EXPERIMENT 1

PASSIVE FILTERS (LCR BASED)

(SIMULATION)

OBJECTIVE

To build highpass, lowpass and bandpass LCR filters[1] using circuit simulation tools.

INTRODUCTION

Ladder networks are filters of the first kind, built in the early part of last century, before op-amp technology was available. The lossless, or reactance ladder, shown in Fig. 1, is still used today in applications where op-amp are not suited. For example, if the required power is beyond the capability of the op-amp, or for high frequency applications. The ladder circuit shown here is considered a prototype lowpass filter, and therefore frequency transformations may be applied to this structure to obtain different responses, and hence more complicated ladder structures.
The synthesis of a passive filter can be accomplished via many different methods. In this lab, you will design lowpass, highpass and bandpass lossless ladder filters using the frequency transformation techniques. The circuits will exhibit Butterworth and Chebyshev responses, in singly and doubly terminated configurations. The voltage transfer functions are all-pole functions and are characterized by:

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{k}{b_n s^n + b_{n-1} s^{n-1} + \dots\dots\dots + b_o}$$

THE LOWPASS LOSSLESS LADDER

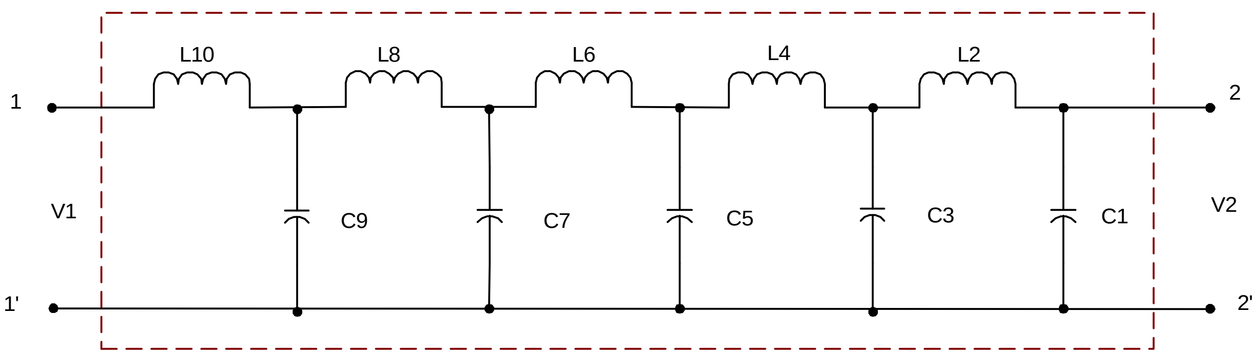

Fig. 1

[1] Please consult the text book , Ch.4 for your understanding.

In the low-pass ladder network all the series elements are inductors and the shunt elements are capacitors. The synthesis for a singly terminated ladder network is outlined here.

SINGLY TERMINATED NETWORKS

Given a singly terminated network with a transfer function which represents a Butterworth or Chebyshev response, we can express V_{out}/V_{in} as a ratio of polynomials Q(s), M(s) and N(s).

$$\frac{V_{out}}{V_{in}} = \frac{Q(s)}{M(s)+N(s)}$$, if the degree of M(s) > the degree N(s), or,

$$\frac{V_{out}}{V_{in}} = \frac{Q(s)}{N(s)+M(s)}$$, if the degree of N(s) > the degree M(s)

Where M(s) = even portion of the denominator, and N(s) = odd portion of the denominator. If the degree of M(s) > degree of N(s), we synthesize Y22 = M(s) / N(s), and the first element of the ladder is a capacitor in shunt. Otherwise we synthesize Z22 = N(s) / M(s) and the first element is an inductor in the series (where Y22 and Z22 are two-port network parameters). The polynomial division can be done by long division, the division algorithm are capable of performing polynomial division. The normalized design is performed for a 1 ohm load and a cut-off frequency of 1 rad/sec. Then the circuit is scaled accordingly. If the denominator has a degree n then there are a total of n L & C elements.

EXAMPLE:

Synthesize the function;

$$H(s) = \frac{1}{s^4 + 2.613s^3 + 3.414s^2 + 2.613s + 1}$$

As a singly terminated LC ladder network. By inspection,

$M(s) = s^4 + 3.414s^2 + 1$ and $N(s) = 2.613s^3 + 2.613s$

Since the degree of M(s) > the degree of N(s) we synthesize Y22. Therefore,

$$Y22 = \frac{s^4 + 3.414s^2 + 1}{2.613s^3 + 2.613s}$$

The results of the division algorithm are;

0.383s	1	3.414	1
1.082s	2.613	2.613	
1.577s	2.414	1	

$$1.531s \,\overline{\big)\, 1.531 \qquad\qquad}$$
$$1$$

Since M(s) > N(s) the first element in the ladder network is a capacitor in shunt, and the element values correspond to the results in the division algorithm. These elements are considered from the load looking toward the source (hence Z22 or Y22).

