Q2 (Alt.) — UG class only

ELEC 441, MT2
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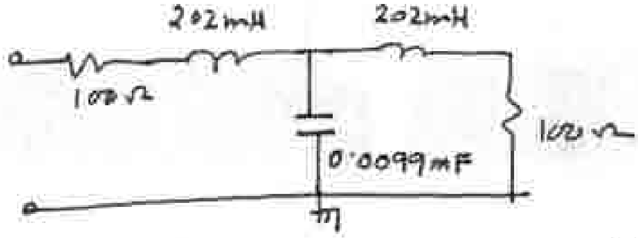
The frequency denormalized and impedance scaled ~~LC to~~ L, C, R ladder filter is:



For FDNR technique we consider an impedance scaling by $\frac{1}{Ks}$. So

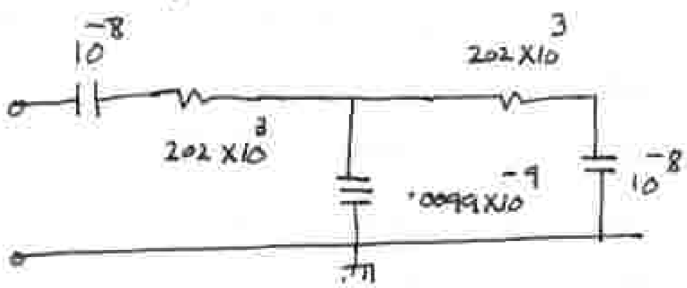
$$R \to \frac{R}{Ks} = \frac{1}{s\left(\frac{K}{R}\right)} = \frac{1}{s\bar{C}} \quad ; \quad \bar{C} = \frac{K}{R}$$

$$L \to sL/Ks = \frac{L}{K} = \bar{R} \quad ; \quad \bar{R} = \frac{L}{K}$$

$$C \to \frac{1}{sC}\frac{1}{Ks} = \frac{1}{s^2}\cdot\frac{1}{KC} = -\frac{1}{\omega^2}\cdot\frac{1}{D} \quad ; \quad D = KC$$

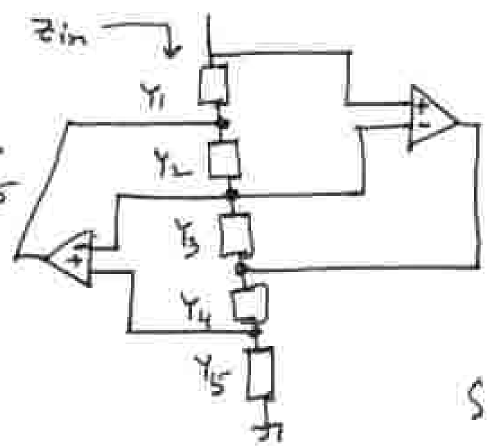
Let $K = 10^{-6}$

The new filter will thus look like:



For the super capacitor D, we use the GIC circuit

$$Z_{in} = \frac{Y_2 Y_4}{Y_1 Y_3 Y_5}$$



~~With Y_2, Y_4, Y_3 ~~AE~~ G_x each~~

~~& $Y_1, Y_2, Y_5 =$ sC_x each~~

~~$Z_{in} = \frac{1}{sC_x^2}\cdot\frac{D}{G_3}G_x$~~

~~$= -\frac{1}{\omega^2}\cdot\frac{1}{C_x}\cdot G_x$~~

~~So $D = KC = \frac{G_x}{G_x}$~~

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But here $D = .0099 \times 10^{-9} = \frac{1}{C_x G_x^3}$

Let $G_x \to \frac{1}{R_x} = 10^{-2}$ i.e. $R_x = 100\,\Omega$

Then $\frac{1}{C_x} = .0099 \times 10^{-9} \times 10^{-6} = .0099 \times 10^{-15}$

