ELEC 312/2-F, Fall 2012, MT test soln.

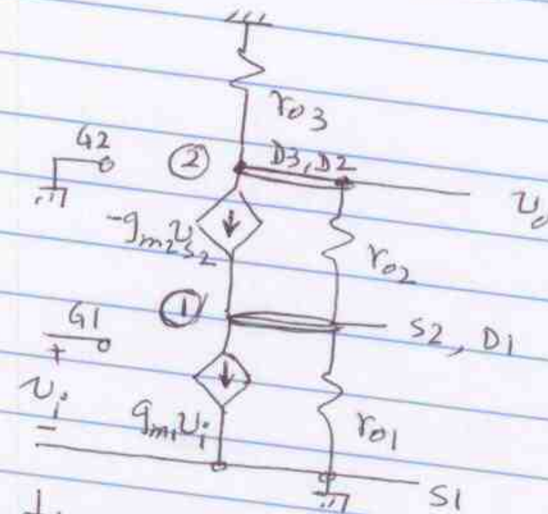
1/7

Q1.

(a) M3 has source and gate at DC (= 0 ac). So it acts as a current-source active load $= r_{o3}$

M2 has gate at DC. So the $g_{m2} v_{gs2}$ source becomes $g_{m2}(0 - v_{s2}) = -g_{m2} v_{s2}$

The ac equivalent circuit is:



soln

(b) This is a two node system ~~with~~ v_i is input node. Since v_i does not have any component attached, we will exclude it in formulating the nodal admittance matrix (NAM). By inspection (and letting $g = \frac{1}{r}$)

$$\begin{array}{c}\begin{bmatrix} g_{o1}+g_{o2} & -g_{o2} \\ -g_{o2} & g_{o2}+g_{o3} \end{bmatrix} \\ \begin{array}{cc} ① & \quad ② \end{array}\end{array} \begin{bmatrix} v_{①} \\ v_{②} \end{bmatrix} = \begin{bmatrix} -g_{m1}v_i - g_{m2}v_{s2} \\ g_{m2}v_{s2} \end{bmatrix}$$

But $v_{s2} = v_{①}$. Substituting and moving on left side (i.e. $g_{m2}v_{s2} = g_{m2}v_{①} \rightarrow -g_{m2}v_{①}$ on left)

$$\begin{bmatrix} g_{m2}+g_{o1}+g_{o2} & -g_{o2} \\ -g_{m2}-g_{o2} & g_{o2}+g_{o3} \end{bmatrix} \begin{bmatrix} v_{①} \\ v_{②} \end{bmatrix} = \begin{bmatrix} -g_{m1}v_i \\ 0 \end{bmatrix}$$ soln.

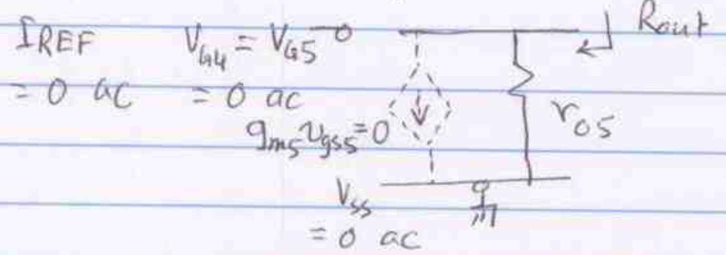
Q2:

For Fig. 2(a), it is a basic current mirror made from M4, M5. R_{out} for M5 is simply r_o of M5.

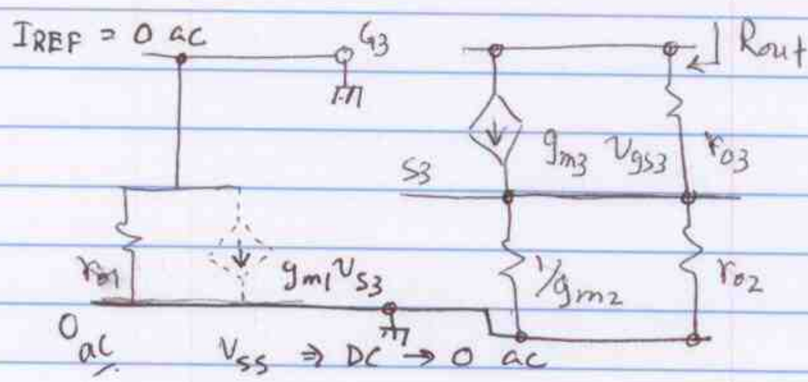
$$r_o = \frac{V_A}{I_{REF}}$$, so I_{REF} need be found out.

