$ with $I_2 = 0$, i.e., port 2 open, $z_{12} = (V_1/I_2)$ with $I_1 = 0$ and so on.

3.5.3 Hybrid (h-) Parameters

In this V_1 is related to I_1 (impedance) and V_2 (transfer ratio) while I_2 is related to I_1 (transfer ratio) and V_2 (conductance). That is why the name hybrid. It became popular to model transistor devices as a two port network. The system of equations are:

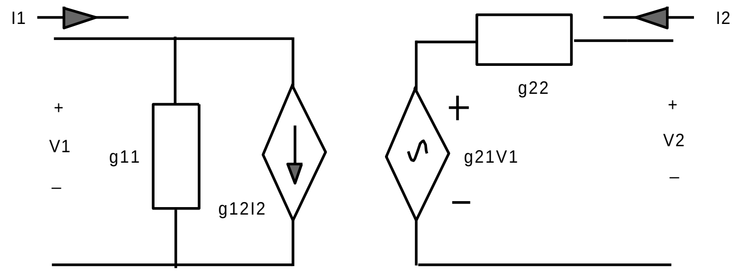
$h_{11}I_1 + h_{12} V_2 = V_1$; $h_{21} I_1 + h_{22} V_2 = I_2$. Thus $h_{11} = (V_1/I_1)$ with $V_2 = 0$, i.e., port 2 short circuited, $h_{12} = (V_1/V_2)$ with $I_1 = 0$, i.e., port 1 open, and so on. The network model is:



3.5.4 Hybrid (g-) Parameters

Here the choice of variables are reversed compared with the h-parameter system. Thus I_1 and V_2 are related to V_1 and I_2. The system equations are:

$g_{11}V_1 + g_{12} I_2 = I_1$; $g_{21}V_1 + g_{22}I_2 = V_2$. Thus $g_{11} = (I_1 / V_1)$ with $I_2 = 0$, $g_{12} = (I_1/I_2)$ with $V_1 = 0$, and so on. The network model is:


I1
I2
+
V1
–
g11
g12I2
g22
+
–
g21V1
+
V2
–

Appendix-IV: Interconnection of Two-port Networks

Various kinds of interconnection are possible between two *two ports*. The principle in obtaining the overall 2-port parameters involve (i) writing down the parameters for the individual 2-ports, (ii) identify the effect of the interconnection on the terminal variables which are being interconnected (they become identical, add up ... etc), (iii) re-write the new set of terminal variables according to the interconnection.

4.1 Parallel connection

In this connection, the terminal voltage pairs remain identical so that the currents at each port get added up because of the interconnection. Y parameter representation is the most preferred choice to begin with. *The overall Y-parameters is the sum of the constituent Y-parameters.* Thus, $[Y] = [Y]_a + [Y]_b$, where networks N_a and N_b are connected in parallel at the input and at the output.

4.2 Series connection

Terminal currents remain same (series) so that the terminal voltage variables add up. Z parameter representation is the preferred choice. *The overall Z-parameter is the sum of the constituent Z-parameters.*Thus, $[Z] = [Z]_a + [Z]_b$ where N_a and N_b are the networks connected in series both at the input and at the output.

4.3 Series (input)- Parallel (output) connection

As the name suggest, the input ports (i.e., port #1) are connected in series while the output ports (#2) are connected in parallel. Thus at port#1 we will have $I_1=I_{1a}=I_{1b}$, but $V_1=V_{1a}+V_{2a}$, while at port#2 we get $I_2=I_{2a}+I_{2b}$, and $V_2=V_{2a}=V_{2b}$. Since V_1 and I_2 needs recalculation, it will be most profitable to start with the h-parameters. The overall H parameter matrix becomes equal to the sum of the component h-parameter matrices. Thus,

$$[H] = [H]_a + [H]_b$$

4.4 Parallel (input)- Series (output) connection

As the name suggest, the input ports (i.e., port #1) are connected in parallel while the output ports (#2) are connected in series. Thus at port#1 we will have $V_1=V_{1a}=V_{1b}$, but $I_1=I_{1a}+I_{2a}$, while at port#2 we get $V_2=V_{2a}+V_{2b}$, and $I_2=I_{2a}=I_{2b}$. Since I_1 and V_2 needs recalculation, it will be most profitable to start with the g-parameters. The overall G parameter matrix is given by:

