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Mid-Term Examination

Analog/IC Filter Design (ELEC 441/6081)

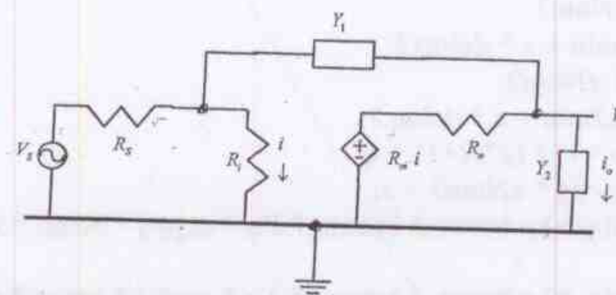
Electrical and Computer Engineering Department
Concordia University

February 28, 2007

Instructor: Dr. R. Raut

Time: 75 minutes

Q.1: In the figure below all voltage and currents are *small ac* signals. The elements Y_1, Y_2 can be chosen to produce different signal processing functions pertaining to electronic filters.



(a) What will be your choice for Y_1, Y_2 so that the function V_o/V_S is a second order transfer function in $s = j\omega$? Prove your point by an analytical expression for V_o/V_S.

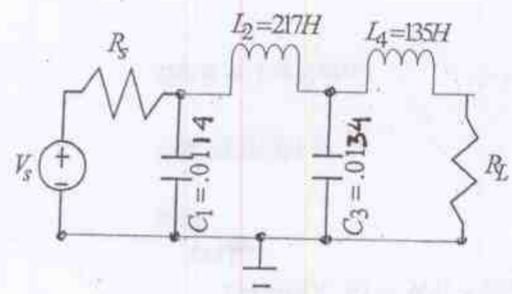
(b) If in the ideal case of $R_i \to 0$, $R_o \to 0$, and $R_m Y_1 >> 1$, show that V_o/V_S becomes an inverting integrator function with $Y_1 = sC_1$

(c) With the same conditions as in (b) above and further that $Y_2 = sC_2$, and $Y_1 = 1/R_1$, show that i_o/V_S represents a differentiator function.

Q.2: You are required to design a BPF with the pass-band extending from $\omega = 10^5$ rad/sec to $\omega = 4\times10^5$ rad/sec. The filter has *equal ripple* characteristic in the pass-band with peak-to-peak value not exceeding 1 dB. At $\omega = 15.263\times10^5$ rad/sec, the response must be at least 60 dB down relative to the pass-band.

(a) Synthesize the normalized low-pass filter function associated with the intended band-pass filter.

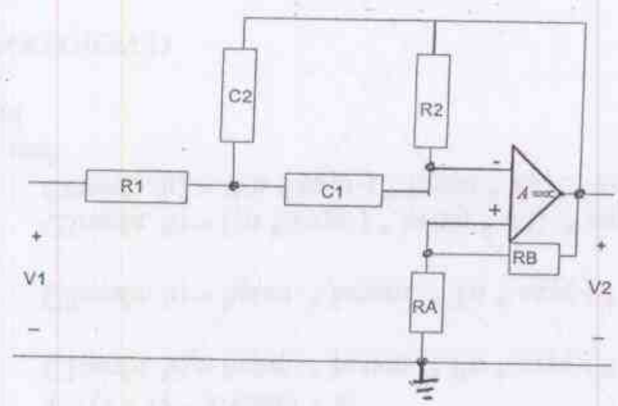
(b) The figure below shows an L,C realization of the low-pass filter function in (a). Based on this, design the band-pass filter as specified in Q.2 above.



Q.3: Consider the band-pass filter network of figure below. It combines both positive and negative feed-back around an ideal OA. The voltage transfer function is given by:

$$\frac{V_2}{V_1} = \frac{-s(K+1)/(R_1C_2)}{s^2 + s[1/(R_2C_2) + 1/(R_2C_1) - K/(R_1C_2)] + 1/(R_1R_2C_1C_2)}, \text{ with } K = R_A/R_B.$$

Assume $K=1$. Design the element values to meet the specifications: f_p=10 kHz, Q_p =10 and C=0.1 uF each.



Filter Function Tables

A.1: Coefficients of denominator polynomial, in the form $s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... + a_{n-2}s^2 + a_{n-1}s + 1$, for Butterworth filter function of order n, with pass-band from 0 to 1 rad/sec[+].

n	a_1	a_2	a_3	a_4	a_5	a_6
2	1.4142					
3	2.0000	2.0000				
4	2.6131	3.4142	2.6131			
5	3.2361	5.2361	5.2361	3.2361		
6	3.8637	7.4641	9.1416	7.4641	3.8637	
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940

A.2: Coefficients of denominator polynomial, in the form $s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... + a_{n-2}s^2 + a_{n-1}s + a_n$, for Chebyshev filter function of order n, with pass-band from 0 to 1 rad/sec[+].

