ELEC 312, Ch 4, Problems

Q1: $A_o = 10^4$, $\omega_H = 100 \times 2\pi$ $\quad T(s) = \dfrac{A_o \omega_H}{s+\omega_H}$

With neg feedback $A_o \to A_f = \dfrac{A_o}{1+A_o\beta} = 50$

$1 + A_o\beta = \dfrac{A_o}{50} = \dfrac{10^4}{50} = 100 \times 2$; $\beta \simeq \dfrac{200}{10^4} = 2 \times 10^{-2}$

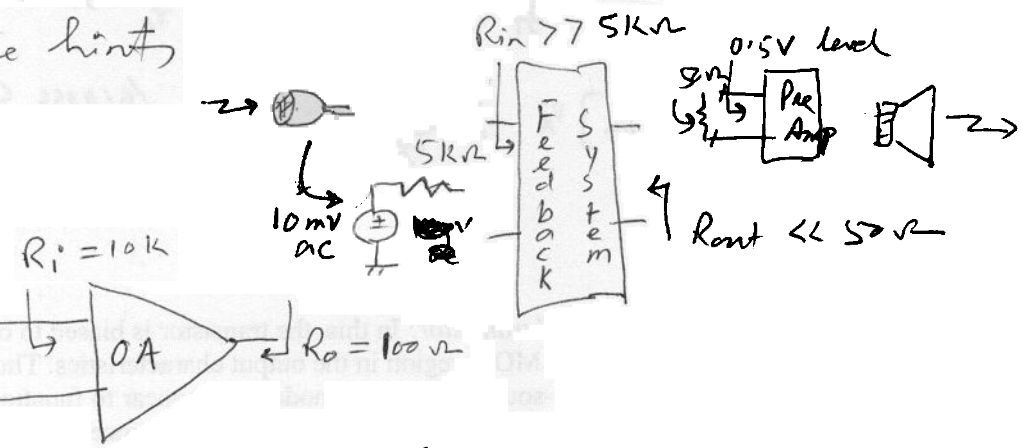
So (a) $T(s) = A(s) = \dfrac{A_o \omega_H}{s+\omega_H}$

(b) new $f_H|_{FB.} = (1+A_o\beta)$ times f_H without feedback.

So $\omega_{Hf} = (1+A_o\beta)\,\omega_H = 200 \times 2\pi \times 100$ rad/sec

(c) Feedback factor at lf $1+A_o\beta = 200$

Q2 Using the hints



We need to achieve $R_{if} \gg 5K$ and even $\gg 10K$

$R_{of} \ll$ 50Ω ie $\ll$ 100Ω

So a series in (to make $R_{in}\uparrow$) and shunt out (to make $R_o \downarrow$) feedback will be needed.

Also under feedback the voltage gain $\dfrac{0.5V}{10mV} = 50 = A_f$

The OA gain (open loop) $= 10^4$

$\dfrac{10^4}{1+10^4\beta} = 50 \qquad 10^4 = 50 + 10^4\, 50.\beta$

$\beta = \dfrac{10^4 - 50}{50 \times 10^4} = 0.0199$

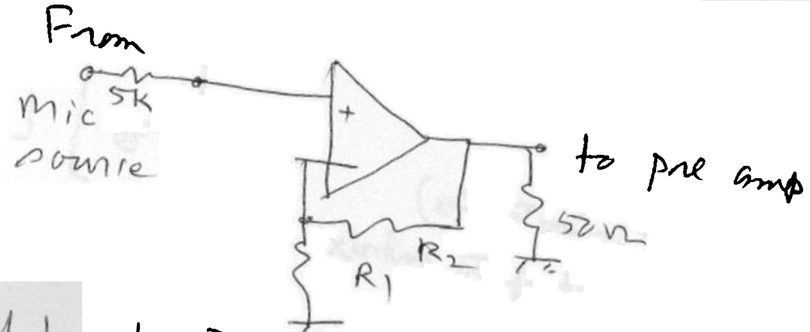
Q2

Further the feedback factor

$$1 + A_0\beta = \frac{10^4}{50} = 200$$

For series-in shunt out feedback the network will be like

and the overall feedback system will be like



We need to design R_1, R_2

Check P 4.7-4.8 in the ~~not~~ lecture note-pack

$h_{11} = R_1 \parallel R_2$; $R_{in} = 5K + 10K + h_{11}$

$\beta = \dfrac{R_1}{R_1 + R_2} = .0199$
$= h_{12}$

$R_{inf} = R_{in}(1 + A\beta) = 200\,(15K + h_{11})$

$R_{out} = R_o \parallel R_L \parallel R_{22}$, $R_{22} = \dfrac{1}{h_{22}}$ & $h_{22} \quad \dfrac{1}{R_1 + R_2}$