$$[G] = [G]_a + [G]_b$$

Appendix-V: Transfer Function and BODE Plot

In the above, the concept of a transfer function has been presented. It is a function of the variable s which is associated with the notion of a complex frequency in electronic circuits and systems. Let us designate this with the symbol T(s). Since T(s) is function of a complex variable (i.e., of s), it can have a magnitude and a phase angle. Bode plot involves plotting the functions 20log |T(s)| and /___T(s) as a function of frequency. Let us consider some simple concepts.

5.1 <u>T(s) = Ks, where K is a constant</u>.

For frequency domain analysis, we shall set s= jω. Thus, 20log |T(s)| = 20log (K) +20log (ω) =M(ω), say, and /____T(s) = arctan (ω/0) = 90 degrees. Since K is a constant, 20log (K) will be a constant. So far frequency dependence is concerned, M(ω) will be influenced by the term 20log (ω). As ω increases, M(ω) will increase linearly, beginning from negative infinity at ω =0. When ω =2, , M(ω) = 6dB, when ω =10, M(ω) = 20dB and so on. In another way, one can say that if there are two frequencies ω_1 and ω_2 , M(ω_2) will be 6 dB higher than M(ω_1) if $\omega_2 = 2\ \omega_1$ and M(ω_2) will be 20 dB higher than M(ω_1) if $\omega_2 = 10\omega_1$. These concepts are spelled out using the phrases: M(ω) changes at a rate of 6 db/octave (ratio =2), or M(ω) changes at a rate of 20 db/decade (ratio =10).

Thus the Bode plot of T(s)= Ks have the characteristics (a) the magnitude part *increases* at a rate of 6db/octave (or 20dB/decade), linearly with a positive slope and (b) the phase part is constant at 90 degrees.

5.2 <u>T(s) = K/s.</u>

Now M(ω)= 20log |T(s)| = 20log (K) - 20log (ω) and and /____T(s) = - arctan (ω/0) =- 90 degrees. Following the concept above, one can conclude that the Bode plot of T(s) has a phase angle of – 90 degrees and a magnitude which *decreases* linearly at a rate of 6 dB/octave (or 20 dB/decade).

5.3 <u>T(s) = K(s+a).</u>

Now the phase angle = arctan (ω/a) and dB magnitude M(ω) = 20log (K) + 20log[$(\omega^2 + a^2)^{1/2}$]= 20log (K) + 10log ($\omega^2 + a^2$). When ω is very small compared with a, M(ω) has a nearly constant value ~ 20 log(K) + 20log(a) dB, increasing slowly with increasing ω. When ω = a, M(ω) = 20log (K) + 20log $(2a^2)^{1/2}$, i.e., , M(ω) = 20log (K) + 20log (a) + 20log $(2)^{1/2}$ =20log (K) + 20log (a) + 3 dB. This is a distinct value. Thus the magnitude of T(s) has a Bode plot which is nearly a constant at small values of ω, increases slowly with ω and becomes 3 dB higher as ω becomes equal to a. Now from the concpet of poles and zeros, it is clear that T(s) has a zero at a. The values of M(ω) when ω has critical values of zero or infinity, are usually referred to as asymptotic values. Thus, one can summarize that *when T(s) has a zero at (say) ω =a, the Bode magnitude plot of T(s) increases by 3 dB at ω =a compared to its asysmptotic value at ω =0.* As ω passes through a and becomes very large compared with a, the function T(s) approximates to T(s) ~ K(s) and hence behaves in the same manner as in case 1 above. Thus *as ω becomes very hgh compared with the zero of T(s),i.e., a, the magnitude plot changes linearly with ω approacing asymptotically a straight line of slope +6dB/octave (or 20dB/decade) erected at the frequency ω =a.* Further, as the phase angle is arctan (ω/a), it is nearly zero for small values of ω and increases slowly with ω. At ω=a, since arctan (a/a) =1, the phase angle is 45 degrees*. Thus at the zero of T(s) the phase angle is +45^o relative to its previous asysmptotic value.* As ω becomes very high compared to a, the phase angle tends towards 90 degrees since arctan (infinity) is 90 degrees (i.e., tan (90^o) is infinity).