For Q2(a)

AC equiv. circuit of Fig 2(a) will be



For (b), remembering I_{REF} is a DC value i.e. zero 'ac' the ac equivalent circuit will be:



$v_{gs3} = 0 - v_{s3} = -v_{s3}$

M2 → gate-drain connected diode

In Q 2(b) we can use $g_m = g_{m3} = g_{m2} = g_{m1}$ etc.

$$I_{REF} = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_{TH})^2 = 100 \times \frac{10}{2} \times 1^2 = 500\,\mu A$$

So r_o for all the MOSFET $= V_A / I_{REF} = \frac{20}{.5}\ k\Omega$

$$r_o = 40\ k\Omega$$

$$g_m = \sqrt{\frac{2\mu C_{ox} W}{L} I_D} \quad \text{Where } I_D = I_{out} = I_{REF}$$

Q2 (b) We take $I_{out} = I_{REF}$ since no specific data
(cont.) are given to make any difference. The data set given implies all transistors are identical.

So $g_m = \sqrt{\underset{\mu C_{ox}}{2 \times 100} \times \underset{W/L}{10} \times \underset{I_D = I_{REF} = I_{out}}{500}}$ ← given formula

$= 1000 \ \mu\mho = 1$ milli mho

So R_{out} for Fig 2(b) circuit is $= 1 \times 10^{-3} \times 40 \times 10^{3} \times 40 \times 10^{3}$
(by given formula)

So $R_{out}\big|_{2b} = 1600 \ k\Omega = 1.6 \ M\Omega$

Comparison: $\left.\begin{array}{l} R_{out}\big|_{2a} = r_o = 40 \ k\Omega \\ R_{out}\big|_{2b} = 1.6 \ M\Omega \end{array}\right\}$

Q3:

$$g_m R_c = \frac{I_c}{V_T} \cdot R_c = \frac{I/2}{V_T} R_c = \frac{2.5\,mA}{25\,mV} \times 2000$$

$$g_m R_c = 200 \;\; V/V \rightarrow \text{Small signal gain.}$$

(i) For $v_D = 10\,mV$

$$\exp(v_D/V_T) = \exp(10/25) = 1.4918$$

$$\exp(-v_D/V_T) = 0.6703$$

Then $$\frac{\exp(-v_D/V_T)}{1+\exp(-v_D/V_T)} = \frac{0.6703}{1+.6703} = 0.401$$

$$\frac{\exp(v_D/V_T)}{1+\exp(v_D/V_T)} = \frac{1.4918}{2.4918} = 0.5987$$

Then $$v_{o1} - v_{o2} = 2000 \times 5\times10^{-3}\left[0.401 - 0.5987\right]$$

$$= -1.977$$

$$\text{Gain} = -\frac{1.977}{10\,mV} = -197.7 \;\; V/V$$

(ii) For $v_D = 1\,mV$

$$\exp(v_D/V_T) = \exp(1/25) = 1.04$$

$$\exp(-v_D/V_T) = 0.96$$

$$\frac{\exp(-v_D/V_T)}{1+\exp(-v_D/V_T)} = \frac{0.96}{1.96} = 0.489$$

$$\frac{\exp(v_D/V_T)}{1+\exp(v_D/V_T)} = \frac{1.04}{2.04} = 0.509$$

Q3: Case $v_D = 1\,mV$
(cont.)

$$v_{o1} - v_{o2} = 2000 \times 5 \times 10^{-3}\left[0.489 - .509\right]$$

$$= -0.2\,V$$

$$\text{Gain} = -\frac{0.2}{1mV} = -200\ V/V.$$

For $v_D = 1\,mV$ which is $<< V_T$, the gain $\rightarrow -200$ exactly matches with that given by the formula $|g_m R_C|$ in magnitude.

