

Q1:

0.707Ω to 707Ω involves a scaling by 1000.

So impedance scaling by 1000 has been done.

This takes $C = 1F$ to $1/1000 = 1mF$ ~~so~~

But final C is $0.1mF = 10^{-7} F$

So 'C' has been further scaled by $10^{-7} / 10^{-3} = 10^{-4}$

Scaling on 'C' alone implies frequency scaling.

Frequency scaling 's' by 'a' i.e. $s \rightarrow s/a$

makes $s_{new} = a \cdot s_{old}$ $\leftarrow 10^{-7}$ $\rightarrow 10^{-3}$

This scales C as $C_{new} = C_{old} / a \rightarrow C_{old} / 10^4$

Thus $s_{new} = 10^4 \cdot s_{old}$

Hence $T_1(s) \rightarrow T_1'(s)$ ^{remains} same due to impedance scaling

$T_1'(s) \rightarrow T_1''(s)$ $\left| s \rightarrow s/10^4$ due to frequency scaling

$$T_1(s) \Big|_{new} = \frac{2}{s^2 + \sqrt{2}s + 1} \Big|_{s \rightarrow s/10^4} = \frac{2}{\frac{s^2}{10^8} + \sqrt{2} \frac{s}{10^4} + 1}$$

$$T_2(s) = \frac{2 \times 10^8}{s^2 + \sqrt{2} 10^4 s + 10^8}$$

Q2:

$$\Omega_{a1} = 4300 \text{ rad/s}, \quad \Omega_{a2} = 6000 \text{ rad/s}$$

$$\Omega_{p1} = 3500 \text{ rad/s}, \quad \Omega_{p2} = 7000 \text{ rad/s}$$

Band-stop filter, so
$$\omega_s = \frac{\Omega_{p2} - \Omega_{p1}}{\Omega_{a2} - \Omega_{a1}} = \frac{3500}{1700}$$

$$\omega_s = 2.059$$

Pass-band has ripple \rightarrow CHEB feature.

So we calculate
$$D = \frac{10^{-0.1 \times 40} - 1}{10^{-0.1 \times 5} - 1} = 81946.6 \approx 81947$$

$$\sqrt{D} = 286.26$$

Then
$$\frac{\cosh^{-1} \sqrt{D}}{\cosh^{-1} \omega_s} = 4.702 \rightarrow \text{order of assoc. normalized LP filter}$$

Take $n = 5$ for the normalized LPF.

We consult table to get

$$H_{N(s)} = \frac{1}{2^{5-1} \cdot \epsilon} \cdot \frac{1}{D(s)} \quad \text{where } \epsilon = \sqrt{10^{-0.1 \times 5} - 1} = 0.3493$$

$$H_{N(s)} = \frac{0.1789}{s^5 + 1.1725s^4 + 1.9374s^3 + 1.3096s^2 + 0.7525s + 0.1789}$$

To synthesize the frequency denormalized BSF, we need to use the frequency transformation

$$s \rightarrow \frac{B}{\Omega_0} \cdot \left[\frac{1}{\frac{s}{\Omega_0} + \frac{\Omega_0}{s}} \right]$$

where
$$B = \Omega_{a2} - \Omega_{a1} = 1700 \text{ rad/s}$$

$$\Omega_0^2 = \Omega_{a1} \Omega_{a2} = 4300 \times 6000 \text{ rad}^2/\text{sec}^2$$

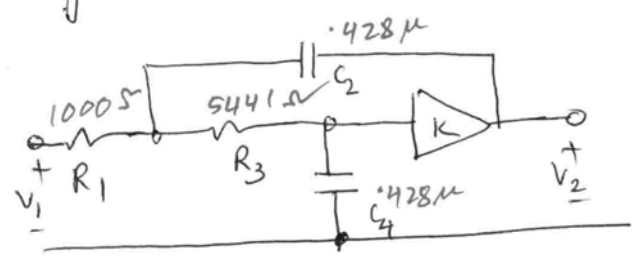
$$\Omega_0 = 5079.37 \text{ rad/sec}$$

Q3.

Given $H_W(s) = T_N(s) = \frac{1.9825}{s^2 + 1.098s + 1.103}$

Finite Gain SAB (FGSAB - S&K) design

The given TF. is a L.P.F. So we consider the circuit



The network TF. is:

$$K / R_1 R_3 C_2 C_4$$

$$s^2 + s \left(\frac{1}{R_3 C_4} + \frac{1}{R_1 C_2} + \frac{1}{R_3 C_2} - \frac{K}{R_3 C_4} \right) + \frac{1}{R_1 R_3 C_2 C_4}$$

The above TF. has a DC gain of K ~~5~~

We set $K = 5$ as required in Q.4.

