Q1: ELEC 441/6081 MT2, W 2013 11/9

$$D(s) = s^2 + \frac{s}{R_1C_1}\cdot\frac{1+R_6/R_5}{1+R_4/R_3} + \frac{R_6/R_5}{R_1R_2C_1C_2}$$

$$\rightarrow s^2 + s\left(\frac{\omega_p}{Q_p}\right) + \omega_p^2$$

Thus:
$$\frac{R_6/R_5}{R_1R_2C_1C_2} = (10^5)^2$$

$$\frac{1}{R_1C_1}\cdot\frac{1+R_6/R_5}{1+R_4/R_3} = \frac{\omega_p}{Q_p} = B.W. = 10^3$$

Let $R_1 = R_2 = R$; $C_1 = C_2 = C$. $\left\{\begin{array}{l}8 \text{ elements}\\ 2 \text{ conditions}\end{array}\right.$

R_3, R_4, R_5, R_6, R, C to be determined.

Then

$$\frac{1}{RC}\cdot\frac{1+R_6/R_5}{1+R_4/R_3} = 10^3$$

$$\left(\frac{1}{RC}\right)^2\frac{R_6}{R_5} = (10^5)^2 = 10^{10}$$

TBD → to be determined.

Let $R_5 = R_6 = R_x$ (so R_3, R_4, R_x, R, C TBD)

$$\frac{1}{RC} = 10^5$$

Let $R = 1$, $C = 10^{-5}$ (so R_3, R_4, R_x TBD).

Going back:
$$\frac{1}{R_1C_1}\cdot\frac{1+R_6/R_5}{1+R_4/R_3} = 10^3$$

i.e.
$$10^5\cdot\frac{1+1}{1+R_4/R_3} = 10^3$$

$$1 + \frac{R_4}{R_3} = 200 \;;\; \frac{R_4}{R_3} = 199.$$

Q1:

ELEC 441/6081 MT2 , W2013

Let $R_3 = 1$, $R_4 = 199$; $R_x = 1$.

So $R_1 = R_2 = 1$; $R_3 = 1$, $R_4 = 199$; $R_5 = R_6 = 1$

$C_1 = C_2 = 10^{-5}$

is the FIRST cut design.

verify: ω_p & Q_p values ..

Practical set of values? Let $R_1 = R_2 = R_3 = R_5 = R_6 = 100\,\Omega$

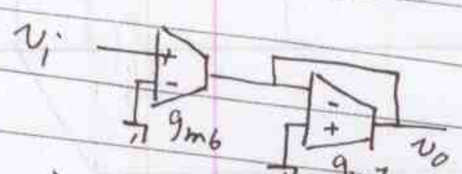
Then $R_4 = 19900 = 19.9\,K\Omega$

$C_1 = C_2 = 10^{-7} = 0.1\,\mu F$.

Q2: Using the general structure (5-OTA) and with $V_A, V_B = 0$, we'll have 3 OTAs only.

$$\frac{V_o}{V_i} = \frac{s^2 C_1 C_2}{s^2 C_1 C_2 + s C_1 g_{m3} + g_{m1} g_{m2}}$$

At $\omega \to \infty$, gain $= 1$. To get $H_o = 3$, we can use an OTA-based gain-stage i.e.



$$\frac{v_o}{v_i} = 3 = \frac{g_{m6}}{g_{m7}} \quad ; \text{ i.e. } \cancel{g_{m6} = 3 g_m}$$

i.e. $\therefore g_{m6} = 3 g_{m7}$.

Further:

$$\frac{g_{m1} g_{m2}}{C_1 C_2} = (2000)^2 \quad , \quad \frac{g_{m3}}{C_2} = \frac{2000}{5} = 400$$

31

Q2: We could chose $C_1 = C_2 = 0.01\,\mu F$.

$g_{m1} = g_{m2} = g_m$; $\frac{g_m}{.01\,\mu F} = 2000$; $g_m = 20\,\mu℧$

$Q_p = 5 = \cancel{\frac{10^{-5}}{g_{m3}}} \cdot 2000$

$\frac{2000}{5} = \frac{g_{m3}}{C}$; $g_{m3} = \frac{2000 \times 10^{-8}}{5}$

$= 4\,\mu℧$

Let $g_{m7} = 4\,\mu℧$, $g_{m6} = 12\,\mu℧$

Total $g_m = \sum g_m = 60\,\mu℧$

For the BJT stage $g_m|_Q = \frac{I_{DC}}{V_T}$, $V_T = 25\,mV$.

So $\sum I_{DC} = 25 \times 10^{-3} \times 60 \times 10^{-6} = 1.5 \times 10^{-6}$

$I_{Bias} = 2 \sum I_{DC} = 3\,\mu A$.

$P_{DC} = (5 - (-5)) \times 3\,\mu W = 30\,\mu W$.