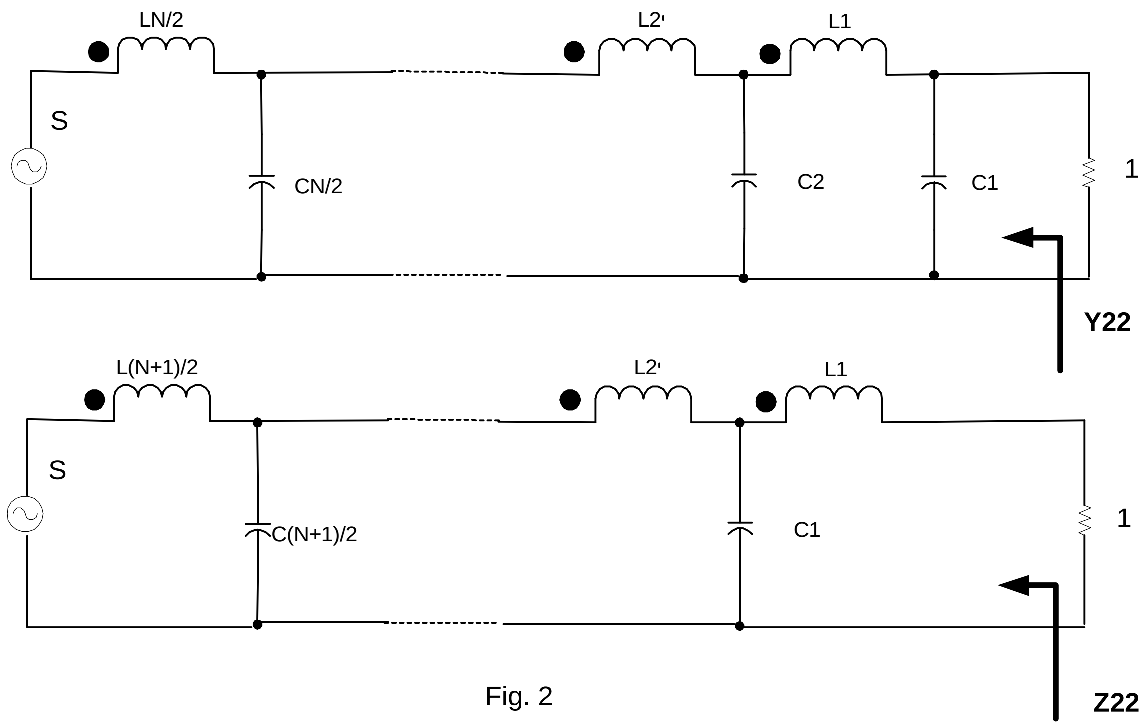

Fig. 2

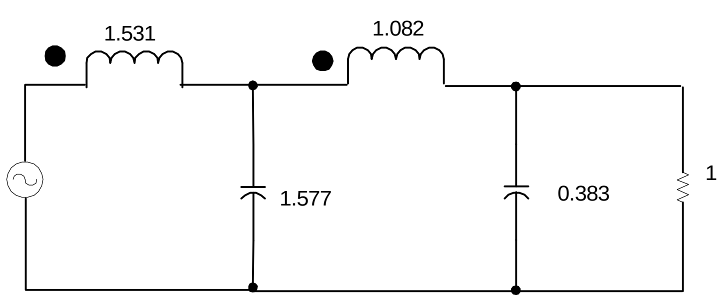

Fig. 3

DOUBLY TERMINATED NETWORKS

The doubly terminated networks are similar to the singly terminated networks in their synthesis. We always design a normalized filter (1 ohm resistors and a cut-off frequency of 1 rad/sec). And then magnitude and frequency scale the components. With doubly terminated networks we synthesize Zin. If the order of the numerator is smaller than the order of the denominator, the network starts with a capacitor in shunt, otherwise if the order of the numerator is greater than the order of the denominator, the ladder starts with an inductor in series. The most mechanical process involved in these designs is obtaining Zin. This procedure will be covered in your lectures. We will work from the assumption that you are given Zin.

