EXPERIMENT 2

ACTIVE FILTERS

(EXPERIMENTAL)

OBJECTIVE

To design second-order low –pass filters using the Sallen & Key (finite positive- gain) and infinite-gain amplifier models. One circuit will exhibit a Butterworth response and the other will be based on a Chebyshev response. Scaling techniques will be used to ensure practical component values.

INTRODUCTION

Active filters are constructed using a combination of resistors, capacitors and an active device. The active device is usually an op-amp. The main advantages of these filters are:

-Expensive and bulky coils are eliminated.
-Arbitrary circuit gain can be realized.
-High input/low output impedances yield good isolating properties.

Some disadvantages include:

-Op-amps require power supplies.
-At higher frequencies op-amp gain is reduced considerably.
-The circuit is sensitive to component change.

Two common filter configurations are the Sallen & Key positive gain and the Negative-feedback class of filters. In this lab you will design and build, a second –order low-pass Butterworth filter using the Sallen & Key model, and a second-order low-pass Chebyshev filter using the Negative-feedback model.

BUTTERWORTH RESPONSE

A Butterworth response exhibits specific characteristics. A Butterworth response has a transfer function

$$|T_n(jw)|^2 = \frac{1}{1+\left(w/w_0\right)^{2n}}$$

For a normalized frequency, $w_o = 1$ the transfer function is:

$$|T_n(jw)|^2 = \frac{1}{\sqrt{1+\left(w\right)^{2n}}}$$

Where w_o = *half -power-frequency*, and n= number of poles

Let $w = w_o$, then $|T_n(jw)|^2 = 0.5$, and $|T_n(jw)| = 0.707$, or
$20\log|T_n(jw)| = 20\log(0.707) = -3dB$

The Butterworth characteristics can be summarized:

1. $|T_n(jw)|$ at $w = 0$ is 1 for all n.
2. $|T_n(jw)|$ at $w = 1$ is 0.707 for all n.
3. For a large w, $|T_n(jw)|$ exhibits n-pole roll-off.
4. The response is maximally flat, since the first *(n-1)* derivatives of the Taylor series expansion of $|T_n(j\omega)|$ is zero at $w = 0$ (i.e., DC).

CHEBYSHEV RESPONSE

The Chebyshev or *equal-ripple* approximation has a transfer function:

$$|T_n(jw)|^2 = \frac{1}{1+\varepsilon^2 C_n{}^2(w)}$$

Where $C_n(w) = \cos(n\cos^{-1} w)$ for $0 \le w \le 1$

$C_n(w) = \cosh(n\cosh^{-1} w)$ for $w > 1$.

w is the normalized frequency with respect to the pass-band edge frequency w_o.
$T_n(jw)$ is the n^{th} order Chebyshev polynomial. Therefore:

$$|T_n(jw)| = \frac{1}{\sqrt{1+\varepsilon^2 C_n{}^2(w)}}$$

The Chebyshev characteristics can be summarized as follows.

1. At $w = 0$;

$$|T_n(j0)| = \frac{1}{\sqrt{1+\varepsilon^2}} \qquad C_n(0) = 1 \text{ for n even}$$

$$|T_n(j0)| = 1 \qquad C_n(0) = 0 \text{ for n odd}$$

2. At $w = 1$;

$$|T_n(j0)| = \frac{1}{\sqrt{1+\varepsilon^2}} \qquad C_n(1) = 1 \text{ for all n}$$

The magnitude of the Chebyshev response decreases monotonically in the stop band. Because of the ripples in the pass-band the Chebyshev filter exhibits a sharper attenuation in the stopband than a Butterworth filter of the same order.

SALLEN & KEY FILTER

The Sallen & Key low-pass filter which uses a positive gain amplifier is shown in figure 1.

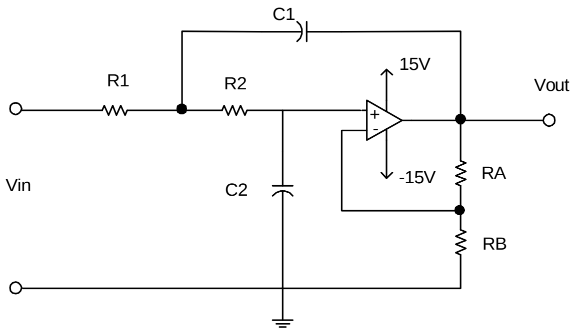

Fig. 1 Sallen & Key Low-pass Model

A special case of the Sallen & Key model is the unity gain S&K shown in figure 2.