But here $D = .0099 \times 10^{-9} = \cancel{\frac{G_x}{C_x}}\ \frac{C_x}{G_x}$

$C_x = G_x \times 99 \times 10^{-13}$

If $G_x = 10^{-2}$, i.e. $R_2, R_3, R_4 = 100\,\Omega$ each

$C_x = 99 \times 10^{-15} = 99\,f = .099\,pF.$

Q2 Alt:

With $Y_2, Y_4, Y_3 = G_x$, $Y_1, Y_5 \to sC_x$ each

$Z_{in} = \frac{G_x}{s^2 C_x^2} = -\frac{1}{\omega^2 C_x^2} G_x$; So $D = \frac{C_x^2}{G_x}$

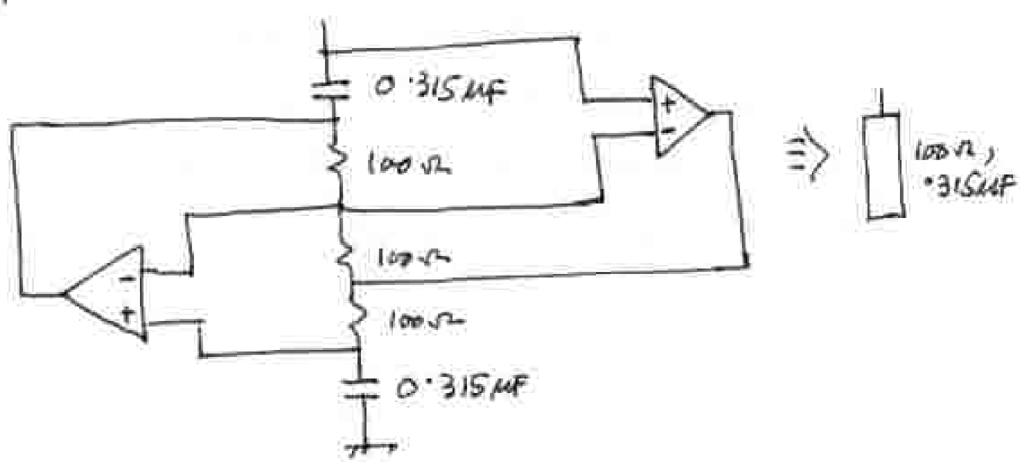
But here, $D = .0099 \times 10^{-9} = \frac{C_x^2}{G_x}$

Let $G_x = 10^{-2}$ i.e. $R_2 = R_4 = R_3 = 100\,\Omega$

So $C_x^2 = .0099 \times 10^{-11} = 99 \times 10^{-15}$

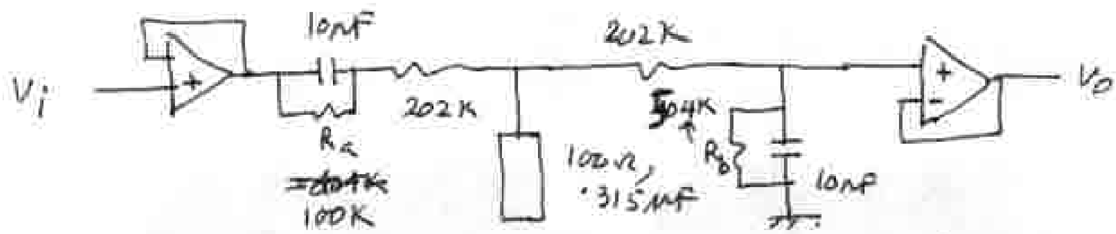
$C_x = \sqrt{99 \times 10^{-15}} = 0.315 \times 10^{-6}$

So design for D is:



Q2 Alt. ← UG class only

The overall complete system will include buffers at input and output, together with resistive paths through the filter structure. Thus:



DC gain of the designed network is:

$$\frac{R_b}{R_b + R_a + 404K} = \text{DC gain of original ladder filter} = 0.5$$

$$R_b = 0.5 R_a + 202K + 0.5 R_b \quad ; \quad \text{If } \cancel{R_a = R_b = 0.5R}, \ \cancel{0.5R = 202K}$$

$$\cancel{R = 404K} \quad R_b = R_a + 404K \quad ; \text{ let } R_a = 100K, \text{ Then } R_b = 504K$$

