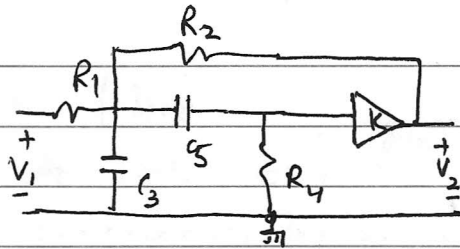
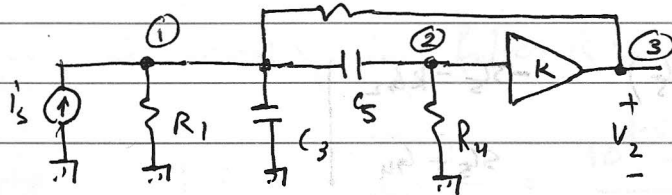


Q1:



After source transformation, $i_s = \frac{V_1}{R_1}$



Three node system, the nodal matrix is: with $G_x = \frac{1}{R_x}$

$$\begin{bmatrix} G_1 + G_2 + sC_3 + sC_5 & -sC_5 & -G_2 \\ -sC_5 & sC_5 + G_4 & 0 \\ -G_2 & 0 & G_2 \end{bmatrix} \begin{bmatrix} V_{(1)} \\ V_{(2)} \\ V_{(3)} \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix}$$

$V_{(1)}, V_{(2)}$ are the voltages at nodes (1), (2)

Observation, $V_{(3)} = K V_{(2)}$. So we can substitute column (3) elements and absorb them in column (2).

Since (3) is the output node of a voltage amplifier, the KCL @ node (3) is arbitrary. So row (3) of the matrix can be discarded.

Hence, the reduced nodal matrix is:

$$\begin{bmatrix} G_1 + G_2 + s(C_3 + C_5) & -sC_5 - KG_2 \\ -sC_5 & sC_5 + G_4 \end{bmatrix} \begin{bmatrix} V_{(1)} \\ V_{(2)} \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

(1) (2)

Q1. (contd.)

By using Kramer's rule

$$V_{(2)} = \frac{D_2}{D} \quad \text{where}$$

$$D = \begin{vmatrix} G_1 + G_2 + s(C_3 + C_5) & -sC_5 - kG_2 \\ -sC_5 & sC_5 + G_4 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} G_1 + G_2 + s(C_3 + C_5) & i_s \\ -sC_5 & 0 \end{vmatrix}$$

Hence $V_{(2)} = \frac{sC_5 \cdot i_s}{[G_1 + G_2 + s(C_3 + C_5)](sC_5 + G_4) - sC_5(sC_5 + kG_2)}$

Then $V_{(3)} = kV_{(2)}$ & $i_s = \frac{V_1}{R_1}$. Substituting

$$\frac{V_{(3)}}{V_1} = \frac{V_2}{V_1} = \frac{1}{R_1} \frac{k s C_5}{s^2 C_3 C_5 + [G_1 C_5 + (1-k)G_2 C_5 + G_4(C_3 + C_5)]s + \cancel{G_1(G_2 + G_4)} G_4(G_1 + G_2)}$$

$$= \frac{k s G_1 / C_3'}{s^2 + \left[\frac{G_1}{C_3} + (1-k) \frac{G_2}{C_3} + G_4 \left(\frac{1}{C_3} + \frac{1}{C_5} \right) \right] s + \frac{\cancel{G_1(G_2 + G_4)}}{C_3 C_5} G_4(G_1 + G_2)}$$

Q2. MFM filter.

$$\text{For } A_p = 1 \text{ dB, } \epsilon = \sqrt{10^{0.1 \times 1} - 1} = \sqrt{10^{0.1} - 1} = 0.50885$$

The attenuation to pass-band frequency ratio is $\omega_s = \frac{40}{10} = 4$.

Then, the order of the filter is: $n = \frac{\log \eta}{2 \log \omega_s}$

$$\text{where } \eta = \frac{10^{1A_p} - 1}{10^{0.1A_p} - 1} = \frac{10^{1.0} - 1}{10^{0.1} - 1} = 1217.45$$

$$n = \frac{\log (1217.45)}{2 \log 4} = \frac{3.086}{2.785} \rightarrow 3$$

We can start with a 3rd order BUT filter function:

$$H_N(s) \Big|_{\text{BUT}} = \frac{1}{s^3 + 2s^2 + 2s + 1} \quad \text{for which } \epsilon = 1$$