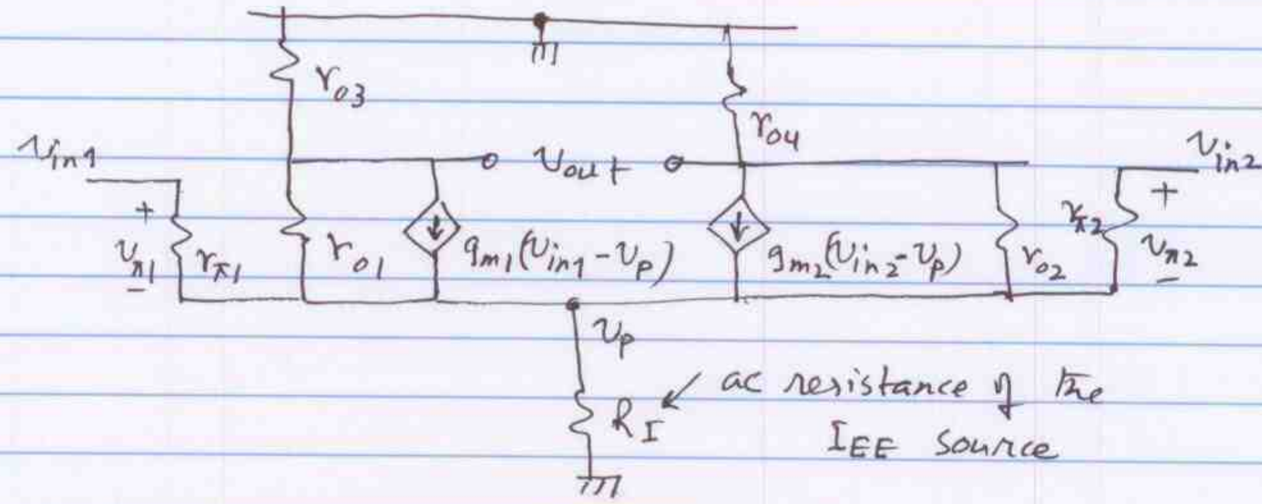
For $v_D = 10\,mV$ which is $< V_T$, the gain is $\rightarrow -197.7$ very close to the theoretical value $|g_m R_C| \rightarrow +200$

6/7

Q4/

Q_3, Q_4 have emitters connected to DC (V_{cc}) and bases connected to DC (V_b). These are functioning like current source active loads.

(a) The ac equivalent circuit is:

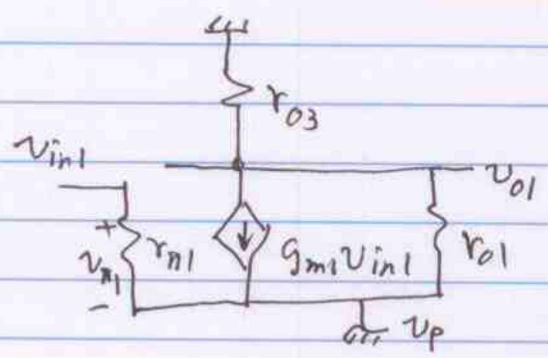


(b) When v_{in1}, v_{in2} are balanced differential signals, $v_p = 0$ (see lecture note derivation)

Each half of the circuit behaves as a CE-BJT amplifier.

Then $v_{o1} = -g_{m1}(v_{in1}) \cdot r_{o1} \| r_{o3}$

Similarly, $v_{o2} = -g_{m2}(v_{in2}) r_{o2} \| r_{o4}$



Assuming the BJTs are matched by pairs (i.e. $Q_3 \equiv Q_4$, $Q_1 \equiv Q_2$)

$v_{out} = v_{o1} - v_{o2}$

$= -g_{mn} \cdot r_{on} \| r_{op} \, v_{in1} + g_{mn} r_{on} \| r_{op} \, v_{in2}$

Q4 (Contd.)

Note: $r_{o1} = r_{o2} = r_{on}$
$r_{o3} = r_{o4} = r_{op}$
$g_{m1} = g_{m2} = g_{mn}$

$$v_{out} = v_{o1} - v_{o2}$$

$$= -g_{mn}\, r_{on} \parallel r_{op}\, (v_{in1} - v_{in2})$$

So $$\frac{v_{out}}{v_{in1} - v_{in2}} = -g_{mn} \cdot r_{on} \parallel r_{op}$$

In the above

$r_{o1} = r_{o2} = r_{on}$; $r_{o3} = r_{o4} = r_{op}$,

$g_{m1} = g_{m2} = g_{mn}$

From the given data: $I_{C1} = I_{C2} = I_{EE}/2 = 1mA$

$$r_{op} = \frac{V_{AP}}{I_C} = \frac{50V}{1mA} = 50\,k\Omega$$

$$r_{on} = \frac{V_{AN}}{I_C} = \frac{25V}{1mA} = 25\,k\Omega$$

$$g_{mn} = \frac{1mA}{25mV} = 0.04 \text{ mho}$$

Voltage gain $= -\,.04 \times 25k\Omega \parallel 50k\Omega = -666.67$ V/V