This completes the system design.

$$\left\langle \frac{504K}{100^K + 404^K + 504^K} \Rightarrow 0.5 \text{ DC gain.} \right\rangle$$

Q3:

$$T_N(s) = \frac{s^2+1}{s^2+1.5s+1}$$

One can frequency denormalize by $\Omega_o = \sqrt{\Omega_L \Omega_H} = 2000$ rad/sec.
or, one can design the filter for

$$T_N = \frac{s^2+1}{s^2+1.5s+1} \quad \text{and}$$

then use frequency denormalization.
After frequency denormalization, one gets

$$T(s) = \frac{s^2/\Omega_o^2 + 1}{\frac{s^2}{\Omega_o^2} + 1.5\frac{s}{\Omega_o} + 1} = \frac{s^2+\Omega_o^2}{s^2+1.5\,s\,\Omega_o+\Omega_o^2}$$

Example design using state variable technique { Possible Design #1 }

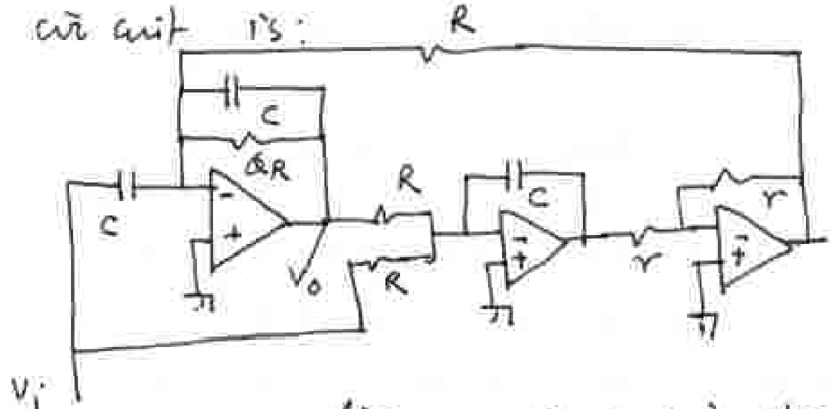
Tow-Thomas biquad

For $T_N(s)$, we see $T_N(s)\big|_{s\to\infty} = 1$; $\Omega_n^2 = \Omega_p^2 = 1$

So $C_1 = C$; $R_1 = $ open (does not exist)

$R_2 = R \cdot \left(\frac{\omega_p}{\omega_n}\right)^2 / \text{hf gain} = R$ $\because$ $\omega_p = \omega_n$ here. & hf. gain $= 1$

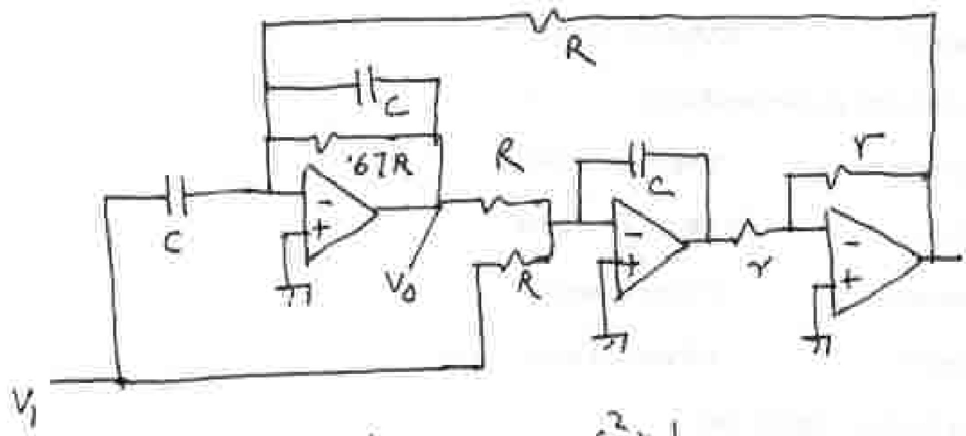
The circuit is:



$\frac{\omega_p}{Q} = $ coeff. of s in denom. $= 1.5$, while $\omega_p^2 = 1$

$$Q = \frac{\omega_p}{1.5} = \frac{1}{1.5} = 0.67$$

Thus the circuit is:



$$T_N(s) = \frac{V_0}{V_1} = \frac{s^2+1}{s^2+1.5s+1}$$

For normalized filter we take $R=1$, $r=1$, $C=1$.

