ELEC 312, Final, Dec'07, Soln

Q.1

$$A(s) = A_M \frac{(1 - s/\omega_z)}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

$$= 10^2 \frac{1 - s/10^9}{(1+s/10^6)(1+s/10^9)}$$

$$|A(s)| = 10^2 \frac{1}{\sqrt{1+\omega^2/10^{12}}}.$$

$$\angle A(s) = -\tan^{-1}\frac{\omega}{10^9} - \tan^{-1}\frac{\omega}{10^6} - \tan^{-1}\frac{\omega}{10^9}$$

$$= -\tan^{-1}\frac{\omega}{10^6} - 2\tan^{-1}\frac{\omega}{10^9} = -\phi$$

For gain margin, we need to find $\angle A(s) = -180^\circ$

Say $\omega = 10^6$, Then $\phi \Rightarrow 45^\circ + 2\tan^{-1}\frac{10^6}{10^9} < 180^\circ$

Say $\omega = 10^9$; Then $\phi \Rightarrow +\tan^{-1} 10^3 + 90^\circ \cong 180^\circ$

At $\omega = 10^9$, $|A(s)| = 10^2 \frac{1}{\sqrt{1+\frac{10^{18}}{10^{12}}}} \approx 10^2 \cdot \frac{1}{\sqrt{10^6}} = \frac{1}{10}$

$|A(s)\beta| = 0.1$, since $\beta = 1$ given

Gain margin $= 0 - 20\log|A(s)\beta| = 20\,dB$

With $A_M = 10^3$, $\beta = 1$, $GM = 0\,dB$

For phase margin, we need to set $|A(s)\beta| = 1$ first

i.e. $10^2 \frac{1}{\sqrt{1+\omega^2/10^{12}}} = 1$ or $1 + \frac{\omega^2}{10^{12}} = 10^4$ 10^6

Q1:

$$\frac{\omega^2}{10^{12}} \approx 10^4 \quad (10^6), \quad \omega^2 = 10^{16} \quad (10^{18}) \qquad \omega = 10^8 \text{ rad/sec} \quad (\omega = 10^9)$$

Then $\phi = \tan^{-1}\frac{10^8}{10^6} + 2\tan^{-1}\frac{10^8}{10^9} =$ (10^9) 180°

Phase margin $180^\circ - |\phi| = 180 - 100.8 = 2 \to 0^\circ$

For a phase margin of 60°, $\phi = 120^\circ$

i.e. $\tan^{-1}\frac{\omega}{10^6} + 2\tan^{-1}\frac{\omega}{10^9} \Rightarrow 120^\circ$, what is ω?

Say $\omega = 10^6$, ~~$\phi \Rightarrow 45^\circ$~~ $\phi = 1 + 2\tan^{-1}(.001) \approx 1.002$

$\omega = 10^8$; $\phi = \tan^{-1}10^2 + 2\tan^{-1}(.1) \Rightarrow 100^\circ.85$

$\omega = 2\times10^8$; $\phi = \tan^{-1}(200) + 2\tan^{-1}(.2) \Rightarrow 112^\circ.3$

$\omega = 4\times10^8$; $\phi = \tan^{-1}(400) + 2\tan^{-1}(0.4) \Rightarrow 133^\circ.45$

$\omega = 3\times10^8$; $\phi = \tan^{-1}(300) + 2\tan^{-1}(.3) \Rightarrow 123^\circ.2$

$\omega = 2.8\times10^8$; $\phi = \tan^{-1}(280) + 2\tan^{-1}(0.28) \Rightarrow 121^\circ.07$