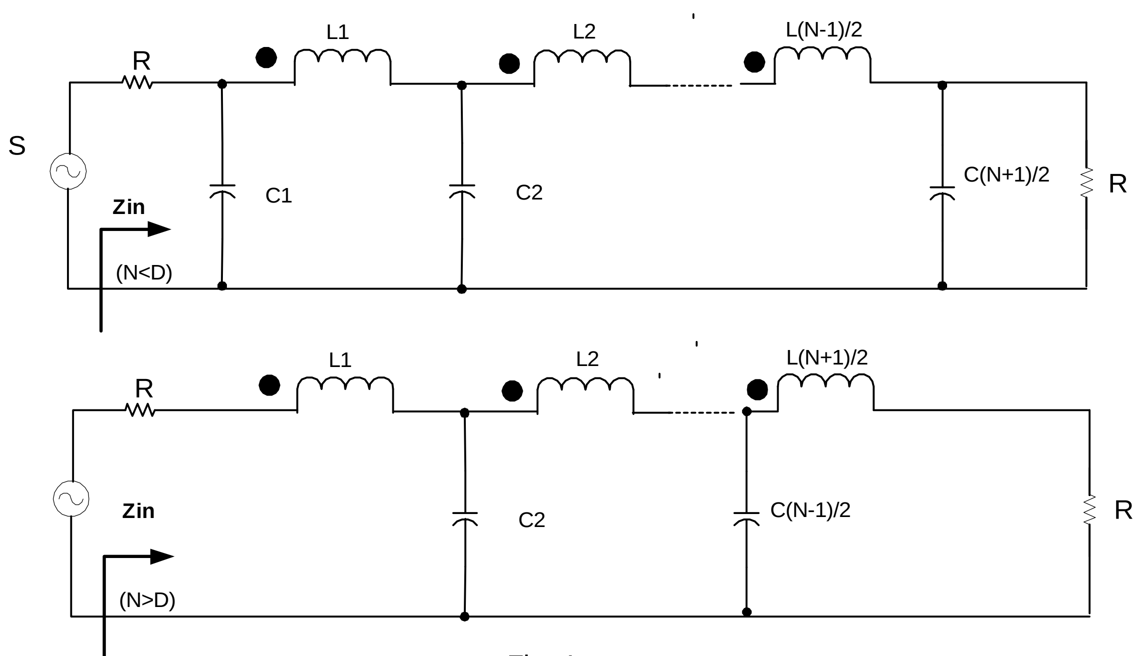

Fig. 4

Given:

$$Z_{IN} = \frac{3.2361s^4 + 5.236s^3 + 5.236s^2 + 3.2361s + 1}{2s^5 + 3.2361s^4 + 5.236s^3 + 5.236s^2 + 3.236s + 1}$$

Polynomial division yields the following components, for the nominal low-pass transfer function.

$R_{Ln} = 1.0\ Ohm$

$C_{1n} = 0.618F$

$L_{2n} = 1.618H$

$C_{3n} = 2F$

$L_{4n} = 1.618H$

$C_{5n} = 0.618F$

Using a termination of $R_L = 100\, Ohm$ and $w_o = 10^4$ rad/sec, $K_m = 100$ and $K_f = 10^4$, and the circuit is shown in Fig. 5 $(L \Rightarrow L_n\ K_m / K_f,\quad C \Rightarrow C_n / K_m K_f,\quad R \Rightarrow R_n K_m)$. Note that $K_m = R_f/R_i$ and $K_f = \omega_f/\omega_i$. Here the subscript f stands for the final value and the subscript I stands for the initial value to which the scaling is applied.

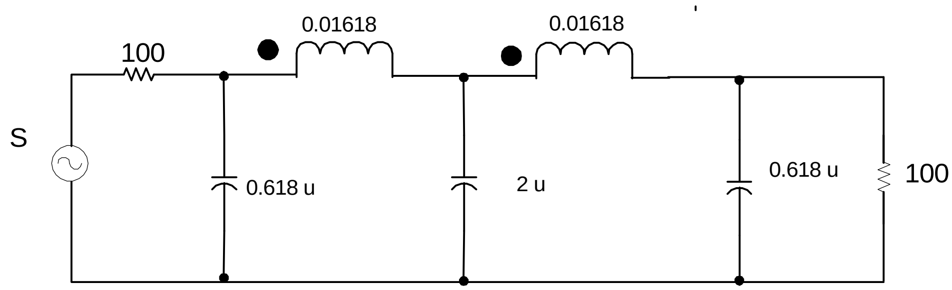

Fig. 5

FREQUENCY TRANSFORMATIONS

The normalized, doubly-terminated, lowpass filter, (R1=R2=1 and $w_o = 1$ rad/sec) can be transformed into a highpass or bandpass filter.