Then by comparing with the theoretical function

$$T_{LP}(s) = \frac{K \omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

We can see $\frac{1}{R_1 R_3 C_2 C_4} = \omega_p^2 = 1.103$

$$\frac{1}{R_3 C_4} + \frac{1}{R_1 C_2} + \frac{1}{R_3 C_2} - \frac{5}{R_3 C_4} = \frac{\omega_p}{Q_p} = 1.098$$

Let $C_2 = C_4 = C = 1 \text{ F}$

Then $\frac{1}{R_1 R_3} = 1.103$; $\frac{1}{R_1} - \frac{3}{R_3} = 1.098$

0.93

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$$R_3 - 3R_1 = 1.098 R_1 R_3 = \frac{1.098}{1.103} = 0.995$$

$$R_3 = 3R_1 + 0.995$$

Subst. back in $\frac{1}{R_1 R_3} = 1.103$

$$R_1 \cdot (3R_1 + 0.995) = \frac{1}{1.103} = 0.9066$$

$$3R_1^2 + 0.995R_1 - 0.9066 = 0$$

$$R_1 = \frac{-0.995 \pm \sqrt{0.995^2 + 4 \times 0.9066 \times 3}}{2 \times 3}$$

acceptable \Rightarrow
positive value

$$\frac{-0.995 + 3.445}{6} = 0.408$$

$$3 \times 0.408 + 0.995 = 2.22$$

Then $R_3 = \cancel{3 \times 0.408 + 0.995} = 3.215$

First cut design values

$$C_3 = C_4 = C = 1 \text{ F}$$

$$R_1 = 0.408 \Omega$$

$$R_3 = 2.22 \Omega$$

satisfies

$$\omega_n = 1.098$$

$$\omega_n^2 = 1.103$$

Actual $\omega_p = 1000 \text{ rad/sec}$.

So we need to scale frequency by $\frac{1000}{\sqrt{1.103}} = 952.165$

This will alter $C \rightarrow \frac{1}{952.165} \cong 1.05 \text{ mF}$.

Second cut design: $R_1 = 0.408 \Omega$ $C_2 = C_4 = C = 1.05 \text{ mF}$
 $R_3 = 2.22 \Omega$

To make $R_1 \rightarrow 1000 \Omega$, impedance scale by 2450.98

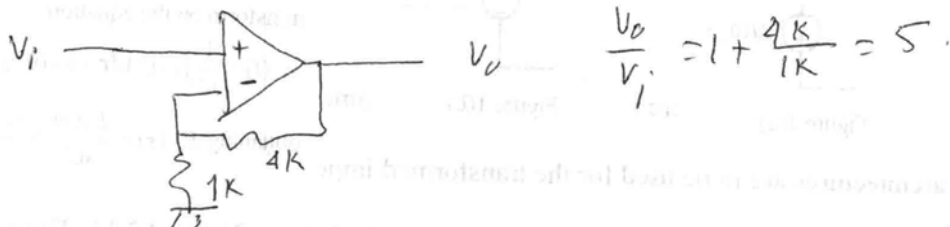
Then

Q3 Then, the final design values come

$$R_1 = 1000 \Omega ; R_3 = 5441.18 \Omega$$

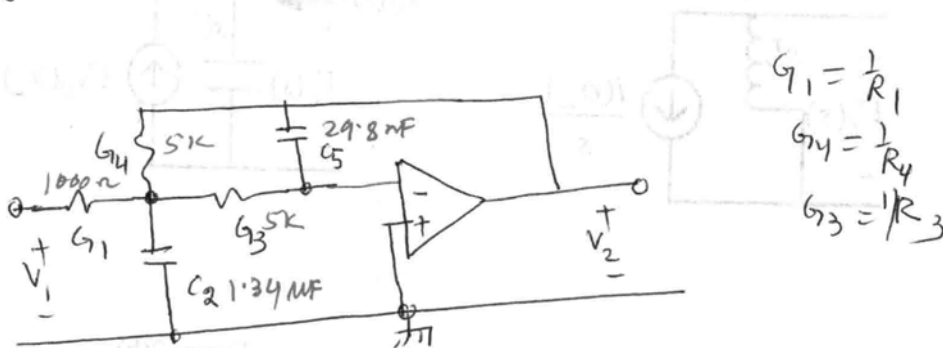
$$C_2 = C_4 = C = \frac{1.05 \text{ mF}}{2450.98} = 0.428 \text{ } \mu\text{F}$$

$K = 5$ is obtained using an OA as:



Q2: IG - SAB design $I_G \rightarrow$ Infinite gain

Now consider the circuit (LPF)



The network TF. is:
$$\frac{-G_1 G_3}{s^2 G_2 C_5 + s C_5 (G_1 + G_3 + G_4) + G_3 G_4}$$

The associated theoretical

TF is:
$$\frac{-H}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

We re-write the network TF. as:

$$\frac{-G_1 G_3 / C_2 C_5}{s^2 + \left(\frac{G_1}{C_2} + \frac{G_3}{C_2} + \frac{G_4}{C_2} \right) s + \frac{G_3 G_4}{C_2 C_5}}$$

Q.3. (16-SAB design contd.)