Since $\epsilon = 0.50885$ here, we can get the normalized LPF function by scaling $s \Big|_{\text{BUT}}$ to $\epsilon^{1/n} s$

$$\text{where } \epsilon^{1/n} = (0.50885)^{1/3} = 0.7984$$

$$s_0 \quad H_N(s) = \frac{1}{(0.7984s)^3 + 2(0.7984s)^2 + 2(0.7984s) + 1}$$

$$H_N(s) = \frac{1}{0.5089s^3 + 1.2749s^2 + 1.5968s + 1}$$

The de-normalized transfer function is obtained by frequency scaling $s \rightarrow s/2\pi f_c = \frac{s}{2\pi \times 10^4}$

Then $H(s)$, the frequency denormalized transfer

Q2 function becomes

$$H(s) = \frac{1}{0.20508 \times 10^{-14} s^3 + 0.32285 \times 10^{-9} s^2 + 0.0002541 s + 1}$$

Q3:

$$\omega_{p1} = 10^5, \quad \omega_{p2} = 4 \times 10^5$$

$$\omega_0 = \sqrt{4 \times 10^{10}} = 2 \times 10^5, \quad \text{geometric mean frequency.}$$

$$\omega_{a1} = 15.263 \times 10^5$$

$$\omega_{a2} = \frac{\omega_0^2}{\omega_{a1}} = \frac{4 \times 10^{10}}{15.263 \times 10^5} = 2.6207 \times 10^4$$

Thus for the normalized LPF associated with the given BPF, the stop-band to pass-band ratio

$$\omega_s = \frac{15.263 \times 10^5 - 2.6207 \times 10^4}{4 \times 10^5 - 10^5} \quad \text{the band width } B = 3 \times 10^5 \text{ rad/sec.}$$

≈ 5

Remember $B = 3 \times 10^5$; $\omega_0 = 2 \times 10^5$

Now $A_p = 1 \text{ dB}$, $A_a = 60 \text{ dB}$, $\omega_s = 5$

The filter has equiripple pass-band. So it is a CHEB filter.

We calculate:

$$\eta = \frac{10^{0.1 A_a} - 1}{10^{0.1 A_p} - 1} = \frac{10^0 - 1}{10^{-1} - 1} = 3862112.23$$

$$\epsilon = \sqrt{10^0 - 1} = 0.5088$$

$$\text{order } n \text{ of the filter } \geq \frac{\cosh^{-1} \sqrt{\eta}}{\cosh^{-1} \omega_s} = \frac{\cosh^{-1} (1965.226)}{\cosh^{-1} (5)}$$

$$n \geq 3.61. \quad \text{Take } n = 4.$$

Hence, using Tables of CHEB filter function

$$H_N(s) = \frac{1}{2^{n-1} \epsilon} \cdot \frac{1}{s^4 + 0.953 s^3 + 1.454 s^2 + 0.743 s + 0.276}$$

Q3 (Contd.)

Ans

$$(a) H(s) = \frac{0.246}{s^4 + 0.953s^3 + 1.454s^2 + 0.743s + 0.276}$$

(b) LC ladder realization

The filter is a small-pole system. Let us

$$\text{write } D(s) = s^4 + 1.454s^2 + 0.276 + 0.953s^3 + 0.743s \\ = M(s) + N(s)$$

$M(s)$ → even degree component of $D(s)$,

$N(s)$ → odd degree component of $D(s)$

With an ideal voltage source i.e. $R_s = 0$ and a 1Ω termination, i.e., $R_L = 1$, the DC gain = 1.

So the realized structure will not meet the DC gain feature of $H(s)$. But response relative to frequency is important in filters.

$$\text{Here } M(s) = s^4 + 1.454s^2 + 0.276 \quad \text{So deg}(M(s)) \\ N(s) = 0.953s^3 + 0.743s \quad > \text{deg}(N(s))$$

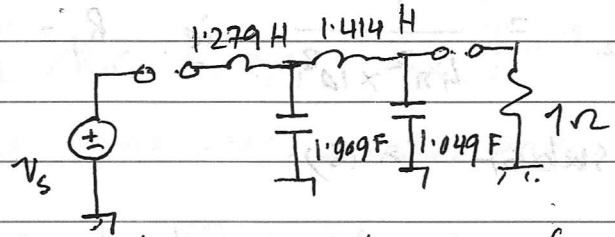
We perform CF expansion of $M(s)/N(s)$

around $s = \infty$ to realize Y_{22}

$$\begin{array}{r} (0.953s^3 + 0.743s) \left) \begin{array}{l} s^4 + 1.454s^2 + 0.276 \\ s^4 + 0.779s^2 \end{array} \right. \begin{array}{l} (1.049s \rightarrow C = 1.049 \\ \hline 0.674s^2 + 0.276 \end{array} \\ \hline (0.674s^2 + 0.276) \left) \begin{array}{l} 0.953s^3 + 0.743s \\ 0.953s^3 + 0.390s \end{array} \right. \begin{array}{l} (1.414s \rightarrow L = 1.414 \\ \hline 0.353s \end{array} \\ \hline (0.353s) \left) \begin{array}{l} 0.674s^2 + 0.276 \\ 0.674s^2 \end{array} \right. \begin{array}{l} (1.909s \rightarrow C = 1.909 \\ \hline 0.276 \end{array} \\ \hline (0.276) \left) \begin{array}{l} 0.353s \\ 0.353s \end{array} \right. \begin{array}{l} (1.279s \rightarrow L = 1.279 \\ \hline 0 \end{array} \end{array}$$