Pass-band ripple A_p	n	a_1	a_2	a_3	a_4	a_5	a_6
0.5 dB, ϵ=0.3493	1	2.863					
	2	1.425	1.516				
	3	1.253	1.535	0.716			
	4	1.197	1.717	1.025	0.379		
	5	1.1725	1.9374	1.3096	0.7525	0.1789	
	6	1.1592	2.1718	1.5898	1.1719	0.4324	0.0948
1.0 dB, ϵ=0.5089	1	1.965					
	2	1.098	1.103				
	3	0.988	1.238	0.491			
	4	0.953	1.454	0.743	0.276		
	5	0.9368	1.6888	0.9744	0.5805	0.1228	
	6	0.9282	1.9308	1.2021	0.9393	0.3071	0.0689
2.0 dB, ϵ=0.7648	1	1.308					
	2	0.804	0.637				
	3	0.738	1.022	0.327			
	4	0.716	1.256	0.517	0.206		
	5	0.7065	1.4995	0.6935	0.4593	0.0817	
	6	0.7012	1.7459	0.8670	0.7715	0.2103	0.0514

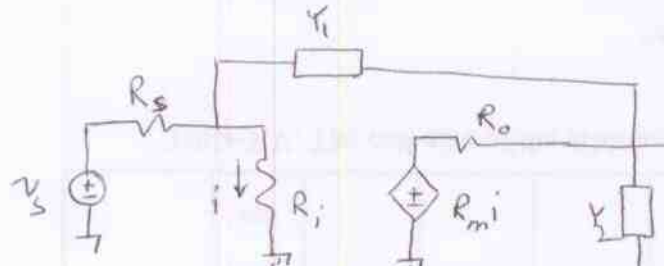
A.3: Coefficients of denominator polynomial, in the form $s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... + a_{n-2}s^2 + a_{n-1}s + a_n$, for Bessel-Thomson filter function of order n[+].

n	a_1	a_2	a_3	a_4	a_5	a_6
1	1					
2	3	3				
3	6	15	15			
4	10	45	105	105		
5	15	105	420	945	945	
6	21	210	1260	4725	10395	10395

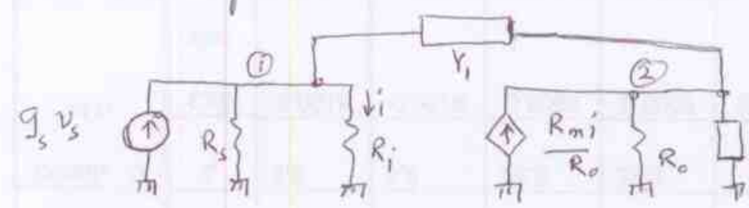
[+] R. Schaumann et al, "Design of Analog Filters- Passive, Active RC, and Switched Capacitor", Prentice-Hall Inc., © 1990

ELEC 441/6081, MT1 Sol/hint, Feb 28, 2007

Q1:



(a) For nodal analysis we re-draw



$$\begin{pmatrix} G_s+G_i+Y_1 & -Y_1 \\ -Y_1 & Y_1+G_o+Y_2 \end{pmatrix}\begin{bmatrix} V_① \\ V_② \end{bmatrix} = \begin{bmatrix} g_s v_s \\ \frac{R_m}{R_o} i \end{bmatrix},$$

But $i = V_① G_i$

$\frac{R_m}{R_o} i = \frac{R_m}{R_o} \cdot G_i V_①$

$$\begin{pmatrix} G_s+G_i+Y_1 & -Y_1 \\ -Y_1-\frac{R_m}{R_o}G_i & Y_1+G_o+Y_2 \end{pmatrix}\begin{pmatrix} V_① \\ V_② \end{pmatrix} = \begin{pmatrix} g_s v_s \\ 0 \end{pmatrix}$$

$$\Delta = (G_s+G_i+Y_1)(Y_1+G_o+Y_2) - Y_1\left(Y_1+\frac{R_m}{R_o}G_i\right)$$

$$V_o = V_② = \frac{1}{\Delta}\begin{vmatrix} G_s+G_i+Y_1 & g_s v_s \\ -Y_1-\frac{R_m}{R_o}G_i & 0 \end{vmatrix} = \frac{g_s v_s\left(Y_1+\frac{R_m}{R_o}G_i\right)}{\Delta}$$

So $$\frac{V_o}{V_s} = \frac{g_s\left(Y_1+\frac{R_m}{R_o}G_i\right)}{(G_s+G_i+Y_1)(Y_1+G_o+Y_2) - Y_1\left(Y_1+\frac{R_m}{R_o}G_i\right)}$$