Comparing with theoretical TF.

$$\frac{G_1 G_3}{G_2 G_5} = H$$

$$\frac{G_1}{C_2} + \frac{G_3}{C_2} + \frac{G_4}{C_2} = \frac{\omega_p}{\omega_p}$$

$$\frac{G_3 G_4}{G_2 G_5} = \omega_p^2$$

DC gain $\frac{G_1 G_3}{G_3 G_4} = \frac{G_1}{G_4} = \frac{H}{\omega_p^2} = 5$ required

So $G_1 = 5G_4$ and $H = 5\omega_p^2$

Suggested design eqns are:

$$G_1 = \frac{H}{\omega_p} ; G_2 = \frac{\omega_p}{\omega_p^2} (2\omega_p^2 + H) ; G_3 = \omega_p$$

$$G_4 = G_3 ; G_5 = \frac{\omega_p}{\omega_p (2\omega_p^2 + H)} = \frac{1}{C_2}$$

In the given TF.

$$\omega_p^2 = 1.103 ; \text{ So } H = 5 \times 1.103 = 5.515$$

$$\omega_p = \sqrt{1.103} = 1.05$$

$$G_1 = \frac{5\omega_p^2}{\omega_p} = 5\omega_p = 5 \times 1.05 = 5.25 \text{ V}$$

$$G_3 = 1.05 \text{ V} = G_4 ; C_2 = \frac{\omega_p}{\omega_p} \left(\omega_p + \frac{H}{\omega_p} \right)$$

$$C_2 = \frac{1}{1.098} \left(1.05 \times 2 + \frac{5.515}{1.05} \right) \because \frac{\omega_p}{\omega_p} = 1.098 \text{ in the given TF. (coeff. of } s \text{ term in D(s))}$$

$$= 6.696 \text{ F}$$

$$G_5 = 1/G_2 = 0.149 \text{ F}$$

First cut design values are:

$$G_1 = 5.25 \text{ S} \rightarrow 0.1905 \Omega$$

$$G_3 = G_4 = 1.05 \text{ S} \rightarrow 0.9524 \Omega$$

$$C_2 = 6.696 \text{ F}$$

$$C_5 = 0.149 \text{ F} \text{ verified } \begin{cases} \omega_p^2 = 1.103 \\ \frac{\omega_p}{\omega_p} = 1.098 \end{cases}$$

R_1 is lowest valued resistance

To make it 1000Ω , impedance scale by $\frac{1000}{0.1905}$
 $= 5249.34$

second cut design.

$$\begin{cases} R_1 = 0.1905 \Omega \rightarrow 1000 \Omega \\ R_3 = R_4 = 0.9524 \Omega \rightarrow 4999.47 \sim 5000 \Omega \\ C_2 = 6.696 \text{ F} / 5249.34 = 1.2756 \text{ mF} \\ C_5 = 0.149 / 5249.34 = 2.8384 \times 10^{-5} = 28.384 \text{ } \mu\text{F} \end{cases}$$

The above is for $\omega_p = 1.05 \text{ rad/sec}$

To achieve $\omega_p = 1000 \text{ rad/sec}$, we frequency scale by $\frac{1000}{1.05} = 952.38$