Q3: Using given data find the order
(hint) $n = 5$ for the normalized LPF. (CHEB filter)

For CHEB filter of order 5 ($A_p = 0.5\,dB$)

$$H_N(s)\Big|_{LP} = \frac{1}{2^4 \cdot \epsilon} \cdot \frac{1}{s^5 + 1.1725 s^4 + 1.9374 s^3 + 1.3096 s^2 + 0.7525 s + 0.1789}$$

$\epsilon = 0.3493$

Use frequency scaling $s \to \frac{\omega_{CH}}{s}$, $\omega_{CH} = 2\pi \times 15 \times 10^{3}$

Apply BLT $s \to 2 \times 150 \times 10^{3} (1 - z^{-1}) / (1 + z^{-1})$ to get $H(z)$

Q 4:

(hint)

$$V_1^{(1)} = V_i^{(1)} \frac{1}{1-z^{-1}} \cdot \frac{-G}{D} + V_i^{(2)} \frac{z^{-\frac{1}{2}}}{1-z^{-1}} \cdot \frac{H}{D} + V_2^{(1)} \cdot \frac{1}{1-z^{-1}} \cdot \frac{-C}{D}$$

$$\because V_i^{(2)} = z^{-\frac{1}{2}} \; V_i^{(1)}$$

$$V_i^{(1)} = V_i^{(1)} \left[\frac{-\frac{G}{D} + z^{-1}\frac{H}{D}}{1-z^{-1}} \right] + V_2^{(1)} \cdot \frac{-C/D}{1-z^{-1}}$$

Similarly:

$$V_2^{(1)} = V_1^{(2)} \cdot \frac{z^{-\frac{1}{2}}}{1-z^{-1}} \cdot \frac{A}{B} - V_2^{(1)} \cdot \frac{1}{1-z^{-1}} \cdot \left(-\frac{F}{B}\right)$$

$$V_2^{(1)} \left[1 + \frac{F/B}{1-z^{-1}}\right] = V_1^{(1)} \frac{z^{-1}}{1-z^{-1}} \left(\frac{A}{B}\right)$$

Then:

$$V_2^{(1)} \left[1 + \frac{F/B}{1-z^{-1}}\right] = \frac{A}{B} \cdot \frac{z^{-1}}{1-z^{-1}} \times \left[\quad \right]$$

$$\left[V_i^{(1)} \frac{-G/D + z^{-1} H/D}{1-z^{-1}} + V_2^{(1)} \frac{-C/D}{1-z^{-1}} \right]$$

Simplify to get

$$\frac{V_2^{(1)}}{V_i^{(1)}} = \dots$$

ELEC 6081 MT2 W2013

Q1. $\omega_p = 10^5$, $BW = 10^3$, $Q_p = 10^5/10^3 = 100$.

A BPF has

$$H(s) = H_0 \frac{(\omega_p/Q_p)s}{s^2 + (\omega_p/Q_p)s + \omega_p^2}$$

~~So we must have in the~~

given $\frac{V_0}{V_1}$, $R_8/R_6 \to 0$ i.e. $R_6 \to \infty$, open.

$R_8 / R_3 R_5 R_7 C_1 C_2 \to 0$ i.e. $R_8 \to 0$, short, or one of $R_3, R_5, R_7 \to \infty$ i.e., open.

$$\frac{1}{R_1 C_1}\left(\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7}\right) = H_0 \cdot (\omega_p/Q_p)$$

So $R_8 \to 0$ is not possible; we take $R_6 \to \infty$.

Then $$\frac{V_0}{V_1} = -\frac{\frac{1}{R_1 C_1} \cdot \frac{R_1 R_8}{R_4 R_7} \cdot s}{s^2 + \frac{1}{R_1 C_1} \cdot s + \frac{R_8}{R_2 R_3 C_1 C_2 R_7}}$$

In the choice R_3, R_5, R_7 one of them $\to \infty$, only choice is $R_5 \to \infty$ $\because R_3, R_7$ decides ω_p.

Now: $$\frac{R_8}{R_2 R_3 C_1 C_2 R_7} = \omega_p^2 = 10^{10}$$

$H_0 \to$ not specifie

$$\frac{1}{R_1 C_1} = \frac{\omega_p}{Q_p} = 10^3$$

Q1: ELEC 6081, MT2, W 2013

To determine ω_p, Q_p: we have

$C_1, C_2, R_2, R_3, R_7, R_8, R_4, R_1 \to 8$ elements

Let $C_1 = C_2 = C$; $\therefore R_2 = R_3 = R_7 = R_8 = R$

Then C, R, R_4, R_1 need be found out.

$\frac{1}{CR} = 10^5$; $\frac{1}{R_1 C} = 10^3$

Let $R = 1$; $C = 10^{-5}$; $\frac{1}{R_1} = 10^3 \times 10^{-5} = 10^{-2}$

$R_1 = 100$

So $R_1 = 100$, $R_2 = R_3 = R_7 = R_8 = R = 1$

$C_1 = C_2 = C = 10^{-5}$; $R_4 \to$ free choice $= 1$ (say)

First cut design values

$R_1 = 100$; $R_2 = R_3 = R_4 = R_7 = R_8 = 1$.