This will be a second order transfer function, ie s^2+bs+C if both Y_1 and Y_2 are capacitive admittances. We shall have

$$D(s) = G_sY_1 + G_sG_o + G_sY_2 + G_iY_1 + G_iG_o + G_iY_2 + Y_1G_o + Y_1Y_2 - Y_1G_i\frac{R_m}{R_o}$$

$$= s^2C_1C_2 + s\left(C_1G_s + C_2G_s + C_1G_i + C_2G_i + C_1G_o - C_1G_i\frac{R_m}{R_o}\right) + G_sG_o + G_iG_o$$

Q1(b) $R_i \to 0,\ R_o \to 0$ implies $G_i, G_o \to \infty$, note $G_o = \frac{1}{R_o}$; $G_i = \frac{1}{R_i}$

$$\frac{V_o}{V_s} = \frac{G_i G_o\, g_s\left(\frac{Y_1}{G_i G_o} + R_m\right)}{G_i G_o\left(\frac{G_s}{G_i} + 1 + \frac{Y_1}{G_i}\right)\left(\frac{Y_1}{G_o} + \frac{Y_2}{G_o} + 1\right) - G_i G_o Y_1\left(\frac{Y_1}{G_i G_o} + R_m\right)}$$

The ratio $\to \dfrac{g_s R_m}{1 - Y_1 R_m}$ as $G_i, G_o \to \infty$

If $Y_1 R_m >> 1$, then this becomes $\simeq -\dfrac{g_s}{Y_1} = -\dfrac{g_m}{sC_1}$ an inverting integrator

(c) With same assumptions and noting

$$i_o = Y_2 V_o$$

$$\frac{i_o}{V_s} = Y_2 \frac{V_o}{V_s} = -\frac{g_s Y_2}{Y_1} = -g_s R_1 s C_2 = -s\, g_s R_1 C_2$$ inverting differentiator.

Q2: $B = \Omega_{p2} - \Omega_{p1} = 3\times10^5 \rightarrow \omega_c$ in the associated LPF.

equal ripple $\rightarrow$ CHEB approximation

Considering the LP $\to$ BP frequency transformation

$$\bar{s} = \frac{\Omega_o}{B}\left(\frac{s}{\Omega_o} + \frac{\Omega_o}{s}\right)$$ leads to $(s \to j\omega$

$$\bar{\omega} = \frac{\Omega_o}{B}\left(\frac{\omega}{\Omega_o} - \frac{\Omega_o}{\omega}\right)$$

Set $\Omega_o = \sqrt{\Omega_{p2}\Omega_{p1}}$, $\omega = 15.263\times10^5$ in the stop-band

$$\bar{\omega} = \frac{2\times10^5}{3\times10^5}\left(\frac{15.263\times10^5}{2\times10^5} - \frac{2}{15.263}\right) = 5.003$$

$\bar{\omega}$ is the normalized stop-band frequency in the associated LPF.

Q 2.(a): Using $A_p = 1$ dB, $A_a = 60$ dB and the above ratio $\omega_a/\omega_c = 5$, we can calculate the order of the associated LPF with CHEB response.

Thus
$$D = \frac{10^{.1A_a} - 1}{10^{.1A_p} - 1} = \frac{10^{6} - 1}{10^{.1} - 1} = 3862112.232$$

$$\sqrt{D} =$$

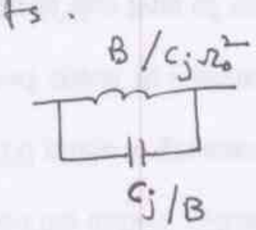
$$n \approx \frac{\cosh^{-1}\sqrt{D}}{\cosh^{-1}(\omega_a/\omega_c)} = 3.61$$ We take $n = 4$

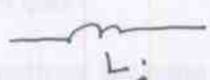
Using Tables
$$H_N(s) = \frac{1/2^3 \times .50885}{s^4 + .9528\,s^3 + 1.4539\,s^2 + .7426\,s + .27563}$$

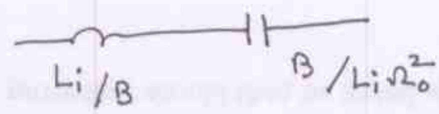
note, for $A_p = 1$ dB, $\epsilon = \sqrt{10^{.1} - 1} = .50885$

2(b) We have to use component transformation formula to derive the BPF components.