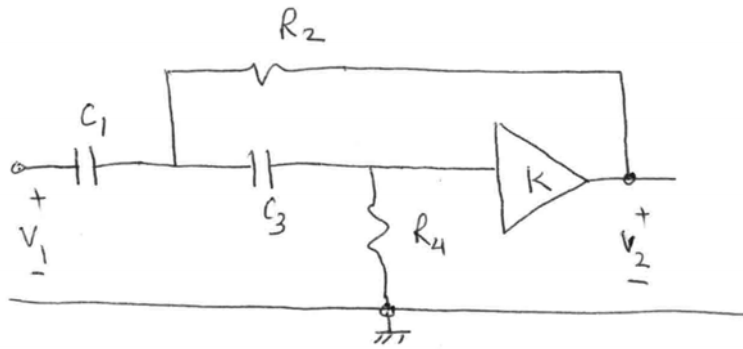
This changes only the capacitors

So final design

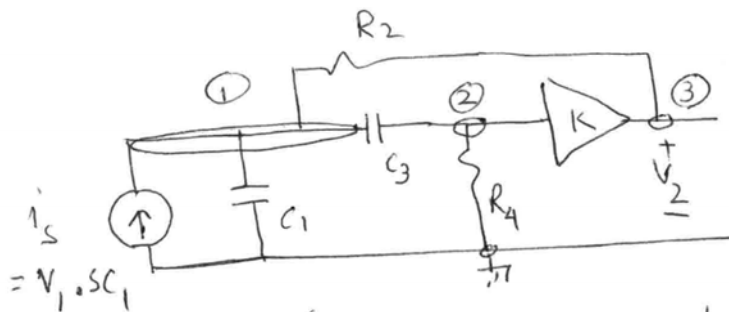
$$\begin{cases} R_1 = 1000 \Omega \\ R_3 = R_4 = 5000 \Omega \\ C_2 = \frac{1.2756 \times 10^{-3}}{952.38} = 1.34 \text{ } \mu\text{F} \\ C_5 = 28.384 \text{ } \mu\text{F} / 952.38 = 29.8 \text{ } \mu\text{F} \end{cases}$$

Q.4:

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For nodal matrix analysis, we re-configure the above circuit as:



All $G = \frac{1}{R}$

The NAM for the 3-node system is:

$$\begin{bmatrix} \textcircled{1} & & \\ \textcircled{2} & & \\ \textcircled{3} & & \end{bmatrix} \begin{bmatrix} \Delta C_1 + \Delta C_3 + G_2 & -\Delta C_3 & -G_2 \\ -\Delta C_3 & G_4 + \Delta C_3 & 0 \\ -G_2 & 0 & G_2 \end{bmatrix} \begin{bmatrix} V_{\textcircled{1}} \\ V_{\textcircled{2}} \\ V_{\textcircled{3}} \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix}$$

Applying the constraint $V_{\textcircled{3}} = K V_{\textcircled{2}}$, we write

$$C_{\textcircled{2}} = C_{\textcircled{2}} + K C_{\textcircled{3}}$$

$$\begin{bmatrix} \Delta C_1 + \Delta C_3 + G_2 & -\Delta C_3 - K G_2 \\ -\Delta C_3 & G_4 + \Delta C_3 \\ -G_2 & K G_2 \end{bmatrix} \begin{bmatrix} V_{\textcircled{1}} \\ V_{\textcircled{2}} \\ V_{\textcircled{3}} \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix}$$

Then, we discard row $\textcircled{3}$ corresponding to node $\textcircled{3}$

Q4

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i.e., the output node of a voltage amplifier

So:

$$\begin{pmatrix} \Delta C_1 + \Delta R_3 + G_2 & -\Delta R_3 - K G_2 \\ -\Delta R_3 & G_4 + \Delta R_3 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_s \\ 0 \end{pmatrix}$$

det of the matrix is

$$(\Delta C_1 + \Delta R_3 + G_2)(G_4 + \Delta R_3) - \Delta R_3(\Delta R_3 + K G_2) = \Delta.$$

Simplifying: $\Delta = \Delta^2 C_1 R_3 + \Delta (C_1 G_4 + R_3 G_4 + R_3 G_2 - K R_3 G_2) + G_2 G_4$

By applying Kramer's method:

$$V_2 = \frac{1}{\Delta} \begin{vmatrix} \Delta C_1 + \Delta R_3 + G_2 & I_s \\ -\Delta R_3 & 0 \end{vmatrix}$$

$$= \frac{\Delta R_3 I_s}{\Delta}$$

So $V_2 = \frac{\Delta R_3}{\Delta} \cdot V_1 \cdot \Delta C_1 \quad \therefore I_s = V_1 \Delta C_1$

$$\frac{V_2}{V_1} = \frac{\Delta^2 C_1 R_3}{\Delta}$$

But $V_2 = K V_1$; So

$$\frac{V_2}{V_1} = K \frac{\Delta^2 C_1 R_3}{\Delta} = \frac{\Delta^2 K C_1 R_3}{\Delta^2 C_1 R_3 + \Delta (C_1 G_4 + R_3 G_4 + R_3 G_2 - K R_3 G_2) + G_2 G_4}$$

Proved = $\frac{K \Delta^2}{\Delta^2 + \Delta \left(\frac{1}{C_1 R_4} + \frac{1}{C_1 R_4} + \frac{1}{C_1 R_2} - K \frac{1}{C_1 R_2} \right) + \frac{1}{R_2 R_4 C_1 R_3}}$