For frequency denormalized filter, $\because \Omega_p = 2000$ rad/sec, we can take $R = \frac{1}{\sqrt{2000}}$ & $C = \frac{1}{\sqrt{2000}}$, so that $\frac{1}{RC}$ becomes 2000. ~~we can keep~~

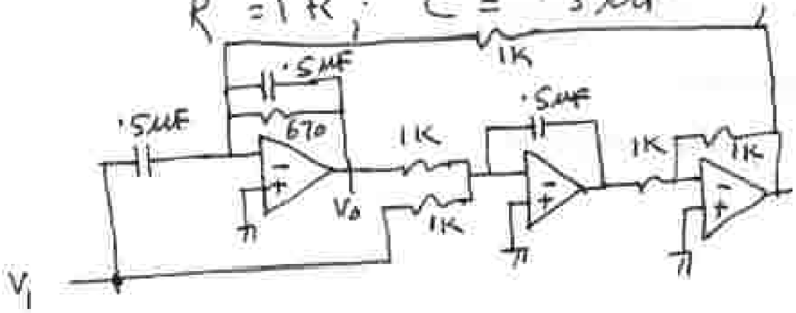
Thus $R = 2.236 \times 10^{-2}$; $C = 2.236 \times 10^{-2}$

For practical $R = 1\ K$, we use impedance scaling by $\frac{1000}{2.236 \times 10^{-2}} = 44721.36$

Then $R = 1\ K$; $C = \frac{2.236 \times 10^{-2}}{44721.36} = 0.499\ \mu F$

Let $r = 1\ K$ also. So final design is:

$R = 1\ K$; $C \simeq .5\ \mu F$, $r = 1\ K$



Q3: (contd.) Using Fleischer-Tow circuit {Possible Design #2}

$$\frac{V_o}{V_1} = - \frac{\left(R_8/R_6\right) s^2 + \frac{1}{R_1C_1}\left[R_8/R_6 - \frac{R_1R_8}{R_4R_7} \right] s + \frac{R_8}{R_3R_5R_7C_1C_2}}{s^2 + \frac{1}{R_1C_1} s + \frac{R_8}{R_2R_3C_1C_2R_7}}$$

To match with

$$|T_N| = \frac{s^2+1}{s^2+1.5s+1}$$

We shall make $R_8/R_6 = 1$; $\frac{R_8}{R_3R_5R_7C_1C_2} = 1$

$$\frac{R_8}{R_6} = \frac{R_1R_8}{R_4R_7}, \quad \& \quad \frac{1}{C_1R_1} = 1.5 \; ; \quad \frac{R_8}{R_2R_3C_1C_2R_7} = 1$$

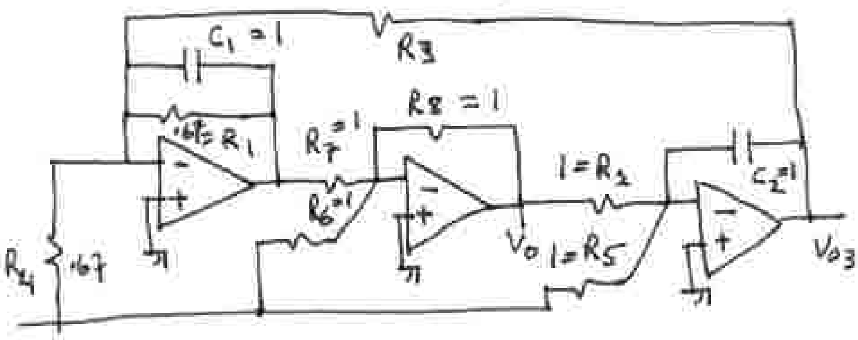
Let $R_8 = 1 = R_6 = R_3 = R_5 = R_7 = C_1 = C_2 = R_2$