$R_{out}|_f = \dfrac{R_{out}}{1 + A\beta} = \dfrac{R_o \parallel R_L \parallel R_{22}}{200}$

Since $A_f = 50$ is required, $\beta \simeq .02$ is required

Let $R_1 = 1K$, then $\dfrac{1K}{R_2 + R_1} \approx .02$, $\dfrac{1K}{R_2 + 1K} = .02$

$1000 - 20 \quad R_2 \times .02$; $R_2 = 49\,000 = 49K$

Then $h_{11} = 1K \parallel 49K \approx 1K$, $R_{in} = 16K$

$R_{inf} \quad 16K \times 200 = 3200K = 3.2M\Omega$ is $>> 10K$

satisfactory

Q2 contd.

$h_{22} \rightarrow R_{22} = R_1 + R_2 = 50K$

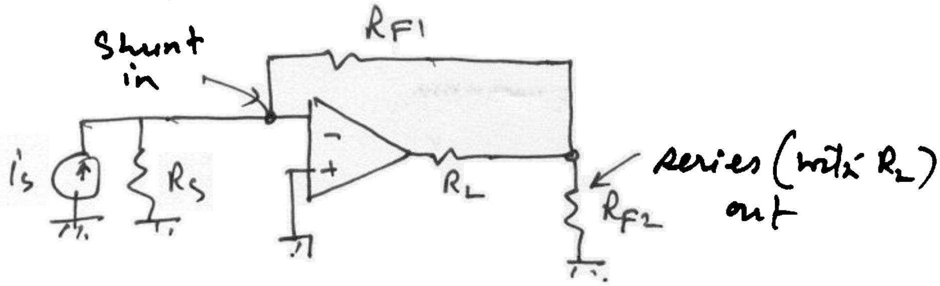
$R_{out} = \cancel{50\Omega} \parallel 100\Omega \parallel 50\Omega \parallel 50K \approx 33.33\Omega$

$R_{out}\big|_f = \dfrac{33.33\,\Omega}{200} = 0.17\Omega << 50\Omega$

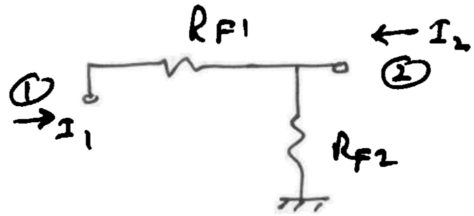
satisfactory.

So a design soln is $\left.\begin{matrix} R_1 = 1K \\ R_2 = 49K \end{matrix}\right\}$

Q3 A preferred feedback schematic will be (p 4.18 of lecture note pack)



The feedback network is



For shunt-in series out connection g-parameters need be calculated

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0} = \frac{1}{R_{F1}+R_{F2}}$$

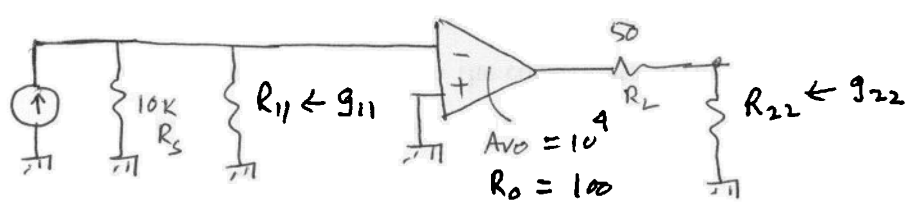
$$R_{11} \rightarrow R_{F1}+R_{F2}$$

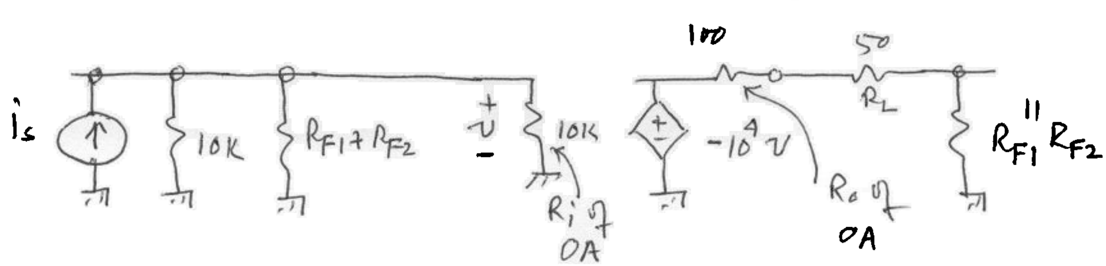
$$g_{12} \rightarrow \beta = \frac{I_1}{I_2}\Big|_{V_1=0} = \frac{-R_{F2}}{R_{F1}+R_{F2}}$$