$C_1 = C_2 = 10^{-5}$

Let $R = 100\,\Omega$; Then

$R_1 = 10^4 = 10\,k\Omega$

$R_2 = R_3 = R_4 = R_7 = R_8 = 100\,\Omega$

$C_1 = C_2 = 10^{-7} = 0.1\,\mu F$.

R_6, R_5 open $\to$ i.e. absent in the circuit.

- Q2, Q3 $\to$ see the ELEC 441 (UG Class) soln/hint

Q 4: Pre-warp: $\hat{\omega}_c = 2f_s \cdot \tan\left(\frac{\omega_c}{2f_s}\right)$

$$= 2f_s \cdot \tan\left(\frac{2\pi \times 3.4\times10^3}{2\times 64\times 10^3}\right)$$

$$= 2f_s \cdot \times 0.168 \qquad = a \times .168$$

$$a = 2f_s.$$

$$F_2 = (2f_s)^2 + \left(\frac{\hat{\omega}_c}{Q_p}\right)\cdot 2f_s + \hat{\omega}_c^2$$

$$= (2f_s)^2\left[1 + \frac{0.168}{1.3} + .168^2\right]$$

Then from the Table:

$$H(z) = h_D \frac{1 + 2z^{-1} + z^{-2}}{1 - a_{1D} z^{-1} + a_{2D} z^{-2}}$$

DC gain of 0 dB $\rightarrow$ $1 = h_D \cdot \frac{2+2}{1 - a_{1D} + a_{2D}}$

where $a_{1D} = \dfrac{2\left[(2f_s)^2 - (2f_s)^2(.168)^2\right]}{(2f_s)^2\left[1 + \frac{.168}{1.3} + .168^2\right]} = \dfrac{2\,(1 - .168^2)}{1 + \frac{.168}{1.3} + .168^2}$

$$a_{2D} = \frac{(2f_s)^2\left[1 - \frac{.168}{1.3} + .168^2\right]}{(2f_s)^2} = \left[1 - \frac{.168}{1.3} + .168^2\right]$$

Thus, $a_{1D} = 1.679$

$a_{2D} = 0.777$

So $h_D = \dfrac{1 - 1.679 + .777}{4} = 0.0245$

Q4:

ELEC 6081, MT2, W2013

Given V_2/V_1 can be re-written as:
(set L, I, J, E = 0, these are not in the given schematic)

$$= -\frac{A(zG-H) + D(z-1)^2 K}{AzC + DF\,z(z-1) + DB(z-1)^2}$$

$$= -\frac{DK}{(DF+DB)} \cdot \frac{\left[1 + \frac{AG-2DK}{DK} z^{-1} + \frac{DK-AH}{DK} z^{-2}\right]}{\left[1 - \frac{2DB+DF-AC}{DF+DB} z^{-1} + \frac{DB}{DF+DB} z^{-2}\right]}$$

$$= -\frac{K}{F+B} \cdot \frac{1 + \left(\frac{AG}{DK} - 2\right) z^{-1} + \left(1 - \frac{AH}{DK}\right) z^{-2}}{1 - \frac{2DB+DF-AC}{DF+DB} z^{-1} + \frac{B}{F+B} z^{-2}}$$

Design for

$$\frac{K}{F+B} = h_D = 0.0245$$

$$\frac{AG}{DK} - 2 = 2$$

$$1 - \frac{AH}{DK} = 1$$

$$\frac{2DB+DF-AC}{DF+DB} = a_{1D} = 1.679$$

$$\frac{B}{F+B} = a_{2D} = 0.777.$$

5 equations
A, B, C, D, F, G, K, H
8 capacitors.

Let $B = D = 1$

Q 4:

$$\frac{1}{1+F} = 0.777 \; ; \; F = \frac{1}{.777} - 1 = 0.287$$

Then
$$\frac{2 + .287 - AC}{0.287 + .287} = 1.679$$

So, $AC = 1.323$

$$\frac{K}{1.287} = .0245 \; , \; K = 0.032$$

$$AG = 4 \times 1 \times .032 = 0.128$$

$1 - \frac{AH}{DK} = 1$ can be satisfied with $H = 0$
(i.e. H cap. is absent)

So $B = D = 1$, $F = 0.287$, $K = 0.032$
A, C, G remaining.
If we set $A = 1$, $C = 1.323$, $G = .128$
completes the first cut design. Thus

$A = B = D = 1$; $C = 1.323$, $F = 0.287$, $G = 0.128$,
$K = 0.032$

Scale by the lowest valued Cap i.e. $K = .032$

$K = 1$, $A = B = D = 31.25$
$C = 41.34$, $F = 8.97$, $G = 4$

If $C \to$ largest Cap must be < 20 pF., scale again by 2.067

Then $K = 0.483$ pF, $A = B = D = 15.12$ pF, $C = 20$ pF
$F = 4.34$ pF ; $G = 1.94$ pF.