Then $\frac{1}{C_1R_1} \rightarrow \frac{1}{R_1} = 1.5$; $R_1 = 0.67$

$\frac{1}{R_2} = 1$; $R_2 = 1$.

$$\frac{R_1R_8}{R_4R_7} = \frac{0.67 \cdot 1}{R_4 \cdot 1} = \frac{R_8}{R_6} = 1 \quad \text{So} \quad R_4 = 0.67 = R_1$$

Thus, the normalized design circuit is:



For frequency denormalized filter, since

Q2 contd.

Q3:

$$\frac{R_8}{R_3 R_5 R_7 C_1 C_2} = \Omega_0^2 = \frac{R_8}{R_2 R_3 C_1 C_2 R_7}$$

We can make $C_1 = C_2 = \frac{1}{\Omega_0} = \frac{1}{2000}$

Second ckt design is:

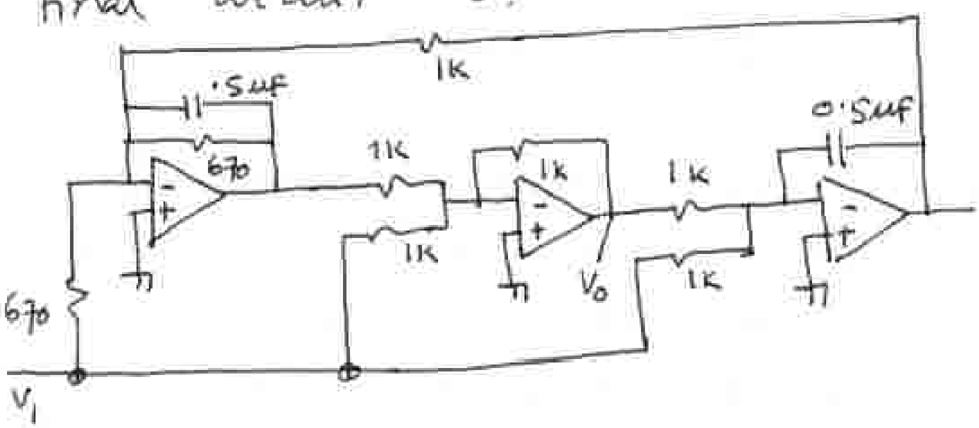
$C_1 = C_2 = 0.0005\ F$, all others as before.

If now we want $R_8 = R_8 = R_3 = R_5 = R_7 = 1K\Omega$
we do impedance scaling. Then

$C_1 = C_2 = .0005 / 1000 = 5 \times 10^{-7} = 0.5\mu f$

$R_1 = R_4 = 670\Omega$, other resistances $= 1K\Omega$.

Final circuit is:



~~Q1~~ Q1: By inspection

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OA1 receives input at phase 1. So

$$V_1^{(1)} = -\frac{1}{D}\cdot V_i^{(1)} G \cdot \frac{1}{1-z^{-1}} + \frac{1}{D}\cdot V_i^{(2)}\cdot H \cdot \frac{z^{-1/2}}{1-z^{-1}} - \frac{1}{D}\cdot V_2^{(1)}\cdot C\cdot \frac{1}{1-z^{-1}}$$

$$= -\frac{G}{D}\cdot\frac{1}{1-z^{-1}}\, V_i^{(1)} + \frac{H}{D}\cdot\frac{z^{-1/2}}{1-z^{-1}}\, V_i^{(2)} - \frac{C}{D}\cdot\frac{1}{1-z^{-1}}\, V_2^{(1)}$$