$g_{21} \approx 0$ for one direction feedback only

$$g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0} = R_{F1} \parallel R_{F2}$$

The loaded A-circuit is now

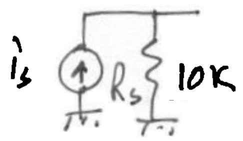




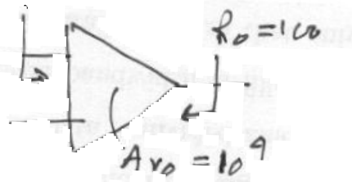
Q3

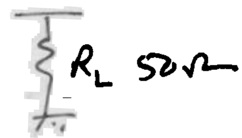
Closed loop $A_{if} = 10 \rightarrow \dfrac{A_{io}}{1+\beta A_{io}}$

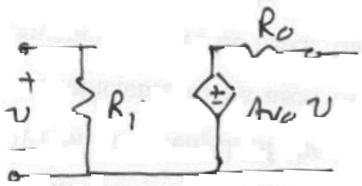
$A_{io} \rightarrow$ open loop current gain (i.e. without feedback)



$R_i = 10K$







OA equiv. cct.

To receive as much of i_s as possible at the input, the feedback system R_{in} should be $<< R_s \rightarrow 10K$.

But OP-AMP R_{in} is $= 10K$ → This has to be lowered

So 'shunt' connection at input is required

The output is a current signal across 50Ω.

The output of the feedback system should have a resistance $>> R_L \rightarrow 50\Omega$

But OA has $R_o = 100\Omega$, this is to be increased

Hence a 'series' connection at the output is

Q3 We yet do not know about R_{F1}, R_{F2} we could
get a clue about them via β ie

$$A_{if} = 10 = \frac{A_{io}}{1+\beta A_{io}}$$

So we need to find A_{io} ie i_o/i_s, i_o output current through $R_L = 50\,\Omega$

$$v = i_s \times Z, \quad Z = 10K \| 10K \| (R_{F1}+R_{F2}) = 5K \| (R_{F1}+R_{F2})$$

$$v = i_s \cdot \frac{(R_{F1}+R_{F2})\,5K}{5K + R_{F1}+R_{F2}}$$

$$i_o = -10^4 v \,/\, (100+50 + R_{F1} \| R_{F2})$$

$$= -10^4 \frac{1}{150 + \frac{R_{F1}R_{F2}}{R_{F1}+R_{F2}}} v$$

$$= -10^4 \frac{R_{F1}+R_{F2}}{150(R_{F1}+R_{F2}) + R_{F1}R_{F2}} \cdot \frac{(R_{F1}+R_{F2})\,5K}{5K+R_{F1}+R_{F2}} i_s$$

Let $X = R_{F1}+R_{F2}$; $Y = R_{F1} \cdot R_{F2} = R_{F1}(X - R_{F1}) = R_{F1}X - R_{F1}^2$

$$\frac{i_o}{i_s} = A_{io} = -\frac{10^4 \cdot X}{150X + Y} \cdot \frac{X \cdot 5000}{5000+X}$$

$$A_{io}\beta = -\frac{10^4 X}{150+Y} \cdot \frac{5000}{5000+X} \cancel{X} \cdot \frac{-R_{F2}}{\cancel{X}}$$

$$\therefore \beta = g_{12} = -\frac{R_{F2}}{R_{F1}+R_{F2}}$$

$$= \frac{10^4 \cdot 5000\, X\, R_{F2}}{(150+Y)(5000+X)}$$

$$A_{if} = \frac{A_{io}}{1+A_{io}\beta} = \frac{10^4 X^2\, 5000}{(150+Y)(5000+X)} \cdot \frac{(150+Y)(5000+X)}{[(150+Y)(5000+X) + 10^4 R_{F2}\, 5000\, X]}$$

Q3

$$A_{if} = 10 = \frac{10^4 X^2\, 5000}{(150+Y)(5000+X) + 10^4 R_{F2}\, 5000\, X}$$

We have two unknowns X, Y (i.e., R_{F1}, R_{F2}), but one equation. So we shall use trial method. Let $X = 10\,K \leftarrow R_{F1} + R_{F2}$ (such choice has limitation viz., g_{21} must be $\to 0$)