5.4 T(s)= K/(s+a).

Now the phase angle = 0 -arctan (ω/a) and dB magnitude M(ω) = 20log (K) -20log[$(\omega^2 + a^2)^{1/2}$]= 20log (K) - 10log ($\omega^2 + a^2$). When ω is very small compared with a, M(ω) has a nearly constant value ~ 20 log(K) - 20log(a) dB, and it decreases slowly with increasing ω. When ω = a, M(ω) = 20log (K) - 20log $(2a^2)^{1/2}$, i.e., , M(ω) = 20log (K) - 20log (a) - 20log $(2)^{1/2}$ =20log (K) +-20log (a) - 3 dB. This is a distinct value. Thus the magnitude of T(s) has a Bode plot which is nearly a constant at small values of ω, decreases slowly with ω and becomes 3 dB lower as ω becomes equal to a. Now from the concpet of poles and zeros, it is clear that T(s) has a pole at a. Thus, one can say that *when T(s) has a pole at (say) ω =a, the Bode magnitude plot of T(s) decreases by 3 dB at ω =a compared to its asysmptotic value at ω =0.* As ω passes through a and becomes very large compared with a, the function T(s) approximates to T(s) ~ K/s and hence behaves in the same manner as in case 2 above. Thus *as ω becomes very high compared with the pole of T(s),i.e., a, the magnitude plot changes linearly with ω approacing asymptotically a straight line of slope -6dB/octave (or 20dB/decade) erected at the frequency ω =a.* Further, as the phase angle is - arctan (ω/a), it is nearly zero for small values of ω and decreases slowly with ω. At ω=a, since - - arctan (a/a) =-1, the phase angle is -45 degrees*. Thus at the pole of T(s) the phase angle is -45^o relative to its previous asymptotic value.* As ω becomes very high compared to a, the phase angle tends towards - 90 degrees since - arctan (infinity) is - 90 degrees.

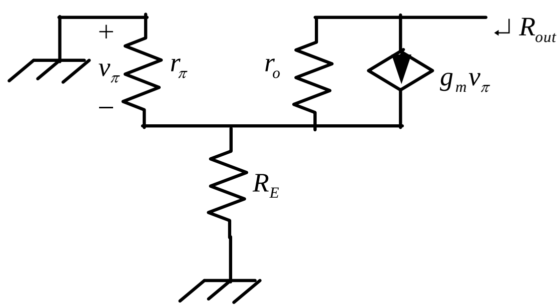
5.5 T(s)= K(s+wz_1)(s+wz_2)……(s+wz_m)/[(s+wp_1)(s+wp_2)…(s+wp_n)]

Now we have a general case where the transfer function has a number of zeros and a number of poles. After taking logarithms, one can derive M(ω) = 20log(K) + 10log($\omega^2 + wz_1^2$) + 10log($\omega^2 + wz_2^2$) + … +10log($\omega^2 + wz_m^2$) –10log($\omega^2 + wp_1^2$) - 10log($\omega^2 + wp_2^2$) -….- 10log($\omega^2 + wp_n^2$). Similarly, the phase angle will be Φ = artctan (ω/wz_1) + artctan (ω/wz_2)+.. + artctan (ω/wz_m) - artctan (ω/wp_1) - artctan (ω/wp_2)- ..- artctan (ω/wp_n). We can now apply the knowledge learned in items 3 and 4 above like the principle of superposition at each zero and at each pole in accordance with their order of numerical values. Thus so far the magnitude (in dB) plot is concerned, M(ω) will start off with a constant value of 20log(K) + 20log(wz_1) + 20log(wz_2) + … +20log(wz_m) –20log(wp_1) - 20log(wp_2) -….- 20log(wp_n). This is the asymptotic value at ω = 0. *As ω increases, at each zero of T(s) the magnitude response changes by +3dB relative to the previous asysmptotic value and at each pole of T(s), the response changes by –3dB relative to the previous asysmptotic value*. Between two susccessive critical points (i.e., zero or pole), the plot tends to follow a straight line of slope +6dB per octave (+20dB per decade) if the current critical point is a zero of T(s). If the current critical point is a pole of T(s), the magnitude plot tends to folow a straight line of slope -6dB per octave (-20dB per decade*). For a succession of zeros and poles of T(s), the values are to be added up successively.*