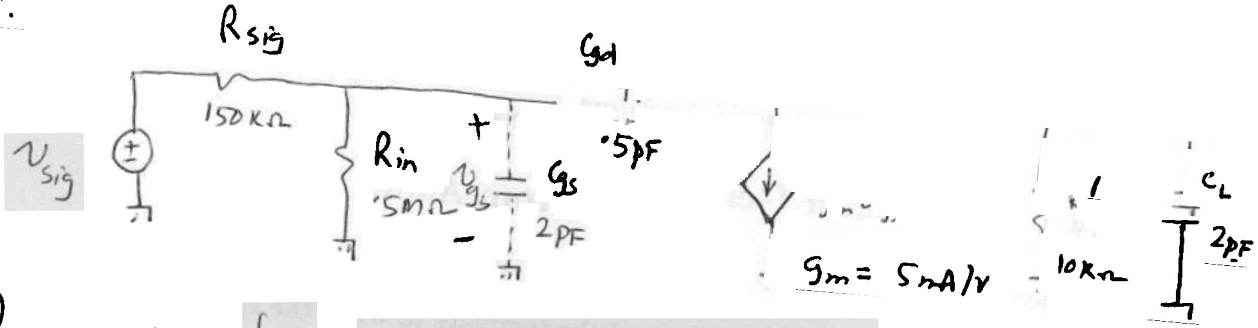
So we take $\omega = 2.8\times10^8$

Then $$|A\beta| = 10^2 \frac{1}{\sqrt{1+\frac{10^{16}\times 2.8^2}{10^{12}}}}\cdot\beta = 1$$

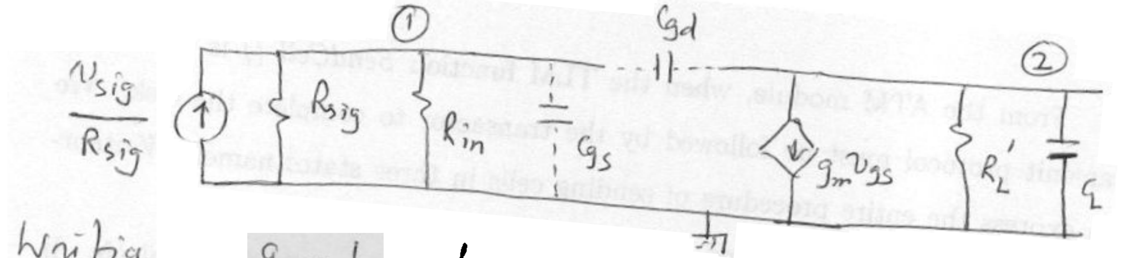
$$\beta = \frac{\sqrt{1+2.8^2\times10^4}}{10^2 \; 10^3} \cong 2.8 \quad \text{(a bit unrealistic)}$$

$\beta \quad 28$

Q2:



Reorganize for Nodal matrix formulation:



Writing $g = \frac{1}{R}$ for all cases

$$\begin{bmatrix} g_{sig} + g_{in} + s(C_{gd} + C_{gs}) & -sC_{gd} \\ g_m - sC_{gd} & sC_{gd} + g_L' + sC_L \end{bmatrix} \begin{bmatrix} V_{(1)} \\ V_{(2)} \end{bmatrix} = \begin{bmatrix} g_{sig} v_{sig} \\ 0 \end{bmatrix}$$

$$V_{(2)} = \frac{1}{\Delta} \begin{vmatrix} g_{sig} + g_{in} + s(C_{gd} + C_{gs}) & g_{sig} v_{sig} \\ g_m - sC_{gd} & 0 \end{vmatrix}$$

$$= -\frac{(g_m - sC_{gd})\, g_{sig} v_{sig}}{\Delta}$$

Where

$$\Delta = \begin{vmatrix} g_{sig} + g_{in} + s(C_{gd} + C_{gs}) & -sC_{gd} \\ g_m - sC_{gd} & g_L' + s(C_L + C_{gd}) \end{vmatrix}$$