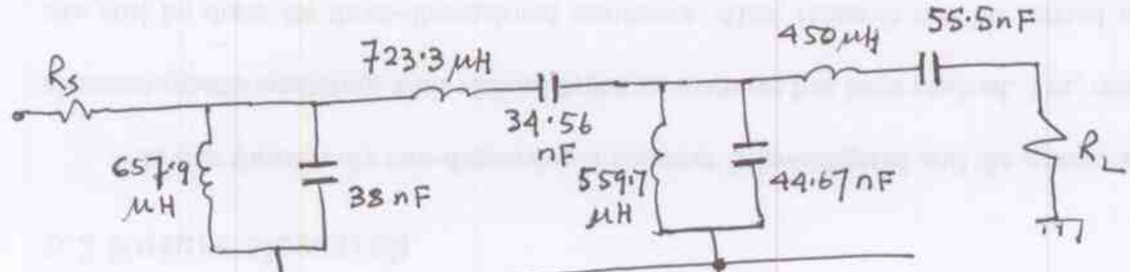
Thus 

becomes 



becomes 

The BPF network will be:



we used $\Omega_0 = 2\times10^5$, $B = 3\times10^5$

Q3:

Compare with the standard BPF expression

$$T(s) = \frac{-H_0\left(\frac{\omega_p}{Q_p}\right)s}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

we see: $H_0 \frac{\omega_p}{Q_p} = (K+1)/R_1C_2 = 2/R_1C_2 \quad \because K=1$ given

3

$$\frac{\omega_p}{Q_p} = \frac{1}{R_2C_2} + \frac{1}{R_2C_1} - \frac{1}{R_1C_2}$$

$$\omega_p = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

Further given $C_1 = C_2 = 0.1\ \mu F$ each.

Thus $\omega_p = \frac{1}{C}\frac{1}{\sqrt{R_1R_2}}$

$$\frac{\omega_p}{Q_p} = \frac{1}{C}\cdot\left(\frac{1}{R_2} + \frac{1}{R_2} - \frac{1}{R_1}\right) = \frac{1}{C}\cdot\left(\frac{2}{R_2} - \frac{1}{R_1}\right)$$

Given specs. $\omega_p = 2\pi\times10^4$, $Q_p = 10$.

So $\frac{1}{C}\frac{1}{\sqrt{R_1R_2}} = 2\pi\times10^4$; $\frac{1}{\sqrt{R_1R_2}} = 2\pi\times10^4\times10^{-7}$
$= 2\pi\times10^{-3} \quad \cdots (1)$

3

$$\frac{\omega_p}{Q_p} = \frac{2\pi\times10^4}{10} = 2\pi\times10^3 = \frac{1}{C}\left(\frac{2}{R_2} - \frac{1}{R_1}\right)$$

$$\frac{2}{R_2} - \frac{1}{R_1} = 2\pi\times10^3\times10^{-7} = 2\pi\times10^{-4} \quad \cdots (2)$$

And $\frac{2}{R_1C_2} = \frac{2}{R_1}\cdot10^7 = H_0\cdot\frac{2\pi\times10^4}{10} = H_0\cdot2\pi\times10^3$

$$\frac{1}{R_1} = H_0\cdot\frac{2\pi\times10^3}{2\times10^7} = H_0\cdot\pi\times10^{-4} \quad \cdots (3)$$

Q3 (Contd.)

Eqs. (1) and (2) are more important in the present case since these relate to ω_p and Q_p.

Let $R_2 = m^2 R_1$

Then $\frac{1}{\sqrt{R_1 . m^2 R_1}} = 2\pi \times 10^{-3}$ or $m R_1 = \frac{10^3}{2\pi}$ .....(4)

and $\frac{2}{m^2 R_1} - \frac{1}{R_1} = 2\pi \times 10^{-4}$; $\frac{1}{R_1}\left(\frac{2}{m^2} - 1\right) = 2\pi \times 10^{-4}$ ..(5)

Multiply (4) and (5)

$$\frac{2}{m} - m = \frac{10^3}{2\pi} \times 2\pi \times 10^{-4} = 10^{-1}$$

$$2 - m^2 = 0.1m \quad , \quad m^2 + 0.1m - 2 = 0.$$

$$m = \frac{-0.1 \pm \sqrt{.01 + 8}}{2} = \frac{-0.1 \pm 2.8302}{2}$$

$$= 1.365 \text{ or } -2.9302$$

Acceptable value of m is 1.365 (a +ve number)

So $R_2 = 1.365^2 R_1 = 1.86323 R_1$

Using (4) $R_1 = \frac{10^3}{2\pi \times 1.365} \simeq 116.6\ \Omega$ $C_1 = C_2 = 0.1\mu F$

Then $R_2 = 217.25\ \Omega$

DESIGN

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The filter will have $H_0 = \frac{10^4}{\pi R_1}$ from eqn (4)

$$= 27.299 \simeq \cancel{27.3} 27.3$$

1

Since $K = \frac{R_A}{R_B} = 1$, we can choose $R_A = R_B = 100\ \Omega$