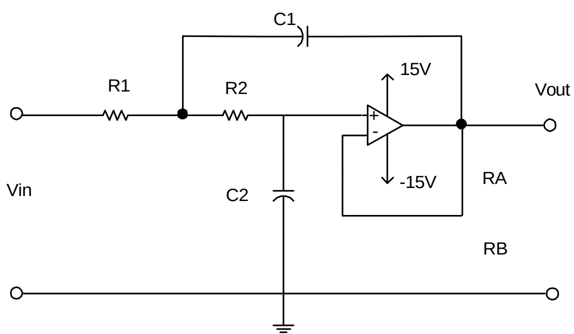

Fig. 2 Sallen & Key Low-pass Model Unity Gain

INFINITE GAIN AMPLIFIER BASED MULTIPLE FEED BACK SINGLE AMPLIFIER BIQUADRATIC (SAB) LOW-PASS FILTER

The multiple feedback (infinite gain amplifier based) low-pass filter is shown in figure 3.

Comment [u1]: Change the figure caption to: “Infinite gain multiple loop SAB (low-pass) filter”

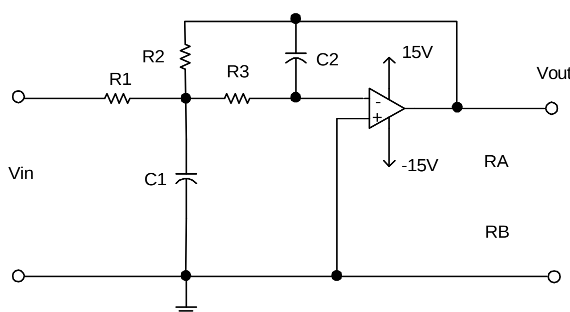

Fig. 3 Sallen & Key Low-pass Model Unity Gain

SCALING (MAGNITUDE & FREQUENCY SCALING)

A circuit can be designed for practical component values employing magnitude scaling. To magnitude scale components the following equations are used.

$$R_{new} = K_m \ R_{old}$$
$$L_{new} = K_m \ L_{old}$$
$$C_{new} = C_{old} \big/ K_m$$

For example if the design value of the capacitor is 0.1μF and you only have 1uF capacitors, then you need a scaling factor, $K_m = C_{old} \big/ C_{new} = 10^{-6} / 10^{-7} = 10$.

A circuit designed for one frequency can be transformed to operate at another frequency by employing frequency scaling. The equations for frequency scaling are similar to the magnitude scaling equations. With the exception that resistors are not frequency scaled.

$$L_{new} = \frac{L_{old}}{K_f}, \qquad C_{new} = \frac{C_{old}}{K_f}$$

Where $K_f =$ ratio of final to initial cut-off frequencies in radians = w_f/w_i.

This technique is particularly useful when you design a filter that is normalized for $w_i = w_o = 1$.

In this case $K_f = w_f$.

PROCEDURE

1. Design a second- order Butterworth low-pass filter using the Sallen & Key model (Fig. 1) for the cut-off frequency of 10KHz. Design the circuit to operate using 1nF capacitors and equal resistors. The pole frequency (cut-off frequency) is given by: $w_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$. Apply a 1V peak-peak sinusoidal input signal, starting at 1 KHz and sweep through the cut-off frequency to 25 KHz, in steps of 500Hz. Make sure to take more readings, at smaller intervals, around the cut-off frequency i.e. from 9 KHz to 15KHz reduce the interval size to 250Hz.
2. Design a second-order Chebyshev low-pass filter with a pass-band ripple of 1 dB, using the configuration in Fig.3, for the pole- frequency of 10KHz. Design the circuit to operate using 10nF capacitors. Apply a 1V peak-peak sinusoidal input signal, starting at 1KHz and sweep through the cut-off frequency to 25KHz, in steps of 500Hz. Make sure to take more readings, at smaller intervals, around the cut-off frequency, i.e.: from 9KHz reduce the interval size to 250Hz.
3. Using the frequency scaling technique, re-design and test the filter in step-2 for a cut-off frequency of 1 KHz

RESULTS & DISCUSSION

1. Derive the transfer function V_{out}/V_{in} for the filter circuits in figures 1, 2, and 3.

2. Plot experimental and theoretical curves (V_{out}/V_{in}) on the same graph for the filters in Figs. 2 and 3.
3. What is the value of Q_p produced by your design in procedure step #1?