Using $V_i^{(2)} = z^{-\frac{1}{2}} V_i^{(1)}$

$$V_1^{(1)} = -\frac{G}{D}\cdot\frac{1}{1-z^{-1}}\, V_i^{(1)} + \frac{H}{D}\cdot\frac{z^{-1}}{1-z^{-1}}\, V_i^{(1)} - \frac{C}{D}\cdot\frac{1}{1-z^{-1}}\, V_2^{(1)}$$

For OA 2, similarly,

$$V_2^{(1)} = -\frac{F}{B}\cdot\frac{1}{1-z^{-1}}\, V_2^{(1)} + \frac{A}{B}\cdot\frac{z^{-\frac{1}{2}}}{1-z^{-1}}\, V_1^{(2)}$$

Using $V_1^{(2)} = z^{-\frac{1}{2}} V_1^{(1)}$

$$V_2^{(1)} = -\frac{F}{B}\cdot\frac{1}{1-z^{-1}}\, V_2^{(1)} + \frac{A}{B}\cdot\frac{z^{-1}}{1-z^{-1}}\, V_1^{(1)}$$

Changing sides

$$V_2^{(1)}\left[1 + \frac{F}{B}\cdot\frac{1}{1-z^{-1}}\right] = \frac{A}{B}\cdot\frac{z^{-1}}{1-z^{-1}}\, V_1^{(1)}$$

$$V_2^{(1)}\left[1 + \frac{F}{B}\cdot\frac{1}{1-z^{-1}}\right] = \frac{A}{B}\cdot\frac{z^{-1}}{1-z^{-1}}\left[\left(-\frac{G}{D} + \frac{H}{D}z^{-1}\right)\frac{1}{1-z^{-1}}V_i^{(1)} - \frac{C}{D}\,\frac{1}{(1-z^{-1})}\,V_2^{(1)}\right]$$

$$\text{or } V_2^{(1)}\left[1 + \frac{F}{B}\cdot\frac{1}{1-z^{-1}} + \frac{AC}{BD}\cdot\frac{z^{-1}}{(1-z^{-1})^2}\right] = \frac{A}{B}\cdot\frac{z^{-1}}{(1-z^{-1})^2}\left(-\frac{G}{D} + \frac{H}{D}z^{-1}\right)V_i^{(1)}$$

$$V_2^{(1)}\;\frac{\left[BD(1-z^{-1})^2 + FD(1-z^{-1}) + ACz^{-1}\right]}{BD\,(1-z^{-1})^2} = \frac{A}{B}\cdot\frac{1}{(1-z^{-1})^2}\cdot\left(-\frac{G}{D}z^{-1} + \frac{H}{D}z^{-2}\right)V_i^{(1)}$$

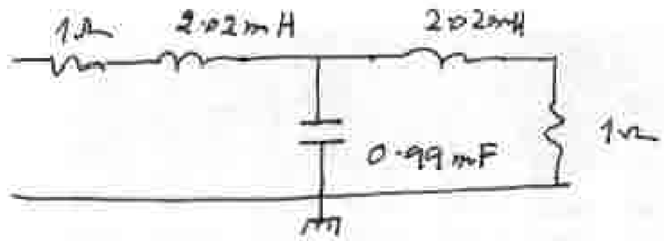
~~Q2.~~ Q1.

$$V_2^{(1)}\left[BD\left(1-z^{-1}\right)^2+ED\left(1-z^{-1}\right)+ACz^{-1}\right]$$

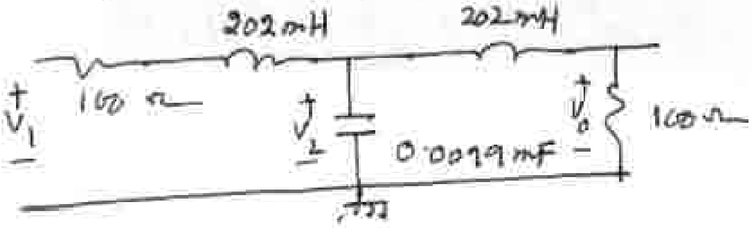
$$=\quad AD\left[-\frac{G}{D}z^{-1}+\frac{H}{D}z^{-2}\right]V_1^{(1)}$$

$$\frac{V_2^{(1)}}{V_1^{(1)}}=\frac{AHz^{-2}-GAz^{-1}}{BD\left(1-z^{-1}\right)^2+ED\left(1-z^{-1}\right)+ACz^{-1}}$$