Q2 contd

Thus

$$\Delta = \left[g_{sig} + g_{in} + s(C_{gd} + C_{gs})\right]\left[g'_L + s(C_L + C_{gd})\right] + sC_{gd}(g_m - sC_{gd})$$

$$= (g_{sig} + g_{in})\, g'_L + s^2 (C_{gd} + C_{gs})(C_L + C_{gd}) + s\left[(C_L + C_{gd})(g_{sig} + g_{in}) + g'_L (C_{gd} + C_{gs})\right] + s\, g_m C_{gd} - s^2 C_{gd}^2$$

$$= (g_{sig} + g_{in})\, g'_L + s\left[(C_L + C_{gd})(g_{sig} + g_{in}) + g'_L (C_{gd} + C_{gs}) + g_m C_{gd}\right] + s^2 (C_{gd} C_L + C_{gs} C_L)$$

Q2 (b) The gain-bandwidth is $= |\text{gain (at DC)}| \times$ 3dB BW

For gain at DC, we put $s \to j\omega = 0$ Then

$$A(0) = \frac{V_o(s)}{V_{sig}} \text{ at } \omega = 0 = \frac{-g_m\, g_{sig}}{(g_{sig} + g_{in})\, g'_L}$$

$$\Rightarrow -\frac{(g_m - sC_{gd})\, g_{sig}}{\Delta} \text{ at } s = 0$$

$$A(0) \quad -\frac{5 \times 10^{-3} \times 1/150 \times 10^3}{\frac{1}{150 \times 10^3} + \frac{1}{500 \times 10^3}}$$

note: $g_{sig} = \frac{1}{R_{sig}}$

$g_{in} = \frac{1}{R_{in}}$

Q (1)

$$|A(0)| = \frac{\quad}{\frac{1}{150 \times 10^3} + \frac{1}{500 \times 10^3}} = 577.73 \text{ V/V}$$

For 3dB BW, we shall take $\omega_{-3dB} \simeq \omega_{Dom}$

where ω_{Dom} = dominant high frequency pole

$$= \frac{g'_L (g_{sig} + g_{in})}{(C_L + C_{gd})(g_{sig} + g_{in}) + g'_L (C_{gd} + C_{gs}) + g_m C_{gd}}$$

Q2 contd

$$\omega_{Dom} = \frac{\frac{1}{10^4} \cdot \left[\frac{1}{150\times10^3} + \frac{1}{520\times10^3}\right]}{\left[(2\times10^{-12} + .5\times10^{-12})\left(\frac{1}{150\times10^3} + \frac{1}{520\times10^3}\right) + \frac{1}{10^4}(.5\times10^{-12} + 2\times10^{-12}) + 5\times10^{-3} \times .5\times10^{-12}\right]}$$

(2)

$$= \frac{8.67\times10^{-10}}{2.17\times10^{-17} + 2.5\times10^{-16} + 2.5\times10^{-15}} \text{ rad/sec}$$

$$= 312804.42 \text{ rad/sec} \rightarrow 49784.37 \text{ HZ}$$

So (1) gain-bandwidth $= 577.73 \times 49784.37$ HZ

$= 28761923.53 \rightarrow 28.76$ MHZ (approx)

Q3:

In class A operation i_{E1} will always remain non zero ie.

$$i_{E1} = i_L + I \geq 0$$

With signal swings at the input, i_{E1} goes down when i_L is <0 ie. v_I and v_o are negative going.

According to P_L and R_L values, we have

$$\frac{\hat{i}_L^2}{2} R_L = P_L = .5 \text{ watts}$$

(2) $\hat{i}_L^2 \quad \frac{1}{100} \quad .01\ A^2$, $\hat{i}_L = 0.1$ Amp

Thus, $\hat{i}_L$ swings between +, − 0.1 Amp.

(1) To keep $i_{E1} \geq 0$, I must be ≥ 0.1 Amp. Let $I = 0.1$ Amp

$v_o = i_L R_L = \pm .1 \times 100 = \pm 10V$.

(2) $v_o|_{min} = -10V$. To keep Q_2 in active region, we must have

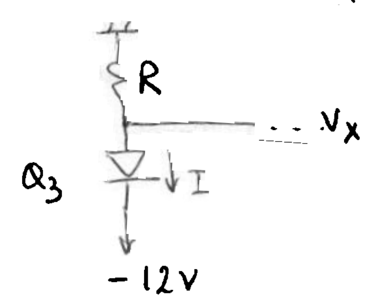
$$v_o|_{min} - V_{cc} \geq V_{CE(sat)} \text{ of } Q_2$$

$$-V_{cc} \geq 0.2 - (-10v) \rightarrow 10.2V$$

$$\Rightarrow V_{cc} \leq -10.2V \quad \text{, let } V_{cc} = -12V$$

(2) Similarly, let $V_{cc} = +12V$.

Based on $-12V$ for $-V_{cc}$, we can design the current mirror as follows



$V_X = -12V + 0.7V = -11.3V$. (2)

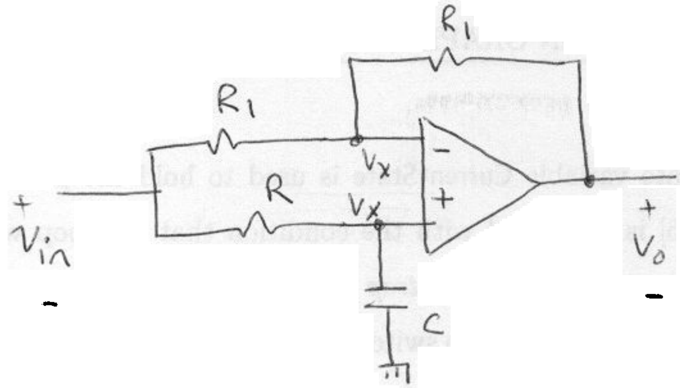
The current mirror carries $I = 0.1$ A

Then IR drop = 11.3V

$$R = \frac{11.3}{.1} = 113\ \Omega$$

(1)

Q4



Ideal OP-Amp has a virtual short between + and − input terminals. Let V_x be the voltage at these terminals. Then the following KCL apply

2 $$\frac{V_x - V_{in}}{R} + V_x sC = 0 \qquad \frac{V_x - V_{in}}{R_1} + \frac{V_x - V_o}{R_1} = 0$$

Thus, $V_x = \frac{V_{in} + V_o}{2}$ and

$$\frac{\cancel{V_{in}} + V_o - \cancel{2}V_{in}}{2R} + sC\cdot\left(\frac{V_{in} + V_o}{2}\right) = 0 \quad ; \quad \frac{V_o - V_{in}}{R} + sC V_{in} + sC V_o = 0$$

$$\frac{V_o}{R} + sCV_o + sCV_{in} - \frac{V_{in}}{R} = 0 \quad ; \quad V_{in}\left(\frac{1}{R} - sC\right) = V_o\left(\frac{1}{R} + sC\right)$$

4 $$V_{in}\frac{1 - sRC}{R} = V_o \frac{1 + sCR}{R} \qquad \frac{V_o}{V_{in}} = \frac{1 - sRC}{1 + sRC} = T(s)$$

2 $$\angle T(s) = -\tan^{-1}\omega RC - \tan^{-1}\omega RC = -2\tan^{-1}\omega RC = \phi$$

For $\phi = -30°$, $\tan^{-1}\omega RC = 15°$

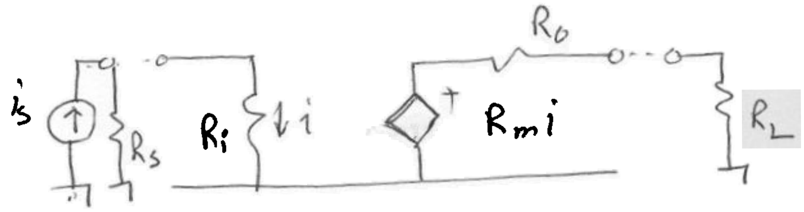
Then $\omega RC = \tan 15° = 0.2679$, Using values of ω and C as given

1 $$R = \frac{.2679}{2\pi \times 10^3 \times 10^{-8}} = 4264.5\ \Omega \times 2\pi \rightarrow 26K$$

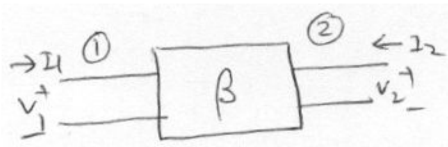
For $\phi = -90°$, $\tan^{-1}\omega RC = 45°$, $\omega RC = \tan 45° = 1$

1 $$R = \frac{1}{2\pi \times 10^3 \times 10^{-8}} = 15915.49\ \Omega \times 2\pi \rightarrow 100K$$

Q5. A TIA has an equivalent circuit



(2)



$R_m = \frac{100V}{mA} = 100 \times 10^3$

$R_i = 1K\Omega$

$R_o = 1K\Omega$

TIA is a CCVS So it has shunt-in, series-out configuration The β-network will have a shunt-in, shunt-out configuration. So it is to be modeled in terms of Y-parameters Thus for the β-network

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 \quad y_{21}V_1 + y_{22}V_2$$

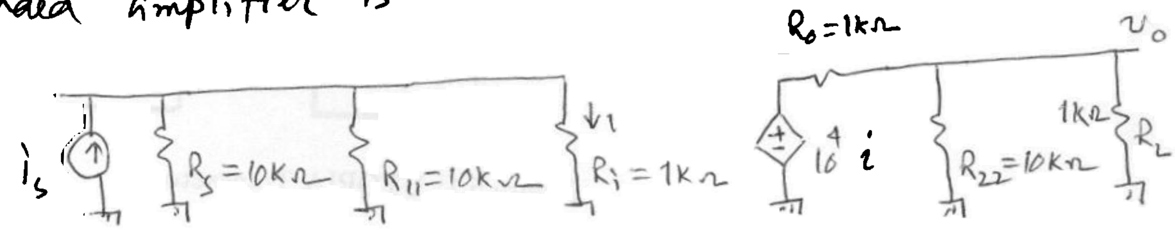
It is given that $\left.\frac{V_2}{I_2}\right|_{V_1=0} = 10K\Omega = \frac{1}{y_{22}} = R_{22}$

and $\left.\frac{V_1}{I_1}\right|_{V_2=0} = \quad R_{11} = \frac{1}{y_{11}} = 10K\Omega$

(2) Feedback factor is the y_{12} value $= \left.\frac{I_1}{V_2}\right|_{V_1=0} = 0.1 \frac{mA}{V}$

i.e. $\beta = 0.1 \times 10^{-3}$ mho

The loaded amplifier is

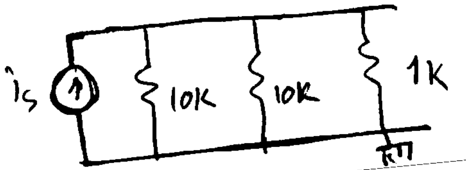


(1)

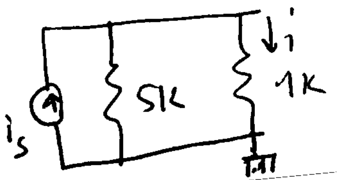
We need to find $A = \frac{v_o}{i_s}$ $\frac{V}{A} \to$ ohm

Q5

At the input side we can draw



as



$\frac{100}{11} = 9.09$

so that $i = i_s \frac{5K}{6K} = \frac{5}{6} i_s$

At the output

$$v_o = (10^4 i) \cdot \frac{R_{22} \parallel R_L}{R_o + R_{22} \parallel R_1}$$ by series division of voltage.

$$= 10^4 \cdot \frac{5}{6} \cdot i_s \frac{(10/11)K}{1K + \frac{10}{11}K} = 10^4 \frac{5}{6} \cdot \frac{0.909}{1.909} i_s$$