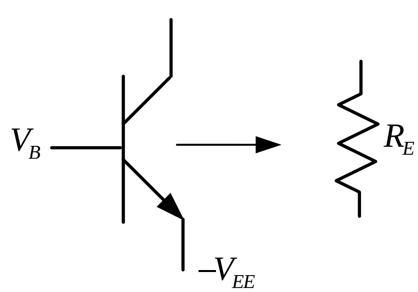
So far the phase angle is concerned, it starts with zero at ω =0. Thereafter, it changes by +45^o at each zero of T(s) relative to its value at the previous critical point (zero or pole) and it changes by – 45^o at each pole relative to its value at the previous critical point (zero or pole). *For a succession of zeros and poles of T(s), the phase angle values are added up successively.*

Appendix-VI: Practice Exercises

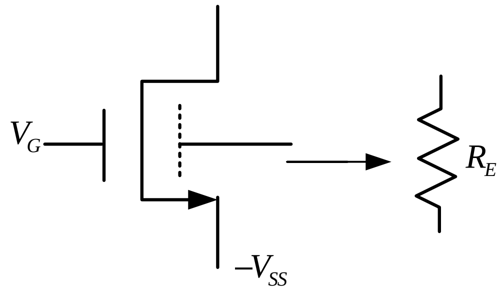
6.1 For the network shown below, find the output resistance for ac small signal case [hint; use dummy source at the output end and find v/I at that end].



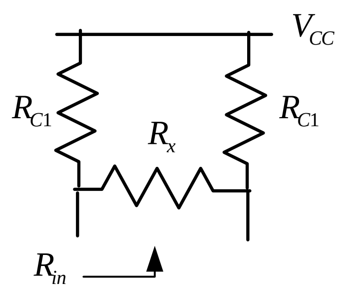
6.2 Repeat 1 with R_E replaced by a BJT device as indicated below. [hint: use the ac equivalent circuit for the BJT device]



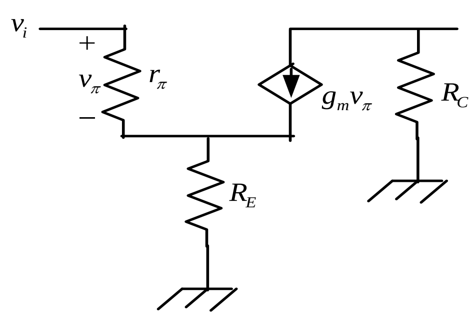
6.3 Repeat 1 with R_E replaced by a MOS device as shown below.



6.4 A sub-network in a differential amplifier circuit appears as below. Find an expression fro R_{in} considering small signal (ac) equivalent circuits. Use <u>nodal matrix</u> method.

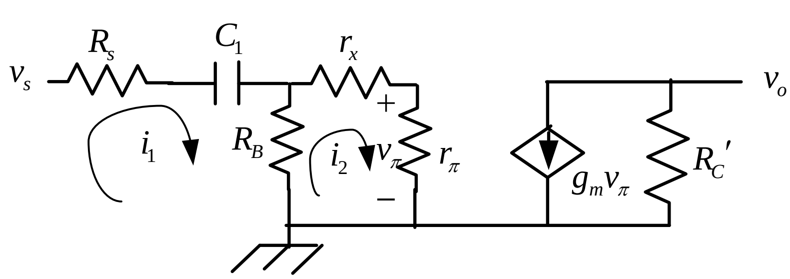


6.5 The ac equivalent circuit for a BJT amplifier with un-bypassed resistance R_E at the emitter is shown below. Find, using nodal matrix analysis method, an expression for the voltage gain v_o/v_i.