Then

$$10 = \cancel{\frac{10^4 X\, 5000 X\, 10^4 \times 10^4}{}}$$

$$10 = \frac{10^4\, 5000 \cdot (10^4)^2}{(150+Y)(5000+10{,}000) + 10^4 \cdot R_{F2} \cdot 5000 \cdot 10{,}000}$$

$$(150+Y)(15000) + 5 \cdot 10^{11} \cdot R_{F2} = 10^3 \cdot 5000 \cdot (10^4)^2 = 10^{14} \cdot 5$$

Now $Y = R_{F1} \cdot 10^4 - R_{F1}^2$

$R_{F2} = 10^4 - R_{F1}$

Subst. for Y & R_{F2} in terms of R_{F1}

$R_{F1} = -.33 \times 10^8,\ 9000.27$

Sdn # 1 $\rightarrow$ Acceptable is $R_{F1} = 9000$ (say), then $R_{F2} = 1000$

Had we taken $X = 30\,K$ (say) $\rightarrow 30{,}000$

Then $A_{if} = 10 = \dfrac{10^4 \cdot X^2\, 5000}{(150+Y)(5000+X) + 10^4 \cdot R_{F2} \cdot 5000 \cdot X}$

$$(150+Y)(5000+X) + 5 \times 10^7 \cdot R_{F2}\, X = X^2 \cdot 5 \cdot 10^6$$

Now using $Y = R_{F1} \cdot X - R_{F1}^2$

$R_{F2} = X - R_{F1}$, $X = 30 \times 10^4$

Sdn #2 $\rightarrow$ $R_{F1} = -0.428 \times 10^8,\ 27001.9$, Accept $R_{F1} \approx 27000$

Then $R_{F2} = 3000$

Q5

R_F is the feedback element, we need Y-parameters

For shunt-shunt feedback, we

for R_F

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

y_{11} $\quad \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{1}{R_F} \rightarrow R_{11} = R_F$ in shunt with input

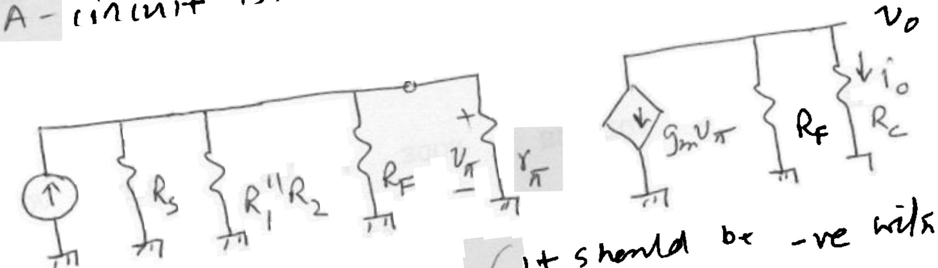
y_{12} $\quad \frac{I_1}{V_1}\Big|_{V_1=0} = -\frac{1}{R_F} \rightarrow \beta$ factor

y_{21} is to ignored

$y_{22} = \frac{1}{R_F} \rightarrow R_{22} = R_F$

So A-circuit is:

$i_s = \frac{v_i}{R_s}$



A (it should be -ve with unit of Ω)

Find v_o / i_s with $\beta = -\frac{1}{R_F}$

Then $A_f = \frac{A}{1+A\beta}$

Q6 i) $\beta =$ is given

We can find 'f' such that ($f = f_x$, say)

$|T(f)| = |A\beta| = 1$ and then

find if $\angle T(f)$ at this f_x = $-180°$ or not

If yes, the is unstable at f_x

If no, the system is stable at f_x

Alternatively, we can find $f = f_y$ where

$$\angle T(f) \rightarrow \angle \frac{100}{\left(1+j\frac{f}{10^5}\right)^3} = -180°$$

(zero phase margin case)

Then investigate $\left| \dfrac{100\beta}{\left(1+j\frac{f}{10^5}\right)^3} \right|$ at $f = f_y$.

Let's try with f_y

$$T'(f) = \frac{100}{\left(1+j\frac{f}{10^5}\right)^3} \rightarrow \angle T'(f) = -3\tan^{-1}\frac{f}{10^5} = -180°$$

(remember: for $\frac{1}{A+jB}$, $\angle$ is $-\tan^{-1}\frac{B}{A}$

for $A+jB$, $\angle$ is $+\tan^{-1}\frac{B}{A}$) review complex variable algebra !!

So, $\tan^{-1}\frac{f_y}{10^5} = 60°$, $\therefore \tan 60° = \frac{f_y}{10^5}$; $\therefore f_y = 1.732 \times 10^5$ Hz.

Then for $\beta = 0.2$ $|T(f)|$ @ $f = f_y$?