(2)

So $A = v_o / i_s = 10^4 \frac{5}{6} \times \frac{.909}{1.909} = \cancel{4145.73 \frac{V}{A}}$ $3968.05 \frac{V}{A}$

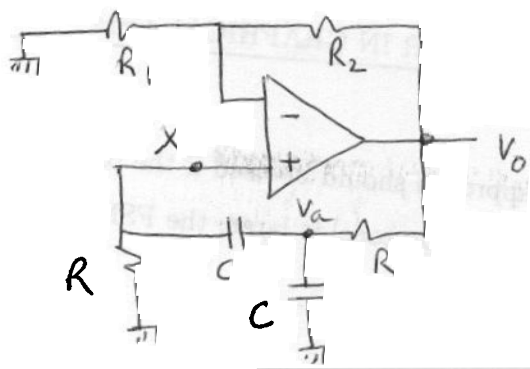
Then by the feedback formula

(1)

$$A_f = \frac{A}{1+A\beta} = \cancel{\frac{4145.73}{1+4145.73\times .1} = 9.9759 \text{ V/A}} = 9.976 \text{ ohm}$$

$$= \frac{3968}{1+3968\times .1} = 9.97$$

Q 6:



For ideal OP-AMP, with reference to node X,

$$V_o = \left(1+\frac{R_2}{R_1}\right)V_x = KV_x \quad \text{(say)}$$

In the lower path between V_x and V_o via V_a node, we have the KCLs

@ V_a node $\quad (V_a - V_x)sC + V_a sC + \frac{V_a - V_o}{R} = 0$

$$V_a\left(sC + sC + \frac{1}{R}\right) - V_x sC - \frac{V_o}{R} = 0 \quad \cdots (1)$$

@ V_x node $\quad \frac{V_x}{R} + (V_x - V_a)sC \quad 0$; or $V_a sC = V_x\left(sC + \frac{1}{R}\right)$

So $\quad V_a = V_x\left(1+\frac{1}{sRC}\right)$ sub in (1)

$$V_x\left(1+\frac{1}{sRC}\right)\left(\frac{1}{R} + 2sC\right) - V_x sC - \frac{V_o}{R} = 0$$

$$V_x\left[\frac{1+sRC}{sRC}\,\frac{1+2sRC}{R} - sC\right] = \frac{V_o}{R} = \frac{KV_x}{R}$$

So $$\frac{(1+sRC)(1+2sRC)}{sRC\cancel{R}} - sCR = \frac{K}{\cancel{R}}$$

$$1 + 3sRC + 2s^2R^2C^2 - s^2C^2R^2 = KsRC$$

Equating Re- and Im- parts of the equation in complex variable $s = j\omega$, we shall get

Q6 :

$K = 3$ ie $1 + \frac{R_2}{R_1} = 3$ is a condition for oscillation

2

$$1 - s^2C^2R^2 + 2s^2C^2R^2 = 0$$

$$\text{or } 1 + \omega^2C^2R^2 - 2\omega^2C^2R^2 = 0 \qquad \text{or, } \omega^2C^2R^2 = 1$$

$\omega = \frac{1}{CR}$ is the frequency of oscillation

Let $\omega_0 = \frac{1}{CR}$

We require $\omega_0 = 2\pi \times 10^4$ with $C = 10\mu F = 10^{-5}$

Then $R = \frac{1}{C\omega} = \frac{1}{10^{-5} \times 2\pi \times 10^4} = \frac{10}{2\pi} = 1.59\,\Omega$

2 For $K = 3$, we can choose $R_2 = 2K\Omega$, $R_1 = 1K\Omega$

making $K = 1 + \frac{R_2}{R_1} = 3$

Design values

$$\left.\begin{aligned} R &= 1.59\,\Omega \\ R_1 &= 1K \\ R_2 &= 2K \\ C &= 10\mu F \end{aligned}\right\}$$