6.6 Use the ac equivalent circuit and <u>loop matrix</u> analysis method to find the expression for the voltage gain v_o/v_s. [Ans. $-\dfrac{g_m R_c' R_B r_\pi}{R_{xx}} \cdot \dfrac{s}{s+1/C_1 R_{c1}}$, where

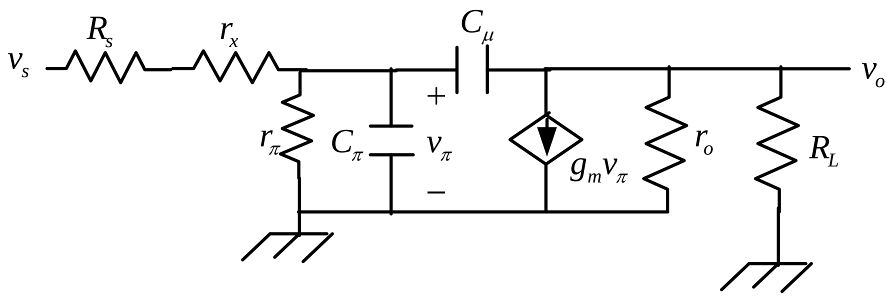
$R_{xx} = R_s R_B + R_s(r_\pi + r_x) + R_B(r_\pi + r_x);\ R_{c1} = R_s + R_B \,||\, (r_x + r_\pi)$]



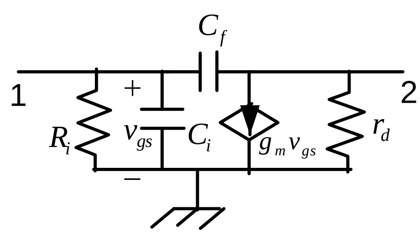
6.7 Consider the ac equivalent circuit of a BJT at high frequencies. Find v_o/v_s using nodal matrix method. [Ans. $-\frac{(g_m - sC_\mu)g_s'}{\Delta}$; where

$\Delta = (g_s' + g_\pi)(g_o + g_L) + s[(g_\pi + g_s')C_\mu + (g_o + g_L)(C_\mu + C_\pi) + g_m C_\mu] + s^2 C_\pi C_\mu$, and

$g_s' = 1/(R_s + r_x)$, all $g_i = 1/r_i$.

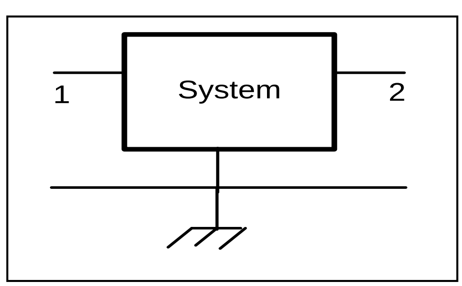


6.8 The high-frequency ac equivalent circuit of a MOSFET ampliflier is shown below. Find the two-port Y-matrix description for the system.[hint: formulate $[I] = [Y][V], [Y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$; then find y_{11}, y_{12}]



6.9 For the system as shown below, the measured y_{11} data are tabulated as below. Find the ac equivalent model for the y_{11} part of the system.[hint: consider y_{11} as a parallel combination of a resistance R and a reactance X. Find the values of R and the component associated (C or L) with X].

Frequency	Re(y_{11})	Im(y_{11})
100	1E-11	6.06E-6
1 K	1E-11	60.6E-6
10 K	1E-11	606E-6



6.10 A frequency domain transfer function is given by $T(s) = \dfrac{10}{s+20}$. Sketch the Bode plot for |*T(s)*|

6.11 A frequency domain transfer function is given by $T(s) = \dfrac{200s}{(s+10)(s+300)}$. Sketch |*T(s)*| using Bode technique.

6.12 A sketch of |*T(s)*| with $T(s) = \dfrac{1500(s+100)}{(s+20)(s+1000)}$, is shown below. If there is any error in the sketch, correct it and re-draw.