Q6

$$|T(f)| = \frac{100 \times 0.2}{\left[1 + \frac{f_y^2}{10^{10}}\right]^{\frac{3}{2}}}$$

$$\left(\text{recall} \quad \left|1 + j\frac{f}{f_a}\right| = \left[1 + \frac{f^2}{f_a^2}\right]^{\frac{1}{2}}\right)$$

$$= \frac{20}{\left[1 + \frac{1.732^2 \times 10^{10}}{10^{10}}\right]^{\frac{3}{2}}} = \frac{20}{\left[1 + 1.732^2\right]^{\frac{3}{2}}}$$

$$= \frac{20}{7.999} = 2.5$$

So $|T(f)| > 1 \rightarrow$ when $\angle T(f) = -180°$ at $f = 1.732 \times 10^5$

Case of potential instability

Investigate at $f = f_x$ with $\beta = 0.2$

$$\frac{100 \times .2}{\left[1 + \frac{f^2}{10^{10}}\right]^{3/2}} = 1 \quad \text{(zero gain margin case)}$$

$$20 = \left(1 + \frac{f^2}{10^{10}}\right)^{3/2} = \left(1 + \frac{f^2}{10^{10}}\right)^{1.5}$$

$$1 + \frac{f^2}{10^{10}} = (20)^{2/3} = 7.368$$

$$\frac{f^2}{10^{10}} = 6.368, \quad f = f_x = 252.35 \times 10^3$$

$$\approx 2.523 \times 10^5 \text{ Hz}$$

Q6

Now $\angle T(f)$ @ $f = f_x$

i.e. $\angle T'(f)$ at $f = f_x$

$$\angle T'(f) = -3\tan^{-1}\frac{f_x}{10^5} = -3\tan^{-1}(2.523)$$

$$= -3 \times (1.193 \text{ radian})$$

$$= -3 \times 68.37^\circ \simeq -205^\circ$$ which is $> 180^\circ$ in value

So the system will be unstable around $f_y = 2.523 \times 10^5$ Hz

So for $\beta = 0.2$, the system has potential instability.

Repeat for $\beta = .02$

Start with zero phase margin case i.e. $\angle T'(f) = -180^\circ$

We already know $f_y = 1.732 \times 10^5$ Hz

Then for $\beta = .02$, $$|T(f)| = \frac{100 \times .02}{\left[1 + \frac{f_y^2}{10^{10}}\right]^{\frac{3}{2}}}$$

$$= \frac{2}{[1 + 1.732^2]^{\frac{3}{2}}} = \frac{2}{7.999} \simeq 0.25$$

So $|T(f)| < 1$

System will be stable around $f = f_y = 1.732 \times 10^5$ Hz

Gain margin? → $0 - 20\log_{10}(0.25) = 0 - (-12)$
$= 12$ dB

↳ $0 - 20\log_{10}|T(f)|$

a positive gain margin ← stable

<Continue yourself>

Q7: $$T(f) = \frac{100\beta}{\left(1+j\frac{f}{10^3}\right)\left(1+j\frac{f}{5\times10^4}\right)\left(1+j\frac{f}{10^6}\right)}$$

For phase margin of 45°, we should get

$\angle T(f) = -135^\circ$ i.e. 45° below 180° (with a − for delay)

$\phi_M = 180^\circ - |\theta|$, $|\theta| = 135^\circ$

For causal system θ is negative

$\angle T(f) \equiv \angle T'(f)$ where $T'(f) = \frac{100}{D(f)}$

$\angle T'(f) = -\tan^{-1}\frac{f}{10^3} - \tan^{-1}\frac{f}{5\times10^4} - \tan^{-1}\frac{f}{10^6}$

This is non-linear We need to use iterations

We start with the second pole i.e. $f \approx 5\times10^4$

Then $\theta_1 = -\tan^{-1}\frac{5\times10^4}{10^3} = -\tan^{-1}(50) = -1.5507$ rad

$= -88.85^\circ$

$\theta_2 = -\tan^{-1}\frac{5\times10^4}{5\times10^4} = -\tan^{-1}(1) = -45^\circ$

$\theta_3 = -\tan^{-1}\left(\frac{5\times10^4}{10^6}\right) = -\tan^{-1}(.05) \rightarrow -2.86^\circ$

So $\theta = -88.85 - 45 - 2.86 = 136.7^\circ$

This is slightly $> 135^\circ$. So we can run another trial with $f < 5\times10^4$, say 4.8×10^4

Find 'θ' again If $\theta \approx 135^\circ$ at this choice of f, let $f = f_x$

Now evaluate $|T(f)| = 1$ @$f=f_x$ then solve for β.

Q 8 Solve in line of solutions to Problem #6 & #7

Q 9: Hint is sufficient