Q3(b)

Beginning from load end



is the L, C ladder realization for $H_N(s)$

Q. 4:

With $k=1$, the transfer function is:

$$\frac{V_2}{V_1} = \frac{-s \cdot 2/R_1 C_2}{s^2 + s \left[\frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} - \frac{1}{R_1 C_2} \right] + \frac{1}{R_1 R_2 C_1 C_2}} \quad \dots (1)$$

The standard band-pass second order function is

$$T(s) = \frac{H_0 \cdot (\omega_p / Q_p) s}{s^2 + (\omega_p / Q_p) s + \omega_p^2} \quad \dots (2)$$

Where ω_p = pole-frequency = $2\pi f_p$
 Q_p = pole-Q.

By comparing (1) and (2)

$$H_0 \cdot \frac{\omega_p}{Q_p} = -2/R_1 C_2 \quad \left(\begin{array}{l} \text{The negative sign is} \\ \text{unimportant for} \\ \text{magnitude} \\ \text{response} \end{array} \right)$$
$$\frac{\omega_p}{Q_p} = \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} - \frac{1}{R_1 C_2}$$
$$\omega_p^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

Given $C_1 = C_2 = 0.1 \mu\text{F}$ each

Let $C_1 = C_2 = 1 \text{ F}$ for now. We can do impedance scaling later.

$$\text{Then } \omega_p^2 = 4\pi^2 \times (10^4)^2 = \frac{1}{R_1 R_2} \quad \dots (3)$$

$$\frac{2\pi \times 10^4}{5} = \frac{2}{R_2} - \frac{1}{R_1} = \frac{2R_1 - R_2}{R_1 R_2} \quad \dots (4)$$

From (3) and (4) by division i.e. (3) \div (4)

$$\frac{4\pi^2 \times 10^8 \times 5}{2\pi \times 10^4} = \frac{1}{2R_1 - R_2} = 2\pi \times 10^4 \times 5$$

Q4: (Contd.)

$$2R_1 - R_2 = \frac{1}{\pi \times 10^5} \quad \dots (5)$$

$$\text{from (3)} \quad R_1 R_2 = \frac{1}{4\pi^2 \times 10^8} \quad \text{ie.} \quad R_1 = \frac{1}{R_2 \times 4\pi^2 \times 10^8} \quad \dots (6)$$

From (6), subst. in (5)

$$\frac{2}{R_2 \times 4\pi^2 \times 10^8} - R_2 = \frac{1}{\pi \times 10^5} \quad ; \quad \text{let } \pi^2 \approx 10.$$

$$\frac{2}{4 \times 10 \times 10^8} - R_2^2 = \frac{R_2}{\pi \times 10^5}$$

$$\text{or } R_2^2 + \frac{R_2}{\pi \times 10^5} - \frac{2}{4 \times 10^9} = 0.$$

Solving for R_2 :

$$R_2 = 0.000021$$

$$\text{then } R_1 = \frac{1}{R_2 \times 4\pi^2 \times 10^8} = 0.000012$$

First cut design values are

$$C_1 = C_2 = 1 \text{ F}$$

$$R_1 = 0.000012$$

$$R_2 = 0.000021$$

$$\text{For } C_1 = C_2 = 0.1 \text{ MF} = 10^{-7} \text{ F,}$$

we divide C_1, C_2 by 10^7
multiply R_1, R_2 by 10^7 } impedance scaling.

Second cut design values: $C_1 = C_2 = 0.1 \text{ MF}$

$$\text{Let } R_A = R_B = 1 \text{ k}\Omega \text{ each.}$$

$$R_1 = 120 \Omega$$

$$R_2 = 210 \Omega$$

Q.4 (Contd.)

The values $R_1 = 120\ \Omega$, $R_2 = 210\ \Omega$,

$C_1 = C_2 = 0.1\ \mu\text{F}$ will satisfy

$f_p = 10^4\ \text{Hz}$ and $Q_p = 5$

$R_A = R_B = 1\ \text{k}\Omega$ is a free choice.