HIGHHPASS TRANSFORMATION

With the normalized circuit;
Replace the inductors by capacitors with the values 1/L.
Replace the capacitors by inductors with the values 1/C.
Once the transformation is done, magnitude and frequency scaling is applied.

BANDPASS TRANSFORMATION

A bandpass filter can be realized by synthesizing a normal lowpass ladder circuit, and then appling the following transformations. Series inductors are replaced by a series combination of an inductorand a capacitor, with the values $L_{bp} = {L_{1p}}/{BW.2\pi}$ and $C_{bp} = {BW.2\pi}/{(L_{1P})(w_o^2)}$ as shown in Fig. 6.

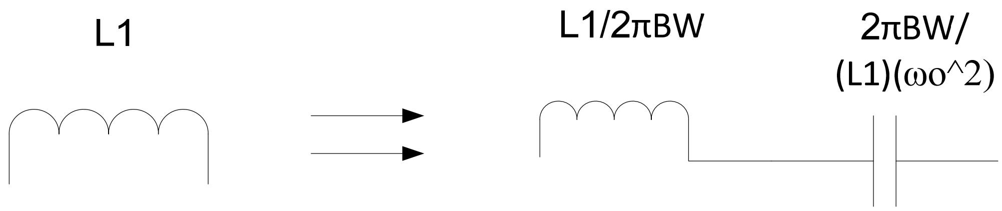

Fig. 6

Shunt capacitros are replaced by a parallel combination of a capacitor and an inductor, with the values $C_{bp} = {C_{lp}}/{2\pi BW}$ and $L_{bp} = {BW 2\pi}/{(C_{lp})(w_o^2)}$ as shown in Fig. 7 . Here ω_o is the center frequency of the band-pass filter.

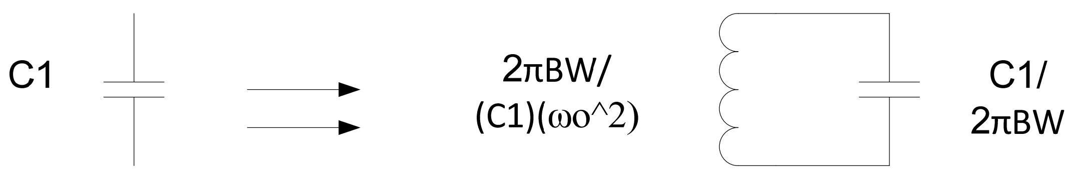

Fig. 7

Note that the bandwidth (BW) is calculated in Hz and the center-frequency (ω_o) is calculated in rad/s.

PRE-LAB

1. Synthesize the following transfer function, of a singly terminated network, for a 1K load resistor. The normalized circuit operates at 1 rad/sec. Scale the circuit for a cut-off frequency of 10KHz. $T(s) = \dfrac{1}{s^5 + 3.236s^4 + 5.236s^3 + 5.236s^2 + 3.236s + 1}$
2. Synthesize the transfer function, of a doubly terminated network terminated in 100 ohm resistors, using Zin given below. Use the closest available capacitros. The normalized circuit operates at 1 rad/sec. Scale the cicuit for a cut-off frequencyof 8.6KHz.
$$Z_{in}(s) = \frac{2s^5 + 1.1725s^4 + 3.1875s^3 + 1.3096s^2 + 1.065s + 0.1789}{1.1725s^4 + 0.6873s^3 + 1.3096s^2 + 0.44s + 0.1789}$$
3. Convert the lowpass filter designed in part 2 of the prelab to a highpass filter using the frequency transformation techniques.

4. Convert the LPF in step 2 to a BPF with a bandwidth (BW) of 5KHz. Assume that the center frequency (ω_o) for the BPF is 20 KHz ($\omega_o=2\pi \times 20\times10^3$ rad/s). All the resistors are 1 Ω.

PROCEDURE

1- Simulate the circuit in part 1 of the pre-lab using sinusoidal signal with 1V peak-to-peak and frequency sweep from 100Hz to 30 KHz.
2- Repeat step 1 for the circuit in part 2 of the pre-lab.
3- Repeat step 1 for the circuit in part 3, and 4 of the pre-lab. Do the simulation.While simulating part 4 of pre-lab, it is better to sweep between 10 KHz and 50 KHz. In the simulation settings use 100 points per decade to get a smooth curve. Verify the functionality. In all these cases attach the circuits and the output with your lab report.