~~Q3~~. Q.2 The frequency denormalised filter is:



The impedance & frequency denormalized filter is:



For operational simulation:

$Y_1 \rightarrow$ 

$$\rightarrow \frac{1}{100 + s \times 202 \times 10^{-3}} = \frac{1}{202 \times 10^{-3}} \cdot \frac{1}{s + \frac{100}{202 \times 10^{-3}}}$$

$$T_1 = RY_1 = \frac{R / 202 \times 10^{-3}}{s + \frac{100}{202 \times 10^{-3}}}$$

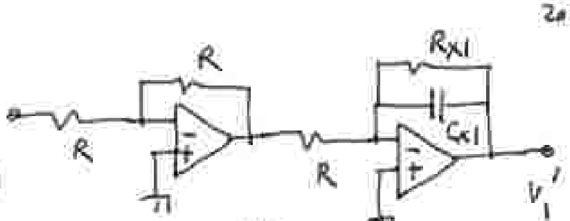
$$-Z_2 = -\frac{1}{s \times .0099\,mF} = -\frac{1}{s \times 99 \times 10^{-7}} = -\frac{1}{s \times 9.9 \times 10^{-6}}.$$

$$-T_2 = -\frac{Z_2}{R} = -\frac{1}{s \times R \times 9.9 \times 10^{-6}}.$$

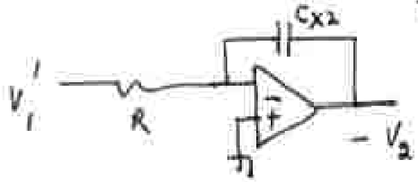
Y_3 

$$\rightarrow T_3 = \frac{R / 202 \times 10^{-3}}{s + \frac{100}{202 \times 10^{-3}}}$$

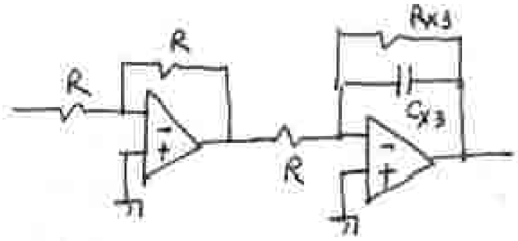
Then $T_1 \rightarrow$



For T_2



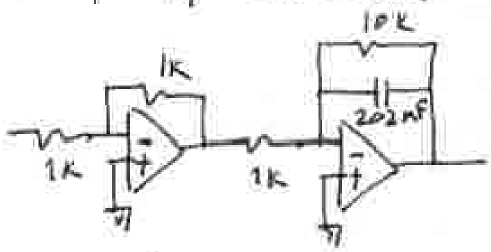
For T_3 :



Let $R = 1\,k\Omega$

The design equations are: $R_x = \dfrac{R^2}{R_s}$ & $C_x = \dfrac{L}{R^2}$

Thus for T_1 we have

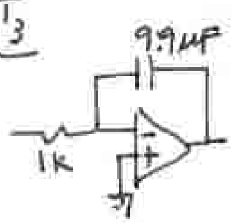


$$R_x = \frac{10^6}{100} = 10^4 = 10K$$

$$C_x = \frac{L}{R^2} = \frac{202\,mH}{10^6} = 202\,~~mH~~\,nF$$

So also for T_3

For $-T_2$ $\Rightarrow$



$$\frac{1}{RC_{x2}} \rightarrow \frac{1}{RC}$$

$$C_{x2} = C = 9.9\,\mu F$$

The interconnection is: