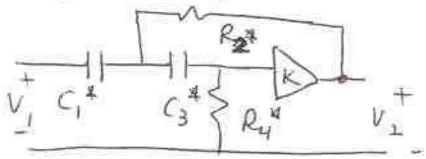
Ch. 5 (Hints & Solns)

5.1

(a) $H(s) = H_o \dfrac{s^2}{s^2 + .01s + 1}$ $C_1 = C_3 = 1F$, $R_2 = R_4$

Use the configuration (Fig 5.5)



From Table 5.4

$$\omega_p^2 = 1 = \frac{1}{R_2^* R_4^* C_1^* C_3^*}$$

$$\frac{1}{Q_p} = .01 = \sqrt{\frac{R_2}{R_4}\frac{C_1}{C_3}} + \sqrt{\frac{R_2}{R_4}\frac{C_3}{C_1}} + (1-K)\sqrt{\frac{R_4}{R_2}\frac{C_3}{C_1}}$$

Using $R_2 = R_4 = R$; $\omega_p^2 = 1 = \dfrac{1}{R^2 \cdot 1}$

or $R = 1\,\Omega$

Then $\dfrac{1}{Q_p} = .01 = 1 + 1 + 1 - K = 3 - K$; $K = 3 - .01 = 2.99$

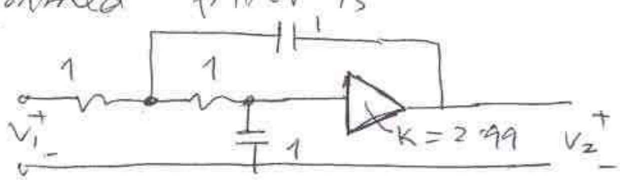
Thus, $H(s) = 2.99 \dfrac{s^2}{s^2 + .01s + 1}$ with $C_1 = C_3 = 1F$, $R_2 = R_4 = 1\,\Omega$, $K = 2.99$

(b)

On using CR : RC transformation

$$H(s) \to H^*(s) = \frac{(\omega_p^2/s)^2}{\left(\frac{\omega_p^2}{s}\right)^2 + .01\frac{\omega_p^2}{s} + 1} = \frac{1}{1 + .01s + s^2}$$

Which is a LPF with $\omega_p = 1$, $1/Q_p = .01$ The transformed filter is



(5.2)

$$\left.\begin{aligned} \omega_p &= 2\pi \times 1000 \\ Q_p &= 10 \\ H_0 &= 4 \end{aligned}\right\} \quad \begin{aligned} &C_3 = C_5 = .01\ \mu F = C \\ &\text{see schematic of Fig. 5 and Table 4.2} \end{aligned}$$

$$H_0 = \frac{K/R_1}{1/R_1 + 1/R_2 + 2/R_4 - K/R_2}$$

$$\frac{1}{Q_p} = \frac{\left[(1-K)(R_1/R_2)+1\right]\sqrt{\frac{R_4 C_3}{R_1 C_5}} + \sqrt{\frac{R_1 C_3}{R_4 C_5}} + \sqrt{\frac{R_1 C_5}{R_4 C_3}}}{\sqrt{1 + R_1/R_2}}$$

$$\omega_p = \frac{\sqrt{1+R_1/R_2}}{C\sqrt{R_1 R_4}} \quad \text{where } C = .01\ \mu F$$

Let $R_1/R_2 = m$; $R_1/R_4 = n$. Then

$$H_0 = \frac{K/R_1}{1/R_1 + \frac{m}{R_1} + \frac{2n}{R_1} - \frac{Km}{R_1}} = \frac{K}{1 + m + 2n - Km} = 4$$

5.2 contd.

$$\frac{1}{Q_p} = \frac{[(1-K)m+1]\sqrt{\frac{1}{n}} + \sqrt{n} + \sqrt{n}}{\sqrt{1+m}} = 0.1$$

$$\omega_p = \frac{\sqrt{1+m}}{C.\sqrt{n}\,R_4} = 2\pi \times 1000$$

Let $K = 2$; $\quad H_0 = 4 = \frac{2}{1+m+2n-2m} = \frac{2}{1+2n-m}$

or $\quad 2+4n-2m = 1 \quad$ or $\quad 4n-2m = -1 \quad m = 2n + .5$

Plug into $1/Q_p = 0.1 = \frac{(1-m)\cdot\frac{1}{\sqrt{n}} + 2\sqrt{n}}{\sqrt{1+m}} = \frac{(1-2n-.5)\frac{1}{\sqrt{n}} + 2\sqrt{n}}{\sqrt{1+2n+.5}}$

or $\quad 0.1\sqrt{1.5+2n} = \frac{.5-2n}{\sqrt{n}} + 2\sqrt{n} = \frac{.5-2n+2n}{\sqrt{n}}$

or $\quad 0.1\sqrt{1.5n+2n^2} = 0.5 \quad$ or $\quad 2n^2+1.5n = 25 \quad , 2n^2+1.5n-25 = 0$

$$n = \frac{1}{4}\cdot\left[-1.5 \pm \sqrt{2.25+200}\right] = \frac{1}{4}\left[-1.5 \pm \sqrt{202.25}\right] \Rightarrow 3.1826 = n$$

$-3.9304 \quad$ (the positive value)

Then $\quad m = 2n + .5 \quad = 6.865 = m$

Plug into $\omega_p = \frac{\sqrt{1+6.865}}{C.R_4.\sqrt{3.1826}} = \frac{2.8045}{1.78398}\cdot\frac{1}{C R_4} = 2\pi \times 1000$

Put $C = .01\,\mu F$, get R_4:

$$R_4 = \frac{2.8045}{1.78398}\cdot 10^8 \cdot \frac{1}{2\pi \times 1000} = 25.0199 \text{ K}$$

Then $R_1 = n R_4 = 79.6283$ K

$R_2 = \frac{R_1}{m} = 11.599$. K

Thus, $K = 2$, $R_1 = 79.628$ K, $R_2 = 11.599$ K, $R_4 = 25.02$ K
$C_3 = C_5 = .01\,\mu F$ is the design. (TRY $K = 3, 4 \cdots$)

5.3 (a) SK, BP filter with

$$H(s) = \frac{600\,s}{s^2 + 600\,s + 3\times10^8} \qquad \omega_p^2 = 3\times10^8$$

So $\omega_p = 10^4\sqrt{3} = 1.732\times10^4$

$\omega_p / Q_p = 600$; $Q_p = \frac{\omega_p}{600} = 1.732\times10^4 / 600 = 28.9$

The design equations are (see Table 5.4)

correction required

5.13 contd.

$$\omega_p^2 = 3\times10^8 = \frac{1+R_1/R_2}{R_1R_4C_3C_5}$$

$$\frac{1}{Q_p} = \frac{1}{28.9} = \frac{\left[1+\frac{R_1}{R_2}(1-k)\right]\sqrt{\frac{R_4C_3}{R_1C_5}} + \sqrt{\frac{R_1C_3}{R_4C_5}} + \sqrt{\frac{R_1}{R_4}\frac{C_5}{C_3}}}{\sqrt{1+\frac{R_1}{R_2}}}$$

$$H_o = \frac{K/R_1C_5}{\frac{1}{R_1C_5} + \frac{1}{R_4C_5} + \frac{1}{R_4C_3} + (1-K)/R_2C_5}$$

Let $C_3 = C_5 = C$; $\frac{R_1}{R_2} = m$; $\frac{R_4}{R_1} = n$, let $R_1 = R$

Then $\omega_p^2 = 3\times10^8 = \frac{1+m}{nR_1^2C^2}$; $n = \frac{(1+m)}{3\times10^8(RC)^2}$

$$\frac{1}{Q_p} = \frac{1}{28.9} = \frac{[1+m(1-K)]\sqrt{n} + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n}}}{\sqrt{1+m}}$$

$R_2C_5 = R_2C = C\cdot\frac{R_1}{m}$

$$= \frac{[1+m(1-K)]n + 2}{\sqrt{n(1+m)}} = \frac{1}{28.9}$$

$$\sqrt{n(1+m)} = 28.9\left[2 + \{1+m(1-K)\}n\right]$$

$$\frac{1+m}{\sqrt{3\times10^8 R^2C^2}} = 28.9\left\{2+n+mn(1-K)\right\}$$

$$1+m = RC\sqrt{3\times10^8}\times28.9\left\{2+\frac{1+m}{3\times10^8\times R^2C^2} + \frac{m(1+m)}{3\times10^8\times R^2C^2}(1-K)\right\}$$

Also $$H_o = \frac{K/RC}{\frac{1}{RC} + \frac{1}{nRC} + \frac{1}{nRC} + (1-K)m/CR}$$

$$H_o = 1 = \frac{nK}{n+1+1+(1-K)mn} = \frac{\frac{m+1}{3\times10^8C^2R^2}K}{2+\frac{m+1}{3\times10^8C^2R^2} + (1-K)\frac{m(m+1)}{3\times10^8C^2R^2}}$$

note $3\times10^8\times(CR)^2 \rightarrow$ is unitless number

Let $\omega_pCR = \alpha$ Then

5.3 cont.

$$1 = \frac{(m+1)\frac{K}{\alpha^2}}{2 + \frac{m+1}{\alpha^2} + (1-K)\frac{m(m+1)}{\alpha^2}}$$

$$1 = \frac{(m+1)K}{2\alpha^2 + (m+1) + (1-K)(m+1)m} \quad \leftarrow \text{from } H_o$$

$$\frac{1+m}{\alpha} = 28.9\left[2 + \{1+m(1-K)\}\frac{1+m}{\alpha^2}\right] \quad \leftarrow \text{from } \frac{1}{a_p}$$

$$\alpha(1+m) = 28.9\left[2\alpha^2 + (1+m)\{1+m-mK\}\right]$$

$$\alpha(1+m) = 28.9\left[2\alpha^2 + (1+m)^2 - m(1+m)K\right]$$

From H_o expression, we can solve for K in terms of 'm' & 'α'

Thus $$K = \frac{1+2m+m^2+2\alpha^2}{1+2m+m^2} = 1 + \frac{2\alpha^2}{1+2m+m^2}$$

$$1-K = -\frac{2\alpha^2}{1+2m+m^2}$$

$$1+m(1-K) = 1 + m\frac{-2\alpha^2}{1+2m+m^2} = 1 - \frac{2m\alpha^2}{1+2m+m^2}$$

Then from $\frac{1}{a_p}$ expression

$$\frac{1+m}{\alpha} = 28.9\left[2 + \left\{1 - \frac{2m\alpha^2}{1+2m+m^2}\right\}\frac{1+m}{\alpha^2}\right]$$

$$(1+m)\alpha = 28.9\left[2\alpha^2 + (1+m)\left(1 - \frac{2m\alpha^2}{1+2m+m^2}\right)\right]$$

We can solve the above eqn and obtain 'm' as a function of $\alpha = \omega_p CR$. Thus, for $\alpha = 33$, $m = 122.9$, $K = 1.148$, $n = 0.1138$

$= \cdot 75 \times 10^{5}$

5.3 cont'd

$\alpha = 33 , = \omega_p CR = 1.732 \times 10^{4}\ CR$

Let $C = R = \sqrt{\dfrac{33}{1.732 \times 10^{4}}} \simeq 0.04365$

Then $C_3 = C_5 = .04365$

$R_1 = R = .04365$, $R_2 = \dfrac{R_1}{m} = \dfrac{.04365}{122.9} = 0.000355$

$R_4 = nR_1 = 0.1138 \times .04365 = .004966$

If we scale R_2 to $100\ \Omega$ (scale factor 281557.8465)

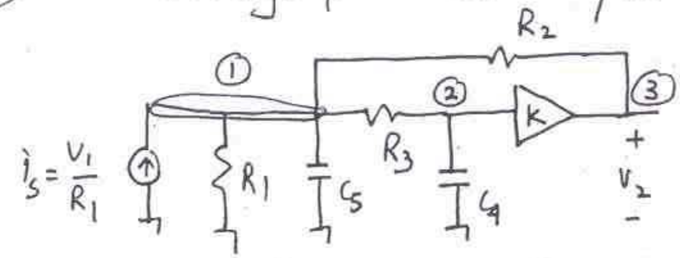
$R_2 = 100\ \Omega$, $R_1 = 12290\ \Omega$, $R_4 = 1398.3\ \Omega$

$C_3 = C_5 = C = .04365 / 281557.8465 = 0.155\ \mu F$

$K = 1.142$

Verified: $\omega_p^2 = 0.2999 \times 10^{9}$; $H_o = 1.013$, $Q_p = 29.284$

5.4 Arrange for nodal analysis:



$$\begin{array}{c} \begin{matrix} \quad\textcircled{1} & \qquad\qquad\textcircled{2} & \qquad\textcircled{3} \end{matrix} \\ \begin{bmatrix} G_1+G_3+G_2+sC_5 & -G_3 & -G_2 \\ -G_3 & G_3+sC_4 & 0 \\ -G_2 & 0 & G_2 \end{bmatrix} \begin{bmatrix} V_{①} \\ V_{②} \\ V_{③} \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix} \end{array}$$

All $G_i = \frac{1}{R_i}$ $i = 1, 2 \cdots$

$\because V_{③} = V_2 = K V_{②}$, eliminate node ③ by subst. for new $Y_{②}$

$y''_{②} = y_{②} + K\,y_{③}$, then discard row ③

$$\begin{bmatrix} G_1+G_2+G_3+sC_5 & -G_3-KG_2 \\ -G_3 & G_3+sC_4 \end{bmatrix} \begin{bmatrix} V_{①} \\ V_{②} \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

Then $V_{②} = \frac{1}{\Delta} \cdot \begin{vmatrix} G_1+G_2+G_3+sC_5 & i_s \\ -G_3 & 0 \end{vmatrix} = \frac{G_3\, i_s}{\Delta} = \frac{G_3 V_1 G_1}{\Delta}$

$\frac{V_{②}}{V_1} = \frac{G_3 G_1}{\Delta}$; $\frac{V_2}{V_1} = \frac{K V_{②}}{V_1} = \frac{K \cdot G_1 G_3}{\Delta} = \frac{N(s)}{D(s)}$,

Since $N(s)$ has no power of 's', it must be a L.P.F. function.

5.5 In 5.4

$$D(s) = \Delta = (G_1+G_2+G_3+sC_5)(G_3+sC_4) - (G_3+KG_2)(G_3)$$

$$= (G_1+G_2+G_3)G_3 + sC_4(G_1+G_2+G_3) + sG_3C_5 + s^2C_4C_5 - G_3^2 - KG_2G_3$$

$$= (G_1+G_2)G_3 - KG_2G_3 + s(G_3C_5 + G_1C_4 + G_2C_4 + G_3C_4) + s^2C_4C_5$$

$$= C_4C_5\left[s^2 + s\left(\frac{G_3}{C_4} + \frac{G_1}{C_5} + \frac{G_2}{C_5} + \frac{G_3}{C_5}\right) + \frac{G_1G_3 + (1-K)G_2G_3}{C_4C_5}\right]$$

If $R_1 = R_2 = R_3 = 1\Omega$, $C_4 = 10F$, the

denominator $\rightarrow s^2 + s\left(0.1 + \frac{3}{C_5}\right) + \frac{1+1-K}{10C_5}$

So $\frac{3}{C_5} = 0.1$ will make the s-coeff $= 0.2$.

$C_5 = 3/.1 = 30$ F

$\frac{2-K}{10C_5} \rightarrow 1$ now. $2-K = 10 \times 30 \times 1 = 300$

$K = 2 - 300 = -298$

So $C_5 = 30F$, $K = -298$ are the solutions

5.8 A second order BT filter has a transfer function

$T(s) = \dfrac{H}{s^2 + a_1 s + a_0}$. The denominator is

$D(s) = s^2 + a_1 s + a_0 \rightarrow a_0 - \omega^2 + j\omega a_1$

$\angle D(j\omega) = \tan^{-1}\dfrac{a_1\omega}{a_0-\omega^2} \rightarrow x - \frac{1}{3}x^3 + \frac{1}{5}x^5$ where

$x = \dfrac{a_1\omega}{a_0-\omega^2}$.

Delay is $\frac{\partial}{\partial\omega}\left[\angle D(j\omega)\right]$, i.e., $-\frac{\partial}{\partial\omega}\left[\angle T(j\omega)\right]$

5.6 contd.

$$\underline{/D(j\omega)} = \frac{a_1}{a_0}\cdot\frac{\omega}{1-\omega^2/a_0} - \frac{1}{3}\cdot\frac{a_1^3}{a_0^3}\cdot\frac{\omega^3}{\left(1-\omega^2/a_0\right)^3} + \cdots$$

$$= f_1(\omega) + f_3(\omega) + \cdots$$

Now $$\frac{\partial}{\partial\omega}\left[f_1(\omega)\right] = \frac{a_1}{a_0}\cdot\frac{\left(1-\omega^2/a_0\right)+\frac{2\omega}{a_0}\cdot\omega}{\left(1-\omega^2/a_0\right)^2} \rightarrow \frac{a_1}{a_0} \text{ at DC i.e. } \omega = 0$$

$$\frac{\partial}{\partial\omega}\left[f_3(\omega)\right] = -\frac{1}{3}\cdot\frac{a_1^3}{a_0^3}\cdot\frac{3\omega^2\left(1-\omega^2/a_0\right)^3 + \cdots}{\left(1-\omega^2/a_0\right)^6} \rightarrow 0 \text{ at DC i.e. } \omega = 0$$

If we set t_0 = delay at DC $= \frac{a_1}{a_0}$, then $a_1 = a_0 t_0$

Further, using the Binomial expansion $(1+x)^n \approx 1+nx$ to the terms $\left(1-\omega^2/a_0\right)^{-1}$ and $\left(1-\omega^2/a_0\right)^{-3}$ in $\underline{/D(j\omega)}$, we can write

$$\underline{/D(j\omega)} = \frac{a_1}{a_0}\cdot\omega\left(1+\frac{\omega^2}{a_0}+\cdots\right) - \frac{1}{3}\cdot\frac{a_1^3}{a_0^3}\cdot\omega^3\cdot\left(1+3\frac{\omega^2}{a_0}+\cdots\right)$$

$$= \frac{a_1}{a_0}\cdot\omega + \left[\frac{a_1}{a_0^2} - \frac{1}{3}\cdot\frac{a_1^3}{a_0^3}\right]\omega^3 + \cdots$$

For linear phase $D(j\omega) \approx \omega t_0$, so

$$\frac{a_1}{a_0^2} - \frac{1}{3}\frac{a_1^3}{a_0^3} = 0$$. Using $a_1 = a_0 t_0$, gives

$$a_0 = 3/t_0^2$$

If $t_0 = 1\,\text{ms}$, $a_0 = \frac{3}{10^{-6}} = 3\times10^6$

$$a_1 = a_0 t_0 = 3\times10^3$$

Thus $$T(s) = \frac{H}{s^2 + 3\times10^3 s + 3\times10^6}$$; $H \to 3\times10^6$ for DC gain = 1.

This is a low-pass filter with
$\omega_p^2 = 3\times10^6$; $\omega_p/Q_p = 3\times10^3$

5.6 contd.

Use the schematic of Fig. 4 for LPF using SK architecture. For equal C, equal R design

$$\omega_p = \frac{1}{RC}$$

$$\frac{1}{Q_p} = 2 + (1-K) = 3 - K$$

Let $C = 1\,F$ to start, then $\omega_p = \sqrt{3\times10^6} = \frac{1}{RC}$; $R \to 577.4\times10^{-6}$ ohm

$$\frac{\omega_p}{Q_p} = 3\times10^3 \quad ; \quad \frac{1}{Q_p} = \frac{3\times10^3}{\sqrt{3\times10^6}} = 1.732 = 3 - K \quad , K \to 1.27$$

$C = 1F$, $R = 577.4\,\mu\Omega$, $K = 1.27$ are initial design values

If $C \to 0.1\,\mu F$, scale R by 10^7 i.e. $R = 5774\,\Omega$

Thus (see Fig. 4)

$$\left.\begin{aligned} C_2 &= C_4 = 0.1\,\mu F \\ R_1 &= R_3 = 5774\,\Omega \\ K &= 1.27 \end{aligned}\right\} \text{are a possible design set}$$

5.7

According to Table 4.3, let $G_1 = H$, $C_3 = C_4 = 1$

$G_2 = 2\omega_p Q_p - H$, $G_5 = \omega_p / 2Q_p \rightarrow \dfrac{2\pi \times 10k}{2 \times 10} = 3.1416 \times 10^3$

H is yet unknown.

The BP-gain (ie. $s = j\omega_p$) is: $\dfrac{Hs}{\frac{\omega_p}{Q_p} s} = \dfrac{HQ_p}{\omega_p} = 1$ (assume)

Then $H = \dfrac{\omega_p}{Q_p} = 6.28319 \times 10^3 = G_1$ (by initial choice) $\therefore G_1 = 6.28319 \times 10^3$

$G_2 = 2 \times 2\pi \times 10^4 \times 10 - 6.28319 \times 10^3 = 1.25035 \times 10^6$

note: $D(s) = s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2 \rightarrow \frac{\omega_p}{Q_p} s$ at $s = j\omega_p$

Thus, initial design solution is:

$G_1 = 6.28319 \times 10^3$ mho, $G_2 = 1.25035 \times 10^6$ mho

$G_5 = 3.1416 \times 10^3$ mho ; $C_3 = C_4 = 1$ F

5.7 (Contd.)

But by given spec. $C_3 = C_4 = 1\,nF$ ie scale factor of 10^9. Then

$$\left.\begin{aligned} R_1 &= \frac{1}{G_1} \rightarrow \frac{1}{G_1} \times 10^9 = 159.15\,K\Omega \\ R_2 &= \frac{1}{G_2} \rightarrow \frac{1}{G_2} \times 10^9 = 799.78\,\Omega \\ R_5 &= \frac{1}{G_5} \rightarrow \frac{1}{G_5} \times 10^9 = 318.309\,K\Omega \\ C_3 &= C_4 = 1\,nF \text{ each} \end{aligned}\right\} \text{A design for B.P.F. according to schematic in Table 4.3.}$$

(5.8) Use nodal analysis and method of constraints. Assuming $K = \frac{R_A}{R_B}$, the voltage transfer function will be:

$$\frac{V_2(s)}{V_1(s)} = \frac{-s(K+1)/R_1C_2}{s^2 + s\left[1/R_2C_2 + 1/R_2C_1 - K/R_1C_2\right] + 1/R_1R_2C_1C_2}$$

Comparing with standard form of biquad B.P.F. transfer function ie.

$$\frac{H_0\,(\omega_p/Q_p)\,s}{s^2 + (\omega_p/Q_p)\,s + \omega_p^2}, \text{ we get}$$

$$\omega_p = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\frac{1}{Q_p} = \sqrt{\frac{R_1}{R_2}} \cdot \left(\sqrt{\frac{C_1}{C_2}} + \sqrt{\frac{C_2}{C_1}} - K\cdot\frac{R_2}{R_1}\sqrt{\frac{C_1}{C_2}}\right)$$

$$|H_0| = \frac{(K+1)/R_1C_1}{1/R_2C_2 + 1/R_2C_1 - K/R_1C_1}$$

Simplified design: Let $C_1 = C_2 = C \rightarrow 0.1\,\mu F$ suggested.

$$\frac{R_1}{R_2} = m \quad ; \quad R_1 = mR_2. \text{ Let } R_2 = R$$

Then $$\omega_p = \frac{1}{C \cdot R\sqrt{m}}$$

5.8

Contd. $\frac{1}{Q_p} = \sqrt{m}\left(1+1-K\cdot\frac{1}{m}\cdot\right) \Rightarrow \frac{1}{\sqrt{m}\,Q_p} = 2-\frac{K}{m}$

Squaring: $\frac{1}{m Q_p^2} = 4+\frac{K^2}{m^2} - 2.2.\frac{K}{m} = 4+\frac{K^2}{m^2}-\frac{4K}{m}$

i.e. $\frac{m}{Q_p^2} = 4m^2+K^2-4Km$; $4m^2 - m\left(4K+\frac{1}{Q_p^2}\right)+K^2 = 0.$

Giving $m = \dfrac{\left(4K+\frac{1}{Q_p^2}\right) \pm \sqrt{\left(4K+\frac{1}{Q_p^2}\right)^2 - 16K^2}}{8}$

OR $m = \frac{1}{8}\cdot\left[\left(4K+\frac{1}{Q_p^2}\right) \pm \sqrt{\frac{1}{Q_p^4}+\frac{8K}{Q_p^2}}\right]$

Assume $K=1$, Then $m = \frac{1}{8}\left[\left(4+\frac{1}{10^2}\right) \pm \sqrt{\frac{1}{10^4}+\frac{8}{10^2}}\right]$

$= \frac{1}{8}\left(4.01 \pm .283\right)$

$\rightarrow .5366$ OR 0.4659

Assume $K=2$, Then $m = \frac{1}{8}\left[\left(8+\frac{1}{10^2}\right) \pm \sqrt{\frac{1}{10^4}+\frac{16}{10^2}}\right]$

$= \frac{1}{8}\left[8.01 \pm .4\right]$

$\rightarrow 1.051$ OR 0.951

Consider:

$K=1$ $\begin{cases} m = .5366; \quad \text{Then } \omega_p = \frac{1}{CR\sqrt{m}} \text{ gives } R = \frac{1}{\sqrt{m}\,C\omega_p} \\ \text{i.e. } R = \frac{1}{\sqrt{.5366}\times 10^{-7}\times 2\pi\times 10^4} = 217.27\,\Omega \\ \text{So } \quad R_2 = 217.27\,\Omega \\ \qquad R_1 = mR_2 = 116.58\,\Omega \end{cases}$

$\left.\begin{matrix} R_2 \\ R_1 \end{matrix}\right\}$ resistance spread $= \frac{217.27}{116.58} \rightarrow 1.86$ i.e. less than 100

Consider $m = .4659$

$K=1$ $\begin{cases} R = \frac{1}{\sqrt{.4659}\times 10^{-7}\times 2\pi\times 10^4} = 233.17\,\Omega = R_2 \\ R_1 = mR_2 = 108.63\,\Omega \end{cases}$ spread $= \frac{1}{.4659} = 2.14..$ i.e., less Than 100

R_A, R_B remains free choice with $R_A = R_B$, since $K=1$

One can take $R_A = R_B = 100\,\Omega$ each.

5.9 See figure 9(b), for a LPF.

$$\frac{V_{o3}}{V_1} = -\frac{\left[\frac{1+R_6/R_5}{1+R_3/R_4}\right]\cdot\frac{1}{R_1R_2C_1C_2}}{D(s)}$$

with $D(s) = s^2 + \frac{s}{R_1C_1}\cdot\frac{1+R_6/R_5}{1+R_4/R_3} + \frac{R_6/R_5}{R_1R_2C_1C_2}$

$$\omega_p = \sqrt{\frac{R_6/R_5}{R_1R_2C_1C_2}} \quad ; \quad \frac{1}{Q_p} = \frac{1+R_6/R_5}{1+R_4/R_3}\cdot\sqrt{\frac{R_5R_2C_2}{R_6R_1C_1}}$$

Let $C_1 = C_2 = 1\ F$; $R_1 = R_2 = 1\ \Omega$

Then $\omega_p = 1$ radian will need $R_6 = R_5$

$\frac{1}{Q_p} = \frac{2}{1+R_4/R_3}\sqrt{1} = \frac{1}{20}$; So $1+\frac{R_4}{R_3} = 40$; $\frac{R_4}{R_3} = 39$

i.e. $R_4 = 39R_3$. One can take $R_3 = 100\ \Omega$, $R_4 = 3900\ \Omega$.

Gain at DC $\rightarrow \frac{1+R_6/R_5}{1+R_3/R_4} = \frac{2}{1+\frac{1}{39}} = \frac{78}{40} = 1.95$

So a possible design is:

- $C_1 = C_2 = 1\ F$; $R_1 = R_2 = 1\ \Omega$ (suggested)
- $R_6 = R_5 = 100\ \Omega$ each
- $R_3 = 100\ \Omega$, $R_4 = 3900\ \Omega$
- Gain at DC $= 1.95$

5.10 See figure 10 and Table 4-4. For a positive gain

BPF: $C_1 \to 0$; $R_1 \to$ open, $R_2 \to$ open

$$R_3 = \frac{Qr}{\text{gain at resonance}} = \frac{15\cdot r}{5} = 3r$$

$C = 1F$ suggested ; $\omega_p = 1$ rad/sec given

$R = 1/\omega_p C = 1\ \Omega$

Let $r = 1\ \Omega$, then $R_3 = 3\ \Omega$

So a design set is: $C = 1F$; $R = 1\ \Omega$; $r = 1\ \Omega$

$R_3 = 3\ \Omega$; $R_x = QR = 15\ \Omega$

5.11 See figure 11. For high-pass filter

let: $b_1 = 0$; $b_0 = 0$; $b_2 = G = 2$ } see eqns. (9), (10)

$$\omega_p = \sqrt{\frac{R_8}{R_2 R_3 C_1 C_2 R_7}} = 1 \text{ rad/sec}.$$

$$Q_p = 8 = \omega_p \cdot R_1 C_1$$

$b_1 = 0$ leads to $\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7} = 0$

$b_0 = 0$ leads to $\frac{R_8}{R_3 R_5 R_7 C_1 C_2} = 0$

$b_2 = 2$ leads to $\frac{R_8}{R_6} = 2$

Let $R_5 \to$ open circuit i.e. ∞. This will make $b_0 \to 0$.

Then $\frac{R_8}{R_6} = 2 = \frac{R_1}{R_4} \cdot \frac{R_8}{R_7}$

Let $\frac{R_1}{R_4} = 2$; $\frac{R_8}{R_7} = 1$, this will make $b_1 = 0$.

Now $\omega_p = \sqrt{\frac{R_8}{R_7}} \cdot \sqrt{\frac{1}{R_2 R_3 C_1 C_2}} = \sqrt{\frac{1}{R_2 R_3 C_1 C_2}}$

Let $R_2 = R_3 = 1\,\Omega$; $C_1 = C_2 = 1F$, then $\omega_p = 1$ rad/sec.

But $Q_p = 8 = \omega_p R_1 C_1$, Then $R_1 = \frac{8}{\omega_p C_1} = 8\,\Omega$

$\because \frac{R_1}{R_4} = 2$ has been assumed, $R_1 = 8\,\Omega$ makes $R_4 = 4\,\Omega$

So a possible design set is:

$R_5 \to$ open
$R_8 = R_7 = 100\,\Omega$ (say)
$R_1 = 8\,\Omega$
$R_4 = 4\,\Omega$
$C_1 = C_2 = 1F$
$R_2 = R_3 = 1\,\Omega$ (say)
$R_6 = R_8/2 = 50\,\Omega$

5.12

See figure 11, For Band reject filter

$$\frac{V_o}{V_1} = -b_2 \cdot \frac{s^2 + b_0/b_2}{s^2 + a_1 s + a_0} \quad \text{with } b_1 = 0$$

and $b_0/b_2 = a_0 = \omega_p^2$ ($\because$ ω_z has not been specified, we take $\omega_z = \omega_p$)

DC gain (i.e. at $s=0$) $\frac{b_0}{a_0} = 1$.

Gain at infinity $= b_2 = 1$.

Thus:

$$\frac{R_8}{R_6} = 1 \leftarrow b_2 = 1$$

$$\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7} = 0 \leftarrow b_1 = 0$$

$$\frac{R_8}{R_2 R_3 C_1 C_2 R_7} = \omega_p^2 = 1 = a_0 = b_0 = \frac{R_8}{R_3 R_5 R_7 C_1 C_2}$$

$$Q_p = \omega_p \cdot R_1 C_1 = 12$$

Let $\frac{R_1}{R_4} = \frac{R_8}{R_7} = 1$ each. This makes $b_1 = 0$; $\because \frac{R_8}{R_6} = 1$.

$$1 = \frac{R_8}{R_7} \cdot \frac{1}{R_2 R_3} \cdot \frac{1}{C_1 C_2} = \frac{R_8}{R_7} \cdot \frac{1}{R_3 R_5} \cdot \frac{1}{C_1 C_2}$$

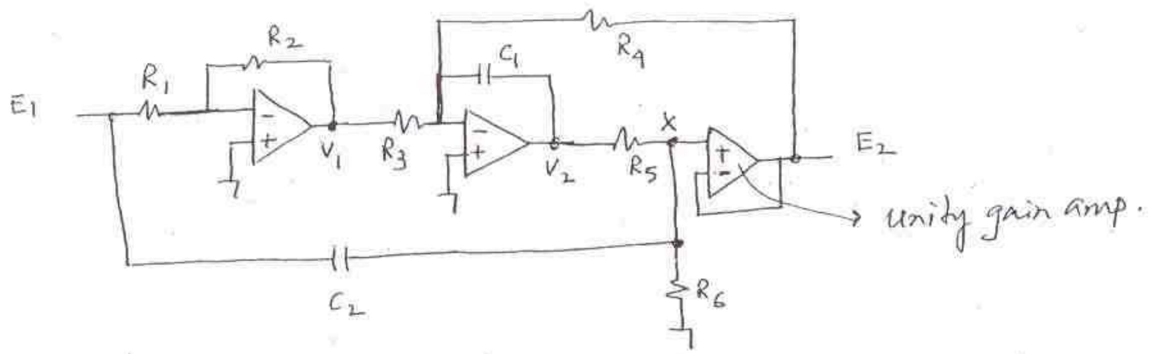
Let $C_1 = C_2 = 1$; $R_2 = R_3 = R_5 = 1$

Then $12 = R_1 \cdot 1 \cdot \omega_p$, makes $R_1 = 12$

Hence, a possible design set is:

$$\left.\begin{aligned}
R_2 = R_3 = R_5 &= 1\,\Omega \\
R_1 &= 12\,\Omega \\
R_4 &= 12\,\Omega \\
R_7 = R_8 &= 1\,\Omega \\
C_1 = C_2 &= 1\,F \\
R_6 &= 1\,\Omega
\end{aligned}\right\}$$

5.13



Working by each OA:

$$V_1 = -\frac{R_2}{R_1} E_1 \quad ; \quad V_2 = -\frac{1}{sC_1R_3} V_1 - \frac{1}{sC_1R_4} E_2$$

$$\frac{V_2 - E_2}{R_5} + (E_1 - E_2)\, sC_2 = \frac{E_2}{R_6} \quad \leftarrow \text{KCL at node X}$$

Re-organizing:

$$-\frac{R_1}{R_2} V_1 = E_1 \qquad \cdots\cdots (1)$$

$$\frac{V_1}{sC_1R_3} + V_2 + \frac{E_2}{sC_1R_4} = 0 \qquad \cdots\cdots (2)$$

$$\frac{V_2}{R_5} - E_2\left(\frac{1}{R_5} + sC_2 + \frac{1}{R_6}\right) = -E_1 sC_2 \quad \cdots (3)$$

From (1) & (2) $\qquad V_2 + \frac{E_2}{sC_1R_4} = -\frac{1}{sC_1R_3} \cdot \frac{-R_2}{R_1} E_1 = \frac{R_2}{R_1} \cdot \frac{1}{sC_1R_3} E_1 \quad \cdots (4)$

From (3) & (4)

$$\left.\begin{aligned} \frac{V_2}{R_5} - \left(\frac{1}{R_6} + \frac{1}{R_5} + sC_2\right) E_2 &= -sC_2 E_1 \\ V_2 + \frac{1}{sC_1R_4} E_2 &= \frac{R_2}{R_1} \frac{1}{sC_1R_3} E_1 \end{aligned}\right\}$$

By Krammer's rule:

$$E_2 = \frac{1}{\Delta} \cdot \begin{vmatrix} \frac{1}{R_5} & -sC_2 E_1 \\ 1 & \frac{R_2}{R_1} \frac{1}{sC_1R_3} E_1 \end{vmatrix}$$

$$\text{where } \Delta = \begin{vmatrix} \frac{1}{R_5} & -\left(\frac{1}{R_5} + \frac{1}{R_6} + sC_2\right) \\ 1 & \frac{1}{sC_1R_4} \end{vmatrix}$$

5.13

The solution is:

$$E_2/E_1 = \frac{s^2 C_2 G_2 C_1 + G_5 G_1 G_3}{G_2\left(G_4 G_5 + s(C_1 G_6 + C_1 G_5) + s^2 C_1 C_2\right)}$$

$$= \frac{G_2 C_1 C_2 \left[s^2 + G_1 G_3 G_5 / G_2 C_1 C_2\right]}{G_2 C_1 C_2 \left[s^2 + \frac{C_1 G_6 + C_1 G_5}{C_1 C_2} s + \frac{G_4 G_5}{C_1 C_2}\right]} = \frac{V_o}{V_s}$$

which is of the form:

$$T(s) = \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2} \quad \text{where}$$

$$\omega_z^2 = \frac{G_1 G_3 G_5}{G_2 C_1 C_2} \; ; \; \omega_p^2 = \frac{G_4 G_5}{C_1 C_2} \; ; \; \frac{\omega_p}{Q_p} = \frac{G_5 + G_6}{C_2}$$

With $C_1 = C_2 = 1F$, the design equations are:

$$\omega_z = \sqrt{\frac{G_1}{G_2} \cdot G_3 G_5} \; ; \; \omega_p = \sqrt{G_4 G_5} \; ; \; \frac{1}{Q_p} = (G_5 + G_6) / \sqrt{G_4 G_5}$$

5.14

$s^2 + \frac{1}{Q} s + 1 = (s+1)^2 - K\alpha s$ can be simplified to:

$$\frac{1}{Q} s = \cancel{s^2} + 2s + \cancel{1} - \cancel{s^2} - \cancel{1} - K\alpha s = 2s - K\alpha s$$

$$\frac{1}{Q} = 2 - K\alpha \quad . \text{ Since } 1/Q = Q^{-1}, \text{ obviously,}$$

$$S_K^Q = -S_K^{1/Q} \quad \text{where } \frac{1}{Q} = 2 - K\alpha = y \text{ (say)}$$

Considering 'K' as the only variable, since 'α' is independent of K

$$dy = -\alpha\, dK \quad ; \quad \frac{dy}{dK} = -\alpha$$

$$\frac{dy/y}{dK/K} = \frac{K}{y} \cdot (-\alpha) = \frac{K}{2 - K\alpha} \cdot \alpha = S_K^{1/Q}$$

Then $$S_K^Q = \frac{K}{K\alpha - 2} \cdot \alpha = \frac{K\alpha}{K\alpha - 2}.$$

5.15 Check problem : 5.8

$$\frac{1}{Q} = \sqrt{\frac{R_1}{R_2}} \cdot \left[\sqrt{\frac{C_1}{C_2}} + \sqrt{\frac{C_2}{C_1}} - K \cdot \frac{R_2}{R_1} \cdot \sqrt{\frac{C_1}{C_2}} \right]$$

$$\sim a \; C_1^{1/2} C_2^{-1/2} + a \cdot C_2^{\frac{1}{2}} C_1^{-\frac{1}{2}} - b \cdot C_1^{\frac{1}{2}} C_2^{-1/2}$$

where $a = \sqrt{\frac{R_1}{R_2}}$, $b = \sqrt{\frac{R_1}{R_2}} \cdot K \cdot \frac{R_2}{R_1}$

Let $y = \frac{1}{Q} = a \cdot C_1^{\frac{1}{2}} C_2^{-\frac{1}{2}} + a \cdot C_2^{\frac{1}{2}} C_1^{-\frac{1}{2}} - b \cdot C_1^{\frac{1}{2}} \cdot C_2^{-\frac{1}{2}}$

Then $S^{Q}_{C_1} = -S^{y}_{C_1}$ etc.

For $S^{y}_{C_1}$, consider $\frac{\partial y}{\partial C_1} \rightarrow a \cdot \frac{1}{2} \cdot C_1^{-\frac{1}{2}} \cdot C_2^{-\frac{1}{2}} - a \cdot \frac{1}{2} \cdot C_2^{\frac{1}{2}} \cdot C_1^{-\frac{3}{2}}$
$- b \cdot C_2^{-1/2} \cdot \frac{1}{2} \cdot C_1^{-1/2}$

$$\frac{\partial y}{\partial C_1} = \frac{1}{2} \cdot \left[\frac{a}{\sqrt{C_1 C_2}} - \frac{a \cdot \sqrt{C_2}}{C_1 \sqrt{C_1}} - b \cdot \frac{1}{\sqrt{C_1 C_2}} \right]$$

$$\frac{C_1}{y} \cdot \frac{\partial y}{\partial C_1} = \frac{1}{2} \frac{1}{y} \cdot \left[a \cdot \sqrt{\frac{C_1}{C_2}} - a\sqrt{\frac{C_2}{C_1}} - b \cdot \sqrt{\frac{C_1}{C_2}} \right] = \frac{1}{2} \frac{1}{y} \left[y - 2 \cdot a \sqrt{\frac{C_2}{C_1}} \right]$$

$$\because y = \frac{1}{Q} = a \cdot \sqrt{\frac{C_1}{C_2}} + a \cdot \sqrt{\frac{C_2}{C_1}} - b \cdot \sqrt{\frac{C_1}{C_2}}$$

So $S^{y}_{C_1} = \frac{\partial y / y}{\partial C_1 / C_1} = \frac{1}{2} \left[1 - a\sqrt{\frac{C_2}{C_1}} \right]$

$S^{Q}_{C_1} = -S^{y}_{C_1} = \frac{1}{2} \left[a\sqrt{\frac{C_2}{C_1}} - 1 \right]$ and so on ...

5.16

LEFT AS AN EXERCISE

5.17 (a) $\frac{V_2(s)}{V_1(s)} = \frac{H}{s^2 + .5714 s + 1}$. For equal R, equal C design:

$\omega_p = \frac{1}{RC} = 1$ $\quad \frac{\omega_p}{Q_p} = \frac{1}{Q_p} = 0.5714 = 3 - K$

So $K = 3 - .5714 = 2.43$

We can let $R = 1$, $C = 1$ and $K = 2.43$ as the design values. With $K = 1.43$, for the LPF, we shall have $H = K = 2.43$.

5.17

(b) The transfer function with ideal OA is:

$$H_{LP}\Big|_{\substack{\text{equal } R,\\ \text{equal } C}} = \frac{K \cdot G_1 G_3 / C_2 C_4}{s^2 + s\left(3\frac{G}{C} - K\frac{G}{C}\right) + \frac{G_1 G_3}{C_2 C_4}}$$

$$= \frac{K.}{s^2 + s.\,(3-K) + 1}$$

$\because G_1 = G_3 = G = 1$, $C_2 = C_4 = 1$ have been assumed.

If OA has a transfer function $A(s) \sim \frac{\omega_t}{s}$, the gain K becomes (see p.66)

$$K(s) \sim K_+ = K \cdot \frac{1}{1+\frac{K}{A}} = \frac{K}{1+\frac{Ks}{\omega_t}} = \frac{K}{1+\left(\frac{K}{\omega_t}\right)s}$$

Then

$$H_{LP} \to \overline{H}_{LP} = \frac{K/\left(1+\frac{K}{\omega_t}s\right)}{s^2 + s.\left(3 - \frac{K}{1+\frac{K}{\omega_t}s}\right) + 1}$$

Let $\beta = \frac{K}{\omega_t}$

$$\overline{H}_{LP} = \frac{\frac{K}{(1+\beta s)}}{s^2 + s\left(3 - \frac{K}{1+\beta s}\right) + 1}$$

$$= \frac{K}{s^2(1+\beta s) + s\left(3(1+\beta s) - K\right) + (1+\beta s)}$$

$$= \frac{K}{\beta s^3 + s^2 + 3s(1+\beta s) - Ks + 1 + \beta s}$$

$$= \frac{K}{\beta s^3 + (1+3\beta)s^2 + s(3-K+\beta) + 1}$$ is the new network function that will be realized.

The new ω_p will be dictated by the Re-part of $D(s) \to 0$

i.e. $(1+3\beta).(-\hat{\omega}_p^2) + 1 = 0$ or $\hat{\omega}_p = \sqrt{\frac{1}{1+3\beta}} = \sqrt{\frac{1}{1+3\frac{K}{\omega_t}}}$

5.17 (b)

Retaining the symbols for R, C

$$D(s) = s^2 + \frac{G}{C}\cdot s\,(3-K) + \frac{G^2}{C^2} \quad \text{with ideal OA.}$$

$$\overline{D(s)} \Rightarrow s^2 + \frac{G}{C}\cdot s\cdot\left(3 - \frac{K}{1+\beta s}\right) + \frac{G^2}{C^2}.$$

$$\rightarrow \beta s^3 + s^2 + 3\frac{G}{C}s + 3\frac{G}{C}\cdot\beta s^2 - K\frac{G}{C}s + \frac{G^2}{C^2} + \frac{G^2}{C^2}\beta s$$

$$\rightarrow \beta s^3 + s^2\left(1 + 3\beta\frac{G}{C}\right) + s\left(3\frac{G}{C} - K\frac{G}{C} + \frac{G^2}{C^2}\beta\right) + \frac{G^2}{C^2}$$

Then $\hat{\omega}_p^2 = \dfrac{G^2/C^2}{1+3\beta\frac{G}{C}}$; $\hat{\omega}_p = \dfrac{G/C}{\sqrt{1+3\frac{G}{C}\beta}} = \dfrac{G/C}{\sqrt{1+3\frac{G}{C}\cdot\frac{K}{\omega_t}}}$

For new Q_p, consider the imaginary part of $\overline{D(s)}$ i.e.

$$\frac{-j\beta\omega^3}{1+3\beta\frac{G}{C}} + j\omega\,\frac{3\frac{G}{C} - K\frac{G}{C} + \beta\frac{G^2}{C^2}}{1+3\beta\frac{G}{C}} \rightarrow j\omega\,\frac{\hat{\omega}_p}{\hat{Q}_p}$$

$$\text{i.e.} \quad -\frac{\beta\hat{\omega}_p^2}{1+3\beta\frac{G}{C}} + \frac{3\frac{G}{C} - K\frac{G}{C} + \beta\frac{G^2}{C^2}}{1+3\beta\frac{G}{C}} = \frac{\hat{\omega}_p}{\hat{Q}_p}$$

Let

$$\frac{1}{\hat{Q}_p} = -\frac{\beta\hat{\omega}_p}{1+3\beta\omega_p} + \frac{3\omega_p - K\omega_p + \beta\omega_p^2}{(1+3\beta\omega_p)\hat{\omega}_p} \qquad \because \frac{G}{C} = \omega_p \text{ in ideal case.}$$

$$= -\frac{\beta\hat{\omega}_p}{1+3\beta\omega_p} + \frac{3-K+\beta\omega_p}{1+3\beta\omega_p}\cdot\frac{\omega_p}{\hat{\omega}_p}$$

$$= -\frac{K\,\omega_p/\omega_t}{1+3\cdot K\frac{\omega_p}{\omega_t}} + \frac{3-K+K\frac{\omega_p}{\omega_t}}{\sqrt{1+3K\frac{\omega_p}{\omega_t}}}$$

5.17 (b) Noting that $\omega_p = \frac{G}{C}$ in the ideal case

$$\hat{\omega}_p = \frac{\omega_p}{\sqrt{1 + 3\frac{\omega_p K}{\omega_t}}} \qquad \because \beta = \frac{K}{\omega_t} \text{ by substitution}$$

$$\frac{1}{\hat{Q}_p} = \frac{3 - K + K\frac{\omega_p}{\omega_t}}{\sqrt{1 + 3K\frac{\omega_p}{\omega_t}}} - \frac{\hat{\omega}_p \cdot \frac{K}{\omega_t}}{1 + 3\frac{\omega_p K}{\omega_t}}$$

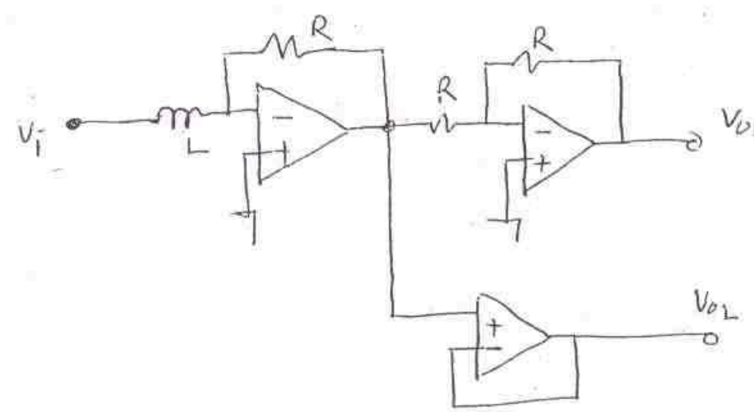
For equal R, equal C case
$K = 2.43$ found out.

So evaluate $\hat{\omega}_p$, $\hat{Q}_p$ for $\frac{\omega_t}{\omega_p} = 7.5$ i.e. $\frac{\omega_p}{\omega_t} = \frac{1}{7.5}$

Note, the flat gain remains unchanged to the value K

(5.17) (c) For convenience of simulation, we shall use $R = 10K$, $C = 0.1\mu F$, so that $\omega_p = \frac{1}{10^{+4} \cdot 10^{-7}} = 10^3$ rad/sec.

Then arrange a non-ideal OA with a gain function $A(s) = \frac{\omega_t}{s}$ where $\omega_t = 7.5 \times 10^3$ rad/sec. Thus:

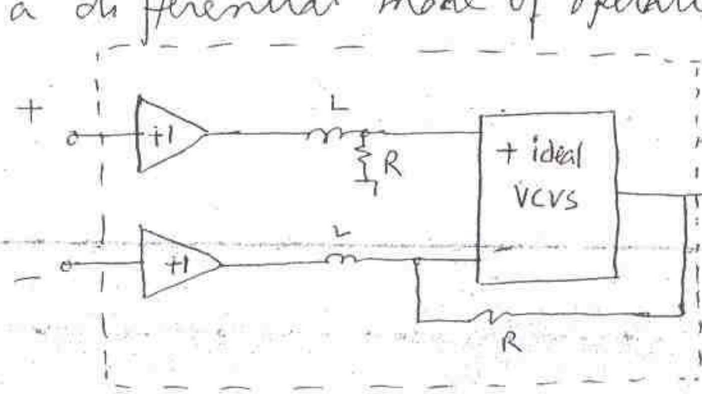


$V_{o1} = +\frac{R}{sL}$, with $\frac{R}{L} = \omega_t$

with all IDEAL OA

$V_{o2} = -\frac{R}{sL}$, with $\frac{R}{L} = \omega_t$

For a differential mode of operation, the system will be:



→ non ideal OA with

$A(s) = \frac{\omega_t}{s}$

$\omega_t = \frac{R}{L}$ $\quad R = 750\,\Omega$, $L = 0.1$ H

for $\omega_t = 7.5 \times 10^3$

5.18 Given $T(s) = \dfrac{Hs}{s^2 + .5714s + 1}$

(a) With equal R, equal C assumption, the network function is:

$$T_N(s) = \frac{sK/RC}{s^2 + s\left[\frac{4}{RC} - \frac{K}{RC}\right] + \frac{2}{R}\cdot\frac{1}{RC^2}}$$

derived from

$$T_N(s) = \frac{sK/R_1C_5}{s^2 + s\left[1/R_1C_5 + 1/R_2C_5 + 1/R_4C_5 + 1/R_4C_3 - K/R_2C_5\right] + \left(1/R_4C_3C_5\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

Then $\omega_p^2 = \dfrac{2}{R^2C^2}$; $\omega_p = \dfrac{\sqrt{2}}{RC}$

$\dfrac{\omega_p}{Q_p} = \dfrac{1}{RC}\cdot(4-K)$; $\dfrac{1}{Q_p} = \dfrac{1}{RC}(4-K)\cdot\dfrac{RC}{\sqrt{2}} = \dfrac{4-K}{\sqrt{2}}$

Given $\omega_p = 1 \leftarrow \dfrac{\sqrt{2}}{RC}$, so $RC = \sqrt{2}$, we can let $\left.\begin{matrix} R = 1\,\Omega \\ C = \sqrt{2}\ F\end{matrix}\right\}$

Given $Q_p = \dfrac{1}{.5714} \cong 1.75 \rightarrow \dfrac{\sqrt{2}}{4-K}$; $K = 3.192$

$$H_o = \frac{K/RC}{\frac{1}{RC}(4-K)} = \frac{K}{4-K} \rightarrow 3.95$$

Initial design values $R = 1\,\Omega$, $C = \sqrt{2}\ F$, $K = 3.192$

(b) Now with $A(s) = \dfrac{\omega_t}{s}$, $K(s) = \dfrac{K}{1 + \frac{K}{\omega_t}s} = \dfrac{K}{1+\beta s}$ as before

Then
$$\bar{D}(s) = s^2 + \frac{1}{RC}s\left(4 - \frac{K}{1+\beta s}\right) + \frac{2}{R^2C^2}$$
$$= \frac{R^2C^2\beta s^3 + (R^2C^2 + 4RC\beta)s^2 + (4RC - KRC + 2\beta)s + 2}{R^2C^2(1+\beta s)}$$

$$\bar{T}(s) = \frac{s.KRC}{s^3\beta R^2C^2 + s^2(R^2C^2 + 4RC\beta) + s(4RC - KRC + 2\beta) + 2}$$

Thus the new ω_p, Q_p and H_o will be

5.18 contd.

(b)

$$\frac{\hat{\omega}_p}{\hat{Q}_p} = -\frac{\beta\hat{\omega}_p^2}{1+\frac{4}{\sqrt{2}}\cdot\beta\omega_p} + \frac{\omega_p}{\sqrt{2}}\cdot\frac{4-K+\frac{2\beta\omega_p}{\sqrt{2}}}{1+\frac{4}{\sqrt{2}}\cdot\beta\omega_p}$$

So
$$\frac{1}{\hat{Q}_p} = -\frac{\beta\hat{\omega}_p}{1+\frac{4}{\sqrt{2}}\beta\omega_p} + \frac{\omega_p}{\hat{\omega}_p\sqrt{2}}\cdot\frac{4-K+2\beta\omega_p/\sqrt{2}}{1+4\beta\omega_p/\sqrt{2}}$$

$$= \frac{\omega_p}{\sqrt{2}\,\hat{\omega}_p}\cdot\frac{4-K+\sqrt{2}\cdot K\,\omega_p/\omega_t}{1+2\sqrt{2}\,K\frac{\omega_p}{\omega_t}} - \frac{K\,\hat{\omega}_p/\omega_t}{1+2\sqrt{2}\,K\frac{\omega_p}{\omega_t}}$$

At $\omega = \hat{\omega}_p$ now

$$\bar{T}(s)\Big|_{\omega=\hat{\omega}_p} = \frac{sKRC}{s^3\beta R^2C^2 + s(4RC-KRC+2\beta)} = \frac{KRC}{4RC-KRC+2\beta+s^2\beta R^2C^2}$$

If we let $s \to j\hat{\omega}_p$

$$\bar{T}(s)\Big|_{\omega=\hat{\omega}_p} = \frac{KRC}{4RC-KRC+2\beta-\hat{\omega}_p^2\beta R^2C^2}$$

$$= \frac{K}{4-K+2\frac{\beta}{RC}-\hat{\omega}_p^2\beta\cdot RC}$$

So
$$\hat{H}_0 = \frac{K}{4-K+2\cdot\frac{\omega_p}{\sqrt{2}}\cdot\frac{K}{\omega_t}-\hat{\omega}_p^2\cdot\frac{K}{\omega_t}\cdot\frac{\sqrt{2}}{\omega_p}}$$

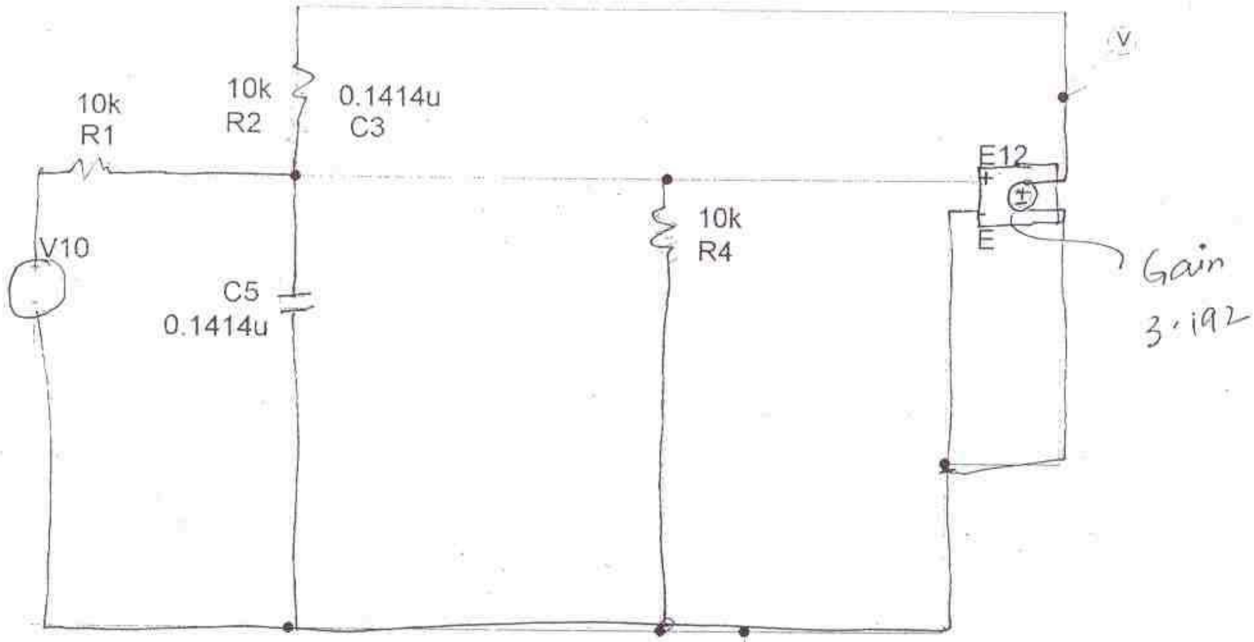
$$= \frac{K}{4-K+\sqrt{2}\,K\frac{\omega_p}{\omega_t}-\sqrt{2}\,K\cdot\frac{\hat{\omega}_p^2}{\omega_p\cdot\omega_t}}$$

With $\omega_p/\omega_t = 7.5$, and nominal $\omega_p = 1$

$\hat{\omega}_p = 0.6736$ rad/sec.

$\frac{1}{\hat{Q}_p} = 0.5415 \rightarrow \hat{Q}_p = 1.846$; $\hat{H}_0 = 2.808$

5.18



$f_0 = 159.33 \rightarrow Hz \rightarrow 1000.79$ rad/sec

$\uparrow \omega_p$

$BW = 90.7302$ Hz

$Q = 159.33/90.7302$

$\rightarrow 1.756$

$H_0 = 3.95$

Simulation with ideal OA.

$\hat{f}_0 = 111.354$ Hz $\rightarrow 699.85$ rad/sec.

$\uparrow \hat{\omega}_p$

$BW = 61.012$ Hz

$\hat{Q} = 1.826$

$\hat{H}_0 = 2.833$

OA of

$A(s) = \frac{\omega_t}{s}$

$\omega_t = 7.5\,\omega_p$

X

5.19

LEFT AS AN EXERCISE

5.20 Peak-to-peak ripple implies a CHEB magnitude approximation. Using the data $A_p = 1\,dB$, $A_a = 50\,dB$, $f_c = 10\,KHz$, $f_a = 60\,KHz$, we find

$n_{CHEB} = 3$

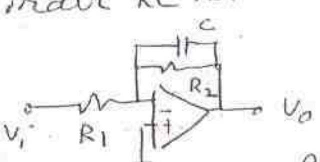
$\epsilon = 0.5088$

Then $H_N(s) = \frac{1}{\epsilon} \cdot \frac{1}{D(s)} = \frac{0.4914}{(s+.494)(s^2+.490s+.994)}$

$$= \frac{.494}{s+.494} \cdot \frac{.994}{s^2+.490s+.994}$$

Cascade of a 1st order and 2nd order RC networks.

For the first order network consider



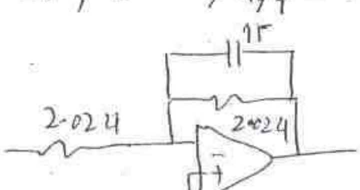
$$\frac{V_o}{V_i} = -\frac{R_2 \| (\frac{1}{sC})}{R_1} = -\frac{\frac{R_2 \frac{1}{sC}}{R_2 + \frac{1}{sC}}}{R_1} = -\frac{R_2/(1+sCR_2)}{R_1}$$

$$= -\frac{R_2}{R_1} \cdot \frac{1}{sCR_2+1} = -\frac{R_2}{R_1 CR_2} \cdot \frac{1}{s+1/CR_2} = -\frac{1}{CR_1} \cdot \frac{1}{s+1/CR_2}$$

if $R_1 = R_2 = R$, $\frac{V_o}{V_i} = -\frac{1/CR}{s+1/CR} \leftarrow -\frac{.494}{s+.494}$

Let $C = 1F$, $R = 1/.494 = 2.024\,\Omega$, making $1/RC = .494$

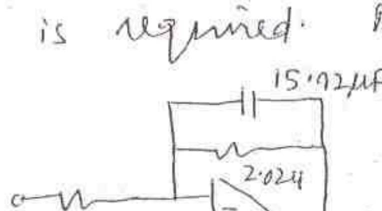
Then



will produce $-\frac{.494}{s+.494}$ transfer function.

This was for $\omega_c = 1$ rad/sec. But given $f_c = 10\,KHz$, $\omega_c = 2\pi \times 10^4$. So frequency scaling is required. Make $C \to 1/2\pi \times 10^4 = 15.92\,\mu F$.

Then



will be one-part of the desired filter.

5.20 Consider $\dfrac{.994}{s^2+.490s+.994}$

this is LPF of DC gain $= 1$, $\omega_p = \sqrt{.994}$ and

$\dfrac{\omega_p}{Q_p} = .490$, $\dfrac{1}{Q_p} = \dfrac{.490}{\omega_p} = .491476$; $Q_p = 2.03$

Consider a SK design (see p. 53-54)

Let $K = 1$. Then

$$\omega_p = \frac{1}{\sqrt{R_1R_3C_2C_4}} \quad ; \quad \frac{1}{Q_p} = \sqrt{\frac{R_3C_4}{R_1C_2}} + \sqrt{\frac{R_1C_4}{R_3C_2}}$$

Take $n = \dfrac{R_3}{R_1}$; $m = C_4/C_2$, Then solve for a good 'm'

from $m < 1/4Q_p^2$ i.e. $m < .061$, let $m = .04$

Then $n = \left(\dfrac{1}{2mQ_p^2} - 1\right) \pm \dfrac{1}{2mQ_p^2}\cdot\sqrt{1-4mQ_p^2}$, gives

$$n = 2.0333 \pm 1.7704 = 3.8037 \text{ OR } 0.2629$$

Take $n = 3.8037$

Then $R_3 = 3.8037 R_1$; $C_4 = .04 C_2$ and

$$\omega_p = \frac{1}{\sqrt{3.8037 \times .04}\; C_2R_1} = \sqrt{.994} = 0.996995$$

Let $C_2 = 1F$, Then $R_1 = \dfrac{1}{.996995\sqrt{3.8037\times .04}} = 2.5714\ \Omega$

Then $C_4 = .04F$, $R_3 = 9.7809\ \Omega$

Confirm: $\omega_p = \dfrac{1}{\sqrt{2.5714\times 9.7809\times 1\times .04}} \Rightarrow .997 \approx .996995$

$$\frac{1}{Q_p} = \sqrt{\frac{R_3C_4}{R_1C_2}} + \sqrt{\frac{R_1C_4}{R_3C_2}} = \sqrt{3.8037\times .04} + \sqrt{\frac{1}{3.8037}\times .04}$$

$$= 0.4926 \approx .4914.$$

Thus $R_1 = 2.5714\Omega$, $R_3 = 9.7809\Omega$, $C_2 = 1F$, $C_4 = .04F$ is a possible design for $\omega_c = 1$ rad/sec. use frequency

... .4 rad/sec as given,

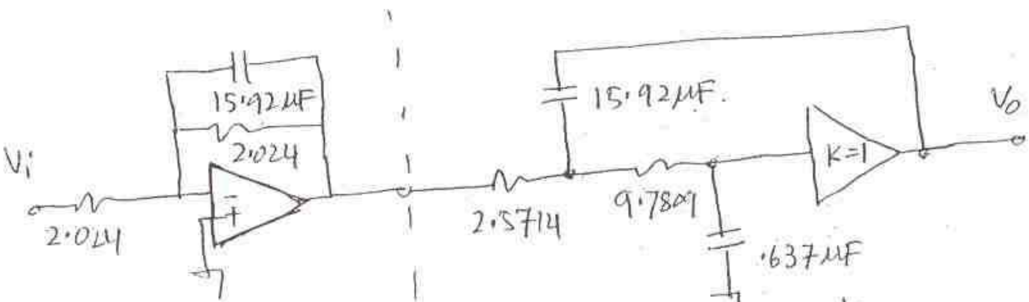
5.20 Let's make $C \to C/(2\pi \times 10^4)$, so

$R_1 = 2.5714\ \Omega$, $R_3 = 9.7809\ \Omega$ will realize

$C_2 = 15.92\ \mu F$; $C_4 = 0.637\ \mu F$

$\omega_c \to 2\pi \times 10^4$ rad/sec.

The overall filter is a cascade:



1st order section | 2nd order section

There is a sign inversion in the overall transfer function.
For magn. approx., this is immaterial.

5.21 Find $n_{CHEB} = 3$ with $\epsilon = 0.7648$

$$H_N(s) = \frac{1}{2^2 \cdot \epsilon} \cdot \frac{1}{D(s)} = \frac{0.327}{(s+.402)(s^2+.369s+.886)}$$

$$= \frac{.402}{s+.402} \cdot \frac{.886}{s^2+.369s+.886}$$ This is the associated normalized LPF. transfer function

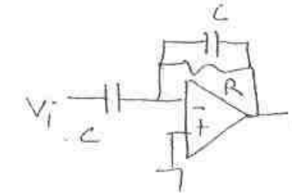
For the normalized HPF., $s \to \frac{1}{s}$

$$H_N\Big|_{HP} = \frac{.402 s}{1+.402 s} \cdot \frac{.886 s^2}{1+.369 s+.886 s^2}$$

$$= \frac{s}{s+2.4876} \cdot \frac{s^2}{s^2+0.4165 s+1.1287}$$

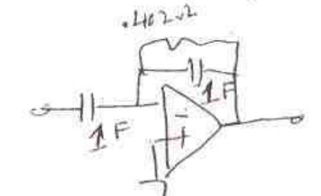
5.21 For the first order HP section consider

Contd.



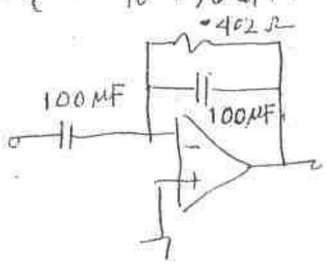
$\therefore \frac{V_o}{V_i} = -\frac{\frac{R}{1+sCR}}{1/sC} = -\frac{sCR}{1+sCR} = \frac{s}{s+1/CR}$

Let $C = 1F$, $R = 1/2.4576$, $= 0.402\,\Omega$

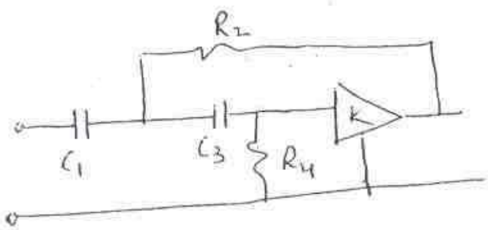


is the network. On denormalization for $\omega_c = 10^4$ rad/sec, the scale factor is 10^4

So make $C = 10^{-4}F = 100\mu F$



For the second order HP section, consider, the following with $k = 1$



Then $\omega_p = \sqrt{1.1287}$

$= \frac{1}{\sqrt{C_1C_3R_2R_4}}$

$\frac{1}{Q_p} = \sqrt{\frac{R_2}{R_4}\frac{C_1}{C_3}} + \sqrt{\frac{R_2}{R_4}\frac{C_3}{C_1}} = \frac{.4165}{\omega_p} = .392$

Following method similar to (4.21), let $1/m = \frac{R_4}{R_2}$, $1/n = \frac{C_3}{C_1}$. Solve for $m < \frac{1}{4Q_p^2}$

$m < \frac{1}{4} \cdot 0.153664 \to .038$. Let $m = .01$

Then $n = \left(\frac{1}{2mQ_p^2} - 1\right) \pm \frac{1}{2mQ_p^2}\sqrt{1 - 4mQ_p^2}$, with $m = .02$

$n = 5.5014$ or 0.1818, Let $n = .1818$

Then $\omega_p = \sqrt{1.1287} = 1.0624 = \frac{1}{\sqrt{nC_3^2 mR_4^2}} = \frac{1}{\sqrt{nm}}\frac{1}{R_4C_3}$

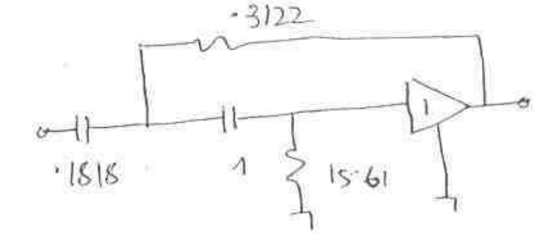
Let $C_3 = 1F$ Then $R_4 = 15.61\,\Omega$

5.21 Contd.

$C_1 = n C_3 = \cdot 1818 \text{ F}$

$R_2 = m R_4 = \cdot 02 \times 15 \cdot 61 = \cdot 3122 \ \Omega$

Thus

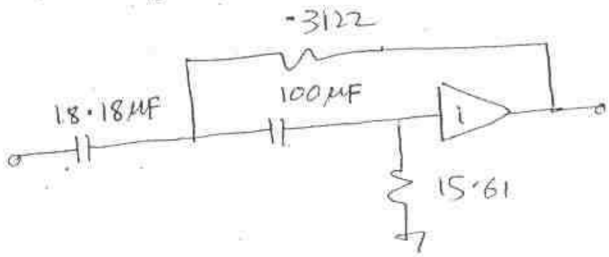


is the normalized network ie for $\omega_c = 1$ rad/sec

For $\omega_c = 10^4$ rad/sec, scale by 10^4

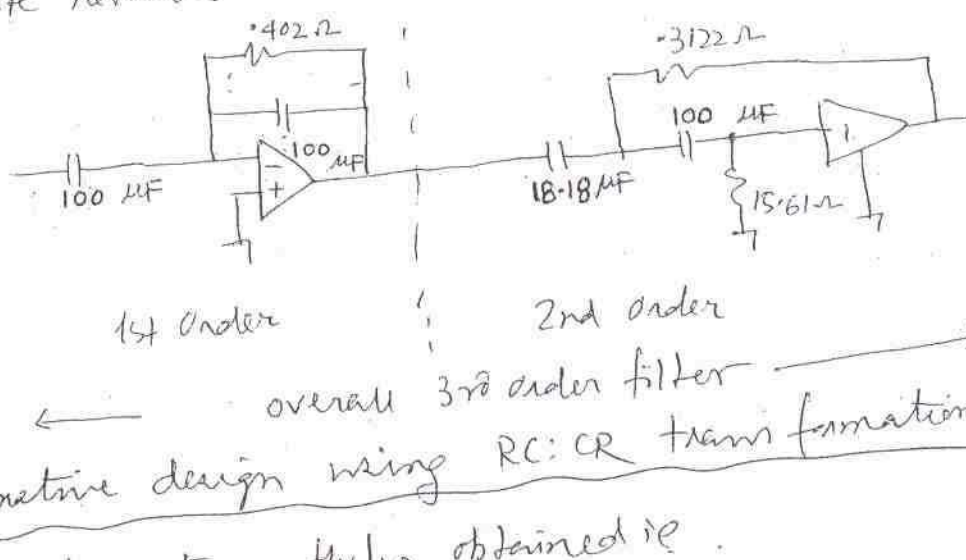
So make $C_3 = 1\text{F} \rightarrow 100 \ \mu\text{F}$

$C_1 = C_3 \times \cdot 1818 = 18 \cdot 18 \ \mu\text{F}$



is the frequency denormalized network

Complete network



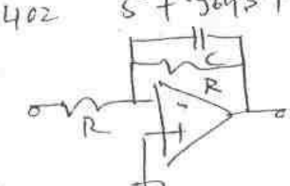
1st Order | 2nd order

← overall 3rd order filter →

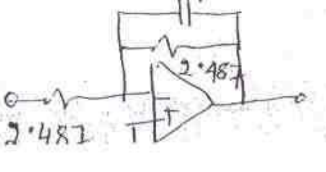
5.21 Alternative design using RC: CR transformation

First realize the $H_N|_{LP}$ obtained ie.

$$H_N(s) = \frac{\cdot 402}{s + \cdot 402} \cdot \frac{\cdot 886}{s^2 + \cdot 369 s + \cdot 886} = H_1 \, H_2$$

$H_1 = \dfrac{\cdot 402}{s + \cdot 402} \rightarrow$ 

with $C = 1\text{ F}$, $R = \frac{1}{\cdot 402} = 2 \cdot 487$

So 

is the normalized H_1 network

5.21

Alt design

$$H_2 = \frac{.886}{s^2 + .369s + .886} \qquad \omega_p = \sqrt{.886} = .9413$$

$$\frac{\omega_p}{Q_p} = .369; \quad \frac{1}{Q_p} = .392, \quad Q_p = 2.551$$

Using $K=1$ and $m = \frac{C_4}{C_2}$; $n = \frac{R_3}{R_1}$, we get as before $n = 5.5014$ or 0.1818.

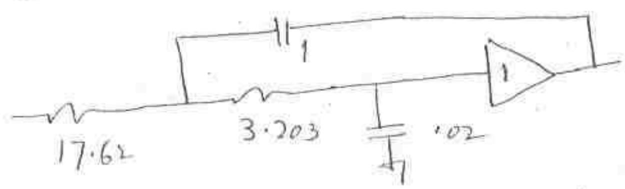
$m < \frac{1}{4Q_p^2} \to .038$. Let $m = .02$, Then

Let $n = 0.1818$, let $C_2 = 1\,F$, R_1 is then $= 17.62\,\Omega$

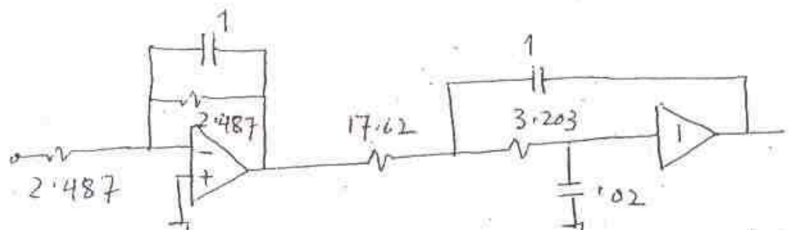
Then $\omega_p = .9413 = \frac{1}{\sqrt{mn}\,R_1C_2}$

Then $R_3 = nR_1 = 3.203\,\Omega$; $C_4 = mC_2 = .02\,F$

So the normalized H_2 is:



Overall normalized 3rd order network is:

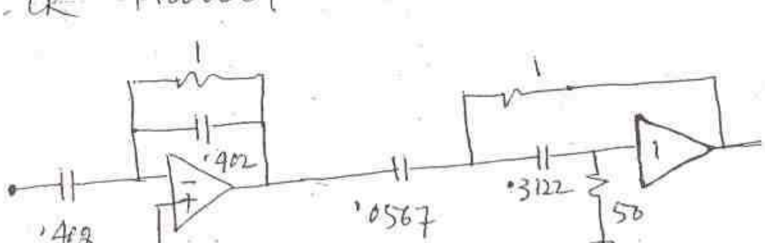


LPF for $\omega_c \to 1$ rad/sec.

Denormalize for $\omega_c = 10^4$ rad/s by making $C \to C/10^4$ ($R \to R/10^4$ will be also O.K.)

First convert the LP to HP using RC : CR transformation i.e. $R_{HP} \to \frac{1}{C_{LP}}$; $C_{HP} \to \frac{1}{R_{LP}}$.

So



This is normalized HP with $\omega_c = 1$ rad/sec.

denormalization will give:

5.22

$\Omega_{p1} = 1000$, $\Omega_{p2} = 4000$, $B = 3000$, $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 2000$

$\Omega_s = 12 \times 10^3$ rad from center of band.

Using $s = \frac{\Omega_0}{B}\left(\frac{S}{\Omega_0} + \frac{\Omega_0}{S}\right) \rightarrow \omega = \frac{\Omega_0}{B}\left(\frac{\Omega}{\Omega_0} - \frac{\Omega_0}{\Omega}\right)$

$s \rightarrow$ nomalized frequency '$j\omega$'

$S \rightarrow$ BP domain frequency $j\Omega$

Let $\Omega = 12 \times 10^3$, $\omega = \frac{2000}{3000}\left(\frac{12000}{2000} - \frac{2000}{12000}\right) = 3.89$

Thus $\omega_a/\omega_c = \omega_s = 3.89$, using this to find the order of the nominal low-pass CHEB filter with $A_p = 0.5\,dB$, we get $n_{CHEB} \geq 2.55$. Take $n_{CHEB} = 3$

For $A_p = 0.5\,dB$, $\epsilon = .3493$

(Alt. If we associate $B \rightarrow \omega_c = 3000$, $\omega_a \rightarrow 12000$, $\omega_s = 4$,
$n_{CHEB} \geq 2.52$. Take $n_{CHEB} = 3$)

So the nominal normalized LPF has

$$H_N = \frac{1}{2^{3-1} \cdot \epsilon} \cdot \frac{1}{D(s)} = \frac{1}{4 \times .3493} \cdot \frac{1}{(s + .626)(s^2 + .626s + 1.142)}$$

$$= \frac{.626}{s + .626} \cdot \frac{1.142}{s^2 + .626s + 1.142}$$

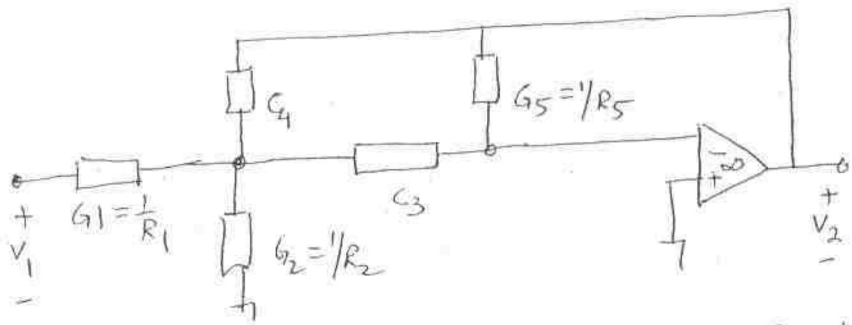
On applying the $s \leftrightarrow \frac{\Omega_0}{B}\left(\frac{S}{\Omega_0} + \frac{\Omega_0}{S}\right)$ transformation and following up the derivations to split a 4^{th} order function into cascade of two second order functions, we get the designable BP transfer function as (frequency denormalized):

$$H(s) = \frac{1878\, s}{s^2 + 1878 s + 4 \times 10^6} \cdot \frac{3.2059 \times 10^3\, s}{s^2 + 361.5718 s + 9.5375 \times 10^5} \cdot$$

$$\frac{3.2059 \times 10^3\, s}{s^2 + 1.5164 \times 10^3 s + 1.6776 \times 10^7}$$

which represents a cascade of 3 second order BPF.

5.22 Each BPF can be designed using the IG-SAB architecture (see Table 4.3)



With the design equations: $C_3 = C_4 = 1$
$G_1 = H$, $G_2 = 2\omega_p Q_p - H$
$G_5 = \frac{1}{2}\frac{\omega_p}{Q_p}$.

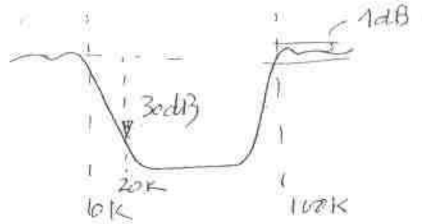
For $\dfrac{1878\, s}{s^2 + 1878\, s + 4\times10^6}$; $H = 1878$
$\omega_p = 2000$
$\frac{\omega_p}{Q_p} = 1878$, $Q_p = 1.065$.

These give $G_1 = 1878$, $G_2 = 2382$, $G_5 = 939$, $C_3 = C_4 = 1$
If $C_3 = C_4 = 0.1\,\mu F$ is made, scale the resistances by 10^7.
Then $C_3 = C_4 = 0.1\,\mu F$; $R_1 = \frac{10^7}{G_1} = 5.325K$, $R_2 = \frac{10^7}{G_2} = 4.198K$,
$R_5 = \frac{10^7}{G_5} = 10.65K$.

Proceeding similarly, we get the following results:

$H(s)$	$C_1 = C_2 = 0.1\mu F$	R_1	R_2	R_5
$\dfrac{1878\,s}{s^2+1878\,s+4\times10^6}$	$0.1\mu F$ each	5.325K	4.198K	10.65K
$\dfrac{3.2059\times10^3\, s}{s^2+361.5718\,s+9.5375\times10^5}$	$0.1\mu F$ each	3.12K	4.83K	55.31K
$\dfrac{3.2059\times10^3\, s}{s^2+1.5164\times10^3 s+1.6776\times10^7}$	$0.1\mu F$ each	3.12K	528.54	13.19K

5.23



$\Omega_0 = \sqrt{10K \cdot 100K} = 31.62\,K \times 2\pi$

$B = 100K - 10K = 90K \times 2\pi$

$s \to \dfrac{B s}{s^2 + \Omega_0^2}$ is the LP → BS transformation.

$$\omega_n = \frac{B}{\Omega_0} \cdot \frac{1}{\frac{\Omega_0}{\omega} - \frac{\omega}{\Omega_0}}$$

For $\omega = 20K \times 2\pi$
$\omega_n \cong 3$.

For $A_p = 1$ dB, $\epsilon = .5088$, $n_{CHEB} = 3$.

$$H_N(s) = \frac{.491}{(s+.494)(s^2 + .49s + .994)}$$

$$= \frac{.494}{s+.494} \cdot \frac{.994}{s^2 + .49s + .994}$$

$$\to \frac{A}{s+A} \cdot \frac{b}{s^2 + as + b}$$

$s \to \dfrac{B s}{s^2 + \Omega_0^2}$ transformation gives

$$\frac{A}{s+A} \to \frac{s^2 + \Omega_0^2}{s^2 + \frac{A}{B} s + \Omega_0^2}$$

$$\frac{b}{s^2+as+b} \to \frac{(s^2+\Omega_0^2)^2}{s^4 + \frac{aB}{b} s^3 + \frac{B^2 + 2b\Omega_0^2}{b} s^2 + \frac{aB\,\Omega_0^2}{b} s + \Omega_0^4}$$

With $A = .494$, $a = .49$, $b = .994$ we shall have $\left(\begin{array}{l} B = 2\pi \times 90 \times 10^3 \\ \Omega_0 = 2\pi \times 31.62 \times 10^3 \end{array} \right)$

$$\frac{.494}{s+.494} \to \frac{s^2 + (2\pi \times 31.62 \times 10^3)^2}{s^2 + 1.1447 \times 10^6 s + (2\pi \times 31.62 \times 10^3)^2} \Rightarrow T_1$$

$$\frac{.994}{s^2 + .49s + .994} \to \underbrace{\frac{s^2 + (2\pi \times 31.62 \times 10^3)^2}{s^2 + 2.561 \times 10^4 s + 3.9931 \times 10^9}}_{T_2} \cdot \underbrace{\frac{s^2 + (2\pi \times 31.62 \times 10^3)^2}{s^2 + 2.5315 \times 10^5 s + 3.9017 \times 10^{11}}}_{T_3}$$

So we have a cascade of T_1, T_2, T_3, three second order band reject filters.

5.23

contd.

For T_1, consider Fleischer - Tow Universal biquad (section 4.6.4)

$$\frac{V_o}{V_1} = -\frac{b_2 s^2 + b_1 s + b_o}{s^2 + a_1 s + a_o}$$

For notch filter as in T_1
$b_1 = 0$, $b_o/b_2 = a_o$.

Then $\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7} = 0$; $b_2 = 1$, $\frac{b_o}{b_2} = a_o$, gives $\frac{R_8}{R_6} = 1$ ... (A)

$\frac{b_o}{b_2} = \frac{R_6}{R_3 R_5 R_7} \cdot \frac{1}{C_1 C_2}$; $a_o = \frac{R_8}{R_2 R_3 R_7 C_1 C_2}$; $\frac{b_o}{b_2} = a_o$ gives

$\frac{R_8}{R_2} = \frac{R_6}{R_5}$; $R_2 = R_5$ ... (B)

$b_1 \to 0$ $\frac{R_8}{R_6} - \frac{R_1}{R_4} \cdot \frac{R_8}{R_7} = 0$; $\frac{R_1}{R_4} \cdot \frac{R_8}{R_7} = 1$ .. $\frac{R_8}{R_6} = 1$ by (A)

$R_8 = R_7$, $R_1 = R_4$ ... (C)

$a_o = \omega_p^2 = \frac{R_8}{R_7 \cdot R_2 R_3 C_1 C_2} \rightarrow \frac{1}{R_2 R_3 C_1 C_2} = \frac{1}{(RC)^2}$

if $R_2 = R_3 = R$; $C_1 = C_2 = C$ ... (D)

$a_1 = \frac{1}{R_1 C} = \frac{\omega_p}{Q_p}$ ... (E)

Note: In T_1
$a_1 = 1.1447 \times 10^6$
$a_o = (2\pi \times 31.62 \times 10^3)^2$

Let $C = 1F$; Then $R = R_2 = R_3 = \frac{1}{C\omega_p} = \frac{1}{2\pi \times 31.62 \times 10^3}$

$R = 5.033 \times 10^{-6} = R_2 = R_3$ .... (1) $C_1 = C_2 = 1F$ .. (2)

$R_1 = \frac{Q_p}{\omega_p C} = \frac{1}{C a_1} = .8736 \times 10^{-6}$ ... (3)

Let us make $C = 0.1 \mu F$, a practical value
Then $R = R_2 = R_3 = 10^7 \times 5.033 \times 10^{-6} = 50.33 \Omega$
$R_1 = R_4 = 8.736 \Omega$
$R_5 = R_2 = 50.33 \Omega$
$R_6 = R_8 = R_7 = 10 \Omega$ (free choice)

one possible design.

5.23 Contd.

For a notch such as T_2, T_3

$b_2 = 1$; $b_1 = 0$; $b_0 = \omega_z^2$, $a_1 = \frac{\omega_p}{Q_p}$; $a_0 = \omega_p^2$

$\frac{R_8}{R_6} = 1$; $R_8 = R_6$ ... (A)

$\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7} = 0$; $\frac{R_1}{R_4} \cdot \frac{R_8}{R_7} = 1$ ... (B)

$\frac{R_8}{R_3 R_5 R_7} \cdot \frac{1}{C_1 C_2} = \omega_z^2$, let $R_8 = R_7$, $R_3 = R_5$, $C_1 = C_2 = C$, then

$\frac{1}{R'} \frac{1}{C} = \omega_z$ where $R' = R_3 = R_5$ with $R_8 = R_7$ ... (C)

$\frac{1}{R_1 C_1} = a_1$ or $R_1 = \frac{1}{a_1 C}$ .... (D)

$\frac{R_8}{R_2 R_3 R_7} \cdot \frac{1}{C_1 C_2} \rightarrow \frac{1}{R_2 R'} \frac{1}{C^2} = \omega_p^2 = a_0$ ... (E)

Let $C = 1F$; From (C), $R' = \frac{1}{\omega_z} = \frac{1}{2\pi \times 31.62 \times 10^3} = 5.033 \times 10^{-6}$ ... (1)

From (E) $\frac{1}{R_2 R' C^2} = \omega_p^2$; $\frac{1}{R_2} = \omega_p^2 \cdot R'$ if $C = 1$ assumed

So $R_2 = \frac{1}{R' \omega_p^2} = \frac{1}{(5.033 \times 10^{-6}) \cdot 3.9931 \times 10^9} = 4.976 \times 10^{-5}$ ... (2)

From (D) $\rightarrow R_1 = \frac{1}{a_1 C} = \frac{1}{a_1} = \frac{1}{2.561 \times 10^4} = 3.905 \times 10^{-5}$ ... (3)

So $R_1 = 3.905 \times 10^{-5}$, $R_2 = 4.976 \times 10^{-5}$, $R_3 = R_5 = R' = 5.033 \times 10^{-6}$

Let $R_4 = R_1 = 3.905 \times 10^{-5}$; $R_6 = R_8 = R_7 = 10\,\Omega$ (say)

Scale $C = 0.1\,\mu F$. Then

$R_4 = R_1 = 390.5\,\Omega$, $R_2 = 497.6\,\Omega$, $R_3 = R_5 = 50.33\,\Omega$

$C_1 = C_2 = C = 0.1\,\mu F$

$R_6 = R_7 = R_8 = 10\,\Omega$

} Possible design for T_2

5.23 contd.

For T_3 we can follow same approach. But now

$$a_1 = \frac{\omega_r}{Q_p} = 2.5315 \times 10^5 \text{ and } a_0 = \omega_p^2 = 3.9017 \times 10^{11}$$

Hence
$$R_1 = \frac{1}{2.5315 \times 10^5} = 3.95 \times 10^{-6}$$
$$R_2 = \frac{1}{(5.033 \times 10^{-6}) \cdot 3.9017 \times 10^{11}} = 5.092 \times 10^{-7}$$
For $C = 1F$

Using $C = 0.1 \mu F$, we get

$R_1 = R_4 = 39.5 \Omega$; $R_2 = 5.092 \Omega$; $R_3 = R_5 = 50.33 \Omega$
$R_6 = R_7 = R_8 = 10 \Omega$ $\quad C_1 = C_2 = C = 0.1 \mu F$

possible design for T_3

x

5.24 Use Tow-Thomas structure (see Table 4.4).

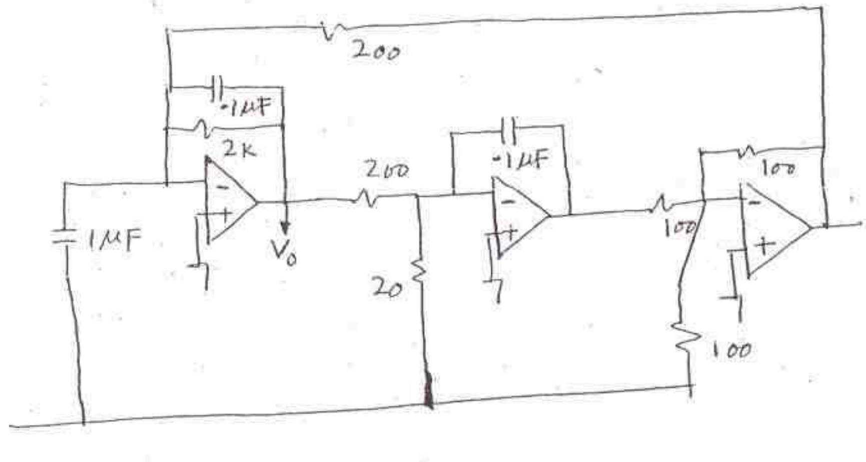
Let $C = 0.1 \mu F$, flat gain = 10 (say).

Then $R = \frac{1}{\omega_p C} = \frac{10^7}{50 \times 10^3} = 200 \Omega$

$R_1 \to$ open, $R_2 = R/10 = 20$; $R_3 = Q r/10 = \frac{10 \cdot r}{10} = 100 \Omega$
if $r = 100 \Omega$

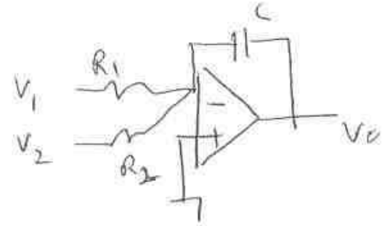
$C_1 = C(10) = 1 \mu F$

So the network is:



OA-s are assumed ideal.

5.25 Take clue from



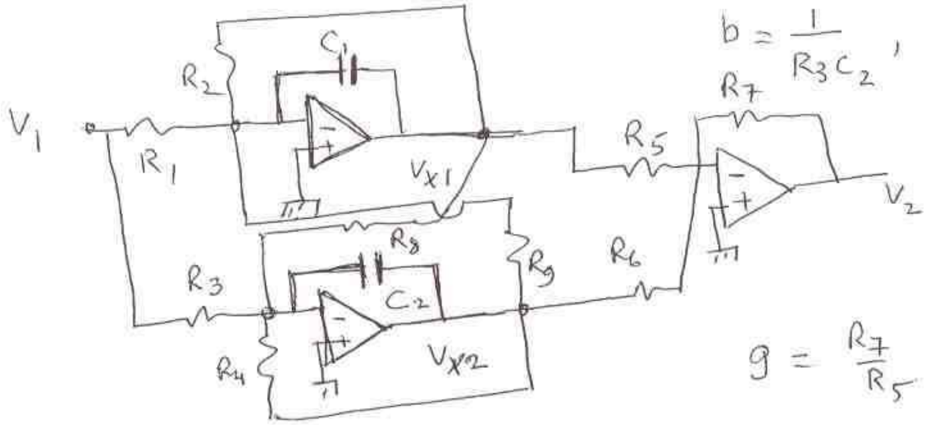
$V_0 = -\frac{1}{R_1C}\frac{1}{s}\cdot V_1 - \frac{1}{R_2C}\frac{1}{s}V_2$

$\frac{1}{R_1C}$, $\frac{1}{R_2C}$ etc. will emulate the branch gains $a, b, \ldots$ and so on.

Each summing junction has two inputs.
So each summing integ. will have two inputs.

$a = \frac{1}{R_1C_1}$; $e = \frac{1}{R_2C_1}$

$b = \frac{1}{R_3C_2}$, $f = \frac{1}{R_4C_2}$



$g = \frac{R_7}{R_5}$; $h = \frac{R_7}{R_6}$

$d = \frac{1}{R_8C_2}$; $c = \frac{1}{R_9C_1}$

$V_{x1} = -\frac{a}{s}V_1 - \frac{e}{s}V_{x1} - \frac{c}{s}V_{x2}$ or $V_{x1}\left(1+\frac{e}{s}\right) = -\frac{a}{s}V_1 - \frac{c}{s}V_{x2}$

$V_{x2} = -\frac{b}{s}V_1 - \frac{f}{s}V_{x2} - \frac{d}{s}V_{x1}$ or $V_{x2}\left(1+\frac{f}{s}\right) = -\frac{b}{s}V_1 - \frac{d}{s}V_{x1}$

$V_2 = -gV_{x1} - hV_{x2}$

Arrange in a matrix form

$$\begin{bmatrix} 1+\frac{e}{s} & \frac{c}{s} \\ \frac{d}{s} & 1+\frac{f}{s} \end{bmatrix}\begin{bmatrix} V_{x1} \\ V_{x2} \end{bmatrix} = \begin{bmatrix} -\frac{a}{s} \\ -\frac{b}{s} \end{bmatrix}[V_1]$$

$$V_{x1} = \frac{V_1}{\Delta}\begin{vmatrix} -\frac{a}{s} & \frac{c}{s} \\ -\frac{b}{s} & 1+\frac{f}{s} \end{vmatrix} \; ; \; V_{x2} = \frac{V_1}{\Delta}\begin{vmatrix} 1+\frac{e}{s} & -\frac{a}{s} \\ \frac{d}{s} & -\frac{b}{s} \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1+\frac{e}{s} & \frac{c}{s} \\ \frac{d}{s} & 1+\frac{f}{s} \end{vmatrix}$$

< continue >

5.26

$H_N(s)$, CHEB order 2 & 2dB ripple is $\frac{1}{2^1 \epsilon} \cdot \frac{1}{s^2+0.804s+0.637}$

where $\epsilon = 10^{\cdot 1 A_p} - 1 = 0.7648$

So $H(s) = H_x \frac{1}{s^2+0.804s+0.637}$ where $H_x = 2 \times .637 = 1.274$

a) By multiple feedback structure

Identify $\omega_p^2 = 0.637$; $\omega_p = 0.798$

$\omega_p / Q_p = 0.804$; $Q_p = \frac{\omega_p}{.804} = 0.993$

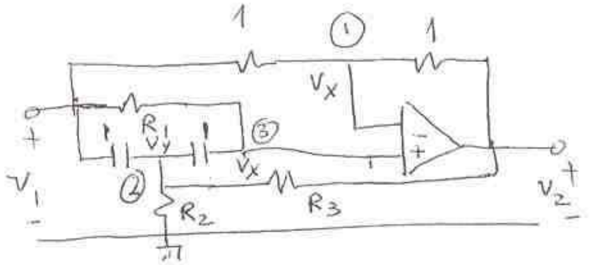
Follow the guidelines in Table 5.5, remembering

$H_o = \frac{R_2}{R_1} = 1.274$.

b) By State variable structure

Use Tow-Thomas universal biquad structure, Table 5.6

5.27



KCL @ ① $\quad V_1 - V_x = V_x - V_2$

So $V_x = \dfrac{V_1 + V_2}{2}$

KCL @ ②

$(V_y - V_1)s + (V_y - V_2)G_3 + (V_y - V_x)s + V_y G_2 = 0$

Solving for V_y ; $\quad V_y = \dfrac{1}{2} \cdot \dfrac{3V_1 s + 2V_2 G_3 + V_2 s}{2s + G_3 + G_2}$

KCL @ ③ $\quad (v_x - v_1)G_1 + (v_x - v_y)s = 0$

Subst. for v_x, v_y ; $\quad v_2 = v_1 \dfrac{s^2 + (2G_1 - G_3 - G_2)s + G_1G_3 + G_1G_2}{s^2 + (2G_1 - G_3 + G_2)s + G_1G_3 + G_1G_2}$

So $\dfrac{V_2}{V_1} = \dfrac{v_2}{v_1} = \dfrac{s^2 + (2G_1 - G_2 - G_3)s + G_1G_3 + G_1G_2}{s^2 + (2G_1 + G_2 - G_3)s + G_1G_3 + G_1G_2}$

To obtain a design $\quad \dfrac{V_2}{V_1} = \dfrac{s^2 - 5s + 10}{s^2 + 5s + 10}$

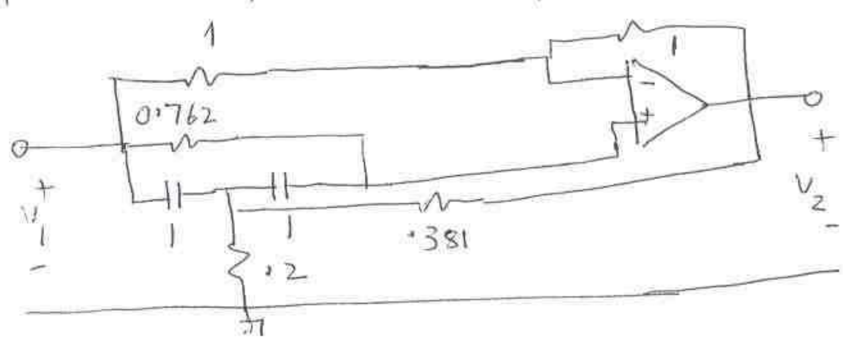
Let $\quad 2G_1 - G_2 - G_3 = -5$; $\quad 2G_1 + G_2 - G_3 = 5$
$G_1G_3 + G_1G_2 = 10$

Solving
$G_2 = 2G_1 - G_3 + 5$
$G_3 = 2G_1$

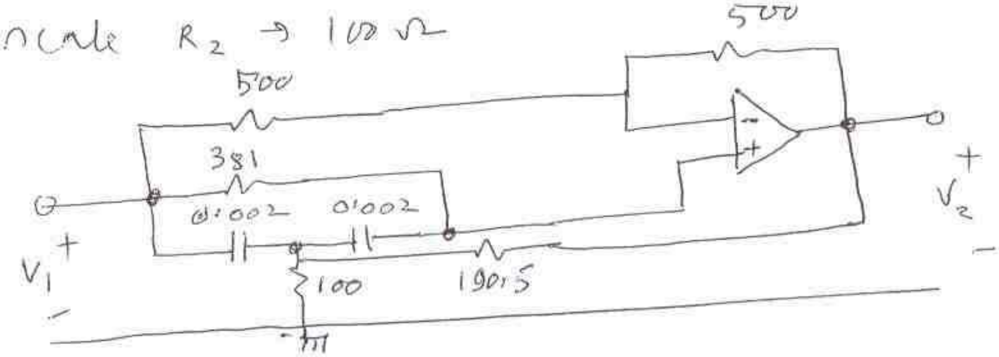
$G_1 = -\frac{5}{4} \pm \frac{1}{4}\sqrt{105} \rightarrow -\frac{5}{4} + \frac{1}{4}\sqrt{105}$ is acceptable.

So $\quad G_1 = -\frac{5}{4} + \frac{1}{4}\sqrt{105} = 1.312$; $\quad G_2 = 5$; $\quad G_3 = 2.624$

$R_1 = 0.762$; $\quad R_2 = 0.2$; $\quad R_3 = 0.381$



If we scale $R_2 \rightarrow 100\,\Omega$



Alternate design

5.28

$$V_{o1} = -V_i\frac{1}{sC_1R} - V_{o1}\frac{1}{sC_1R_1} - V_{o2}\frac{1}{sC_1R_2}$$

$$V_{o1}\left(1+\frac{1}{sC_1R_1}\right) = -\frac{V_i}{sC_1R} - \frac{V_{o2}}{sC_1R_2}$$

around OA # 1.

$$V_{o1}\, g + V_{o2}\left(-\frac{r_2}{r_1}\right)\cdot sC_2 = 0 \qquad \text{around OA \#2, \#3.}$$

$$V_{o2} = \frac{r_1}{r_2}\frac{1}{sC_2}\, g\, V_{o1}$$

Putting back: $V_{o1}\left(1+\frac{1}{sC_1R_1}\right) = -\frac{V_i}{sC_1R} - \frac{1}{sC_1R_2}\left(\frac{r_1}{r_2}\frac{1}{sC_2 r}\right)V_{o1}$

or $V_{o1}\left[1+\frac{1}{sC_1R_1} + \frac{r_1}{r_2}\cdot\frac{1}{s^2C_1C_2R_2 r}\right] = -\frac{V_i}{sC_1R}$

$$V_{o1}\left[\frac{s^2C_1C_2R_2 r r_2 R_1 + sC_2R_2 r r_2 + r_1}{s^2C_1C_2R_2 r R_1 r_2}\right] = -\frac{V_i}{sC_1R}$$

$$\frac{V_{o1}}{V_i} = -\frac{s^{\cancel{2}}\cancel{C_1}C_2R_2R_1 r r_2}{\cancel{s}\cancel{C_1}R}\cdot\frac{1}{r_1 + sC_2R_2 r r_2 + s^2C_1C_2R_1R_2 r r_2}$$

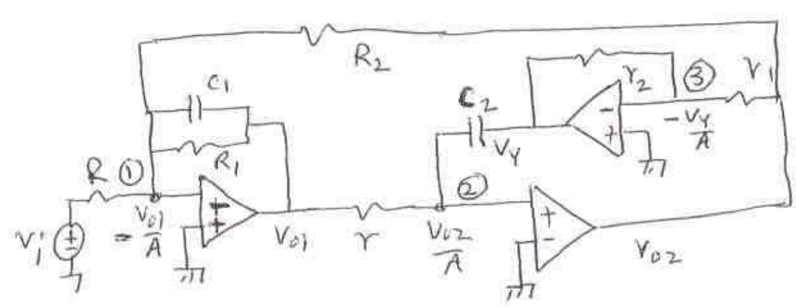
$$= -\frac{s\cancel{C_2}\cancel{R_2}\cancel{R_1}\cancel{r}\cancel{r_2}/R}{C_1\cancel{C_2}\cancel{R_1}\cancel{R_2}\cancel{r}\cancel{r_2}\left[s^2 + s\frac{\cancel{C_2R_2rr_2}}{C_1\cancel{C_2}R_1\cancel{R_2rr_2}} + \frac{r_1}{C_1C_2R_1R_2 r r_2}\right]}$$

or

$$\frac{V_{o1}}{V_i} = -\frac{s/RC_1}{s^2 + s/C_1R_1 + r_1/(C_1C_2R_1R_2 r r_2)}$$ a BPF function

— X —

5.29 For finite gain A of the OA consider



5.29 (Cont.) KCL @ ①

$$\left(v_i + \frac{v_{o1}}{A}\right)G + (G_1+sC_1)\left(v_{o1} + \frac{v_{o1}}{A}\right) + G_2\left(v_{o2} + \frac{v_{o1}}{A}\right) = 0$$

KCL @ ②

$$\left(v_{o1} - \frac{v_{o2}}{A}\right)g + \left(v_y - \frac{v_{o2}}{A}\right)sC_2 = 0$$

KCL @ ③

$$\left(v_{o2} + \frac{v_y}{A}\right)g_1 + \left(v_y + \frac{v_y}{A}\right)g_2 = 0$$

From KCL ③, $v_y = -\dfrac{g_1}{g_2\left(1+\frac{1}{A}\right) + \frac{g_1}{A}}\, v_{o2}$

Subst. in KCL ② we get

$$v_{o1} = v_{o2}\left[\frac{1}{A} + \frac{sC_2/g}{A} + \frac{sC_2\, g_1/g}{g_2\left(1+\frac{1}{A}\right) + \frac{g_1}{A}}\right]$$

Subst. in KCL ①

$$Gv_i + \frac{v_{o1}}{A}G + v_{o1}\left(1+\frac{1}{A}\right)(G_1+sC_1) + v_{o2}G_2 + v_{o1}\frac{G_2}{A} = 0$$

$$Gv_i = -\left[G_2 + \left(\frac{1}{A}\left(1+\frac{sC_2}{g}\right) + \frac{sC_2\, g_1/g}{g_2\left(1+\frac{1}{A}\right) + g_1/A}\right)\left(\frac{G+G_2}{A} + (G_1+sC_1)\left(1+\frac{1}{A}\right)\right)\right]v_{o2}$$

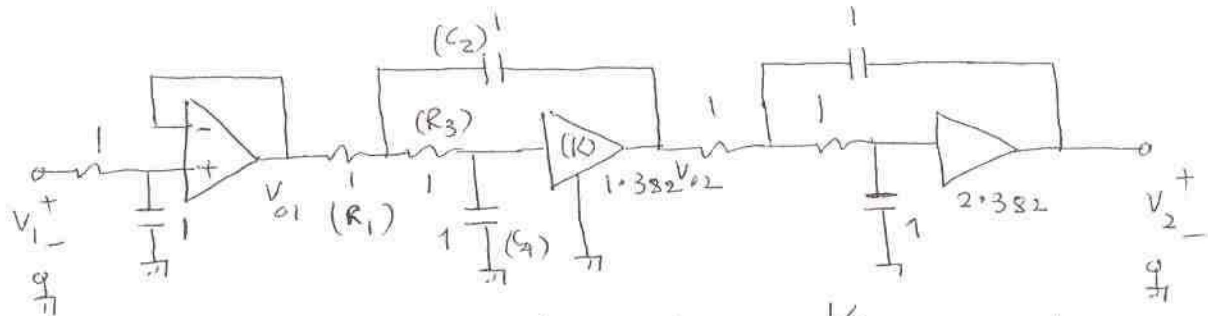
So

$$\frac{v_{o2}}{v_i} = -\frac{G}{G_2 + \left(\frac{1}{A}\,\frac{g+sC_2}{g} + \frac{sC_2\, g_1}{g\left(g_2\left(1+\frac{1}{A}\right) + \frac{g_1}{A}\right)}\right)\cdot\left(\frac{G+G_2}{A} + (G_1+sC_1)\left(1+\frac{1}{A}\right)\right)}$$

$$= -\frac{G\,g}{G_2 g + \left(\frac{g+sC_2}{A} + \frac{sC_2\, g_1}{g_2 + \frac{g_1+g_2}{A}}\right)\left(\frac{G+G_2}{A} + (G_1+sC_1)\left(1+\frac{1}{A}\right)\right)}$$

$$= -\frac{1/Rr}{\frac{1}{R_2 r} + \left(\frac{\frac{1}{R}+\frac{1}{R_2}}{A} + \left(1+\frac{1}{A}\right)\left(\frac{1}{R_1}+sC_1\right)\right)\left(\frac{sC_2/r_1}{\frac{1}{r_2} + \frac{1/r_1+1/r_2}{A}} + \frac{\frac{1}{r}+sC_2}{A}\right)}$$

5.30



The first order network has $V_{o1} = \dfrac{1/s}{1+1/s} V_1 = \dfrac{1}{s+1}$

The second network has $\dfrac{V_{o2}}{V_{o1}} = \dfrac{K(G_1G_3/C_2C_4)}{s^2 + s\left\{\frac{G_1}{C_2} + \frac{G_3}{C_2} + (1-K)\frac{G_3}{C_4}\right\} + \frac{G_1G_3}{C_2C_4}}$

$G_1 = G_3 = C_2 = C_4 = 1$

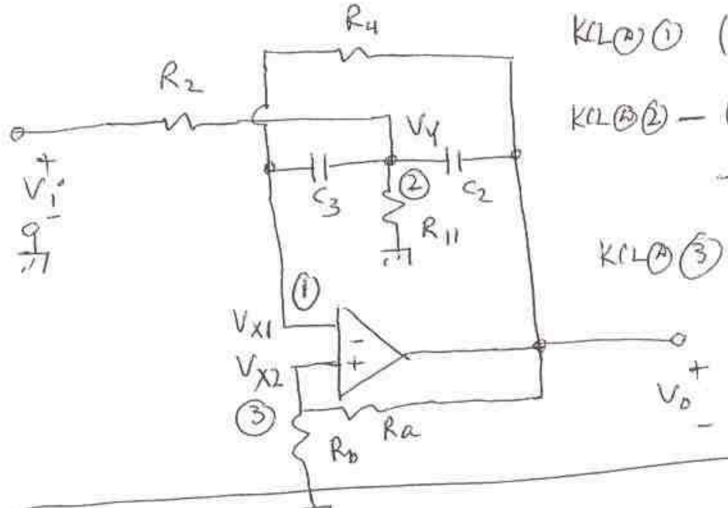
$$\frac{V_{o2}}{V_{o1}} = \frac{1.382}{s^2 + s(1+1+1(1-1.382)) + 1} = \frac{1.382}{s^2 + 1.618s + 1}$$

Similarly for $\dfrac{V_2}{V_{o2}}$

So overall TF is: $\dfrac{1}{s+1} \quad \dfrac{1.382}{s^2 + 1.618s + 1} \cdot \dfrac{2.382}{1+.618s+1}$

We need to adjust $H_1 H_2 H_3 = 1.382 \times 2.382$

5.31



KCL @ ① $(V_{X1} - V_Y)sC_3 + (V_{X1} - V_o)G_4 = 0$

KCL @ ② $-(V_i - V_Y)G_2 + (V_Y - V_{X1})sC_3 + V_Y G_{11} + (V_Y - V_o)sC_2 = 0$

KCL @ ③ $V_{X2} G_b + (V_{X2} - V_o)G_a = 0$

Using $G_i = \frac{1}{R_i}$ here

From KCL @ ① $V_{X1} = \dfrac{V_Y sC_3 + V_o G_4}{sC_3 + G_4}$

From KCL @ ③ $V_{X2} = \dfrac{V_o G_a}{G_a + G_b}$

For ideal OA, $V_{X1} = V_{X2}$ giving

$$V_Y = V_o \frac{-G_4 G_b + s G_a C_3}{(G_a + G_b) s C_3}$$

5.32 (cont.) If $A = \frac{\omega_t}{s}$, we will get

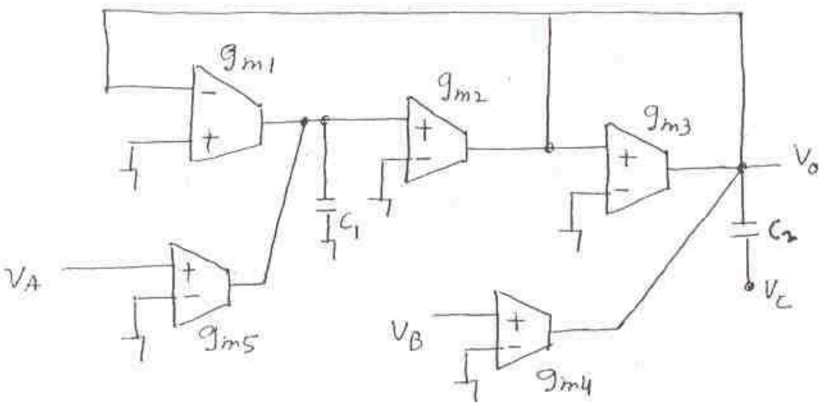
$$\frac{V_o}{V_i} = \frac{N(s)}{D(s)}$$

with $N(s) = -G_2(G_a+G_b)\,\omega_t C_3\, s$

$$D(s) = (C_2C_3G_a + C_2C_3G_b)s^3 + (C_2G_4G_b + C_2G_4G_a + G_{11}C_3G_a + G_4C_3G_b + G_4C_3G_a + G_2C_3G_b + G_2C_3G_a + C_2C_3G_b\omega_t + G_{11}C_3G_b)s^2 + (G_2G_4G_b + G_2G_4G_a + C_2G_4G_b\omega_t + C_3G_4G_b\omega_t + G_{11}G_4G_a + G_{11}G_4G_b - G_2C_3G_a\omega_t - G_{11}C_3G_a\omega_t)s + G_2G_4G_b\omega_t + G_{11}G_4G_b\omega_t$$

5.33

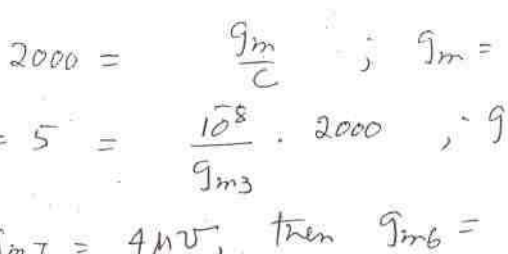
Consider the general biquad structure (i, i)



$$V_o = \frac{g_{m2} g_{m5} V_A - s g_{m4} C_1 V_B + s^2 C_1 C_2 V_C}{s^2 C_1 C_2 + s g_{m3} C_1 + g_{m1} g_{m2}}$$

a) HP, $V_A = V_B = 0$ $\qquad \omega_p = \sqrt{\frac{g_{m1} g_{m2}}{C_1 C_2}}$; $Q_p = \frac{C_2}{g_{m3}} \omega_p$

Flat gain at $\omega = \infty$ 1. To get a gain of 3 we need a voltage amplifier using OTA. Thus



$v_o' = V_o \cdot \frac{g_{m6}}{g_{m7}} = 3 V_o$ if $g_{m6} = 3 g_{m7}$.

Let $C = 0.01 \mu F = C_1 = C_2$; $g_{m1} = g_{m2} = g_m$.

Then $2000 = \frac{g_m}{C}$; $g_m = 2000 \times 10^{-8} = 20 \mu \mho$

$Q_p = 5 = \frac{10^{-8}}{g_{m3}} \cdot 2000$, $\therefore g_{m3} = \frac{2000}{5} \cdot 10^{-8} = 400 \times 10^{-8} = 4 \mu \mho$

Let $g_{m7} = 4 \mu \mho$, then $g_{m6} = 12 \mu \mho$ for a flat gain $H = 3$.

Total $g_m = g_{m1} + g_{m2} + g_{m3} + g_{m6} + g_{m7} = (20 + 20 + 4 + 4 + 12) = 60 \mu \mho$

For the BJT-based OTA, $g_m \big|_{out} = \frac{I_{DC}\big|_{OUT}}{V_T} \Rightarrow 60 \times 10^{-6}$ total

$\sum I_{DC}\big|_{OUT} = 60 \times 10^{-6} \times 25 \times 10^{-3}$ $\therefore V_T = 25 mV$ at room temp.

$= 1500 \times 10^{-6}$ mA $\qquad P = 3\underline{000} \times 10^{-6} \times 1\underline{0}$ mW

$\ldots = 3000 \times 10^{-6}$ mA $\qquad \ldots = 30 \mu W$

contd.

5.33 (b) Let $C_1 = C_2 = .01\,\mu F$; $g_{m1} = g_{m2} = g_m$

$5000 = \frac{g_m}{C}$; $g_m = 10^{-8} \times 5000 = 5.10^{-5} = 50\,\mu℧$

$g_{m1} = g_{m2} = 50\,\mu℧$; $Q_p = \frac{10^{-8}}{g_{m3}} \cdot 5000$; $g_{m3} = \frac{10^{-8} \times 5000}{10}$
$= 50 \times 10^{-8} = 5\,\mu℧$

Gain of '10' $H = \frac{g_{m4}}{g_{m3}}$, so $g_{m4} = 50\,\mu℧$

$\sum g_m = 100 + 5 + 50 = 155\,\mu℧ \leftarrow g_{m1} + g_{m2} + g_{m3} + g_{m4}$

$\sum I_{OUT}|_{DC} = 10^{-6} \times 155 \times 25$ mA ; $\sum I_{BIAS} = 2 \sum I_{OUT}|_{DC} = 50 \times 155 \times 10^{-6}$ mA

$P_{DC} = \sum I_{BIAS} \times 10V = 50 \times 155 \times 10^{-6} \times 10$ mW $= 77.5\,\mu W$

(c) APF ; $g_{m1} = g_{m2} = g_m = C \times \omega_p = 10^{-8} \times 300 = 3\,\mu℧$

$Q_p = \frac{C}{g_{m3}} \omega_p$; $g_{m3} = \frac{C\omega_p}{Q_p} = \frac{10^{-8} \times 300}{3} = 10^{-6} = 1\,\mu℧$

$H = 1$ will require $g_{m5} = g_{m1}$; $g_{m4} = g_{m3}$

So $\sum g_m = 3 + 3 + 1 + 3 + 1 = 11\,\mu℧$

$\sum I_{DC}|_{OUT} = 25 \times 10^{-3} \times 11 \times 10^{-6}$; $\sum I_{BIAS} = 2 \times \sum I_{DC}|_{OUT} = 50 \times 10^{-3} \times 11 \times 10^{-6}$

$P_{DC} = 10 \times 50 \times 10^{-3} \times 11 \times 10^{-6}$ W $= 10 \times 50 \times 11 \times 10^{-3}\,\mu W = 5.5\,\mu W$

(d) LPN : $\dfrac{s^2 C_1 C_2 + g_{m2} g_{m5}}{s^2 C_1 C_2 + s\, g_{m3} C_1 + g_{m1} g_{m2}}$ with $V_B = 0$, $V_A = V_C$

(5.33) contd.

(d) $\dfrac{g_{m1} g_{m2}}{C_1 C_2} = \omega_p^2$; let $C_1 = C_2 = C = 10^{-8}$

$g_{m1} g_{m2} = 10^{-16} \times 2000 \times 2000$ ... (1)

$Q_p = 5$; $g_{m3} = \dfrac{C\omega_p}{Q_p} = \dfrac{10^{-8} \times 2000}{5} = 400 \times 10^{-8} = 4\mu℧$

$H = 1$ means (Low-pass notch) $\rightarrow g_{m5} = g_{m1}$

$\omega_z = \sqrt{\dfrac{g_{m2} g_{m5}}{C^2}} = 3000$; $\sqrt{g_{m2} g_{m5}} = 3000\,C = \sqrt{g_{m2} g_{m1}}$

But $\sqrt{g_{m1} g_{m2}}$ is also $= \omega_p C = 2000 \cdot C$

So $C_1 = C_2$ is not a good assumption OR $g_{m5} = g_{m1}$ is not a good design.

Let $g_{m5} \neq g_{m1}$, Then $H = 1$ will be arranged later.

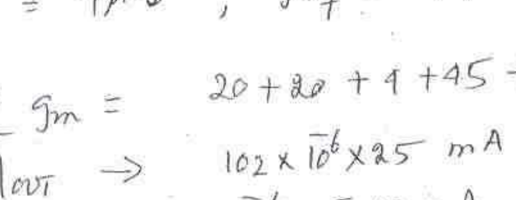
Then $\left.\begin{aligned} \sqrt{g_{m2} g_{m5}} &= 3000\,C \\ \sqrt{g_{m2} g_{m1}} &= 2000\,C \end{aligned}\right\}$ $\sqrt{\dfrac{g_{m5}}{g_{m1}}} = \dfrac{3}{2}$; $g_{m5} = \dfrac{9}{4} g_{m1}$

$g_{m5} = 2.25\, g_{m1}$

Make $g_{m1} = g_{m2} = g_m = \omega_p C = 2000 \times 10^{-8} = 20\mu℧$

So $g_{m1} = g_{m2} = 20\mu℧$; $g_{m3} = 4\mu℧$, $g_{m5} = 20 \times 2.25 = 45\mu℧$

To make $H = 1$, we need



$v_o' = \dfrac{g_{m6}}{g_{m7}} v_o = \dfrac{4}{9} v_o$

i.e. $g_{m6} = \dfrac{4}{9} g_{m7}$

or $g_{m7} = 2.25\, g_{m6}$.

Let $g_{m6} = 4\mu℧$; $g_{m7} = 9\mu℧$

Then $\sum g_m = 20 + 20 + 4 + 45 + 4 + 9 = 102\mu℧$

$\sum I_{DC|OUT} \rightarrow 102 \times 10^{-6} \times 25$ mA

$\sum I_{BIAS} = 102 \times 10^{-6} \times 25 \times 2$ mA ; $P_D = 10 \times 102 \times 10^{-6} \times 25 \times 2$ mW $= 51 \times 10^{-3}$ mW $= 51\mu W$

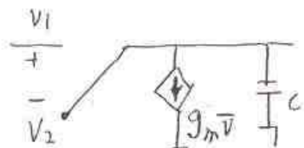
5.33

(e) left as an exercise

5.34

work with ac equivalent circuits.

(a)



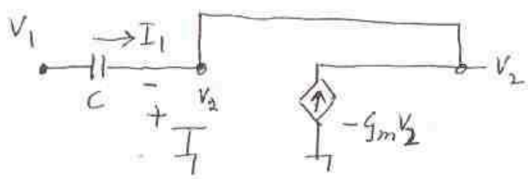
$\bar{v} = V_1 - V_2$

$V_2 = -g_m \bar{v} \cdot \frac{1}{sC} = -g_m (V_1 - V_2) \frac{1}{sC}$

$V_2 \left(1 - \frac{g_m}{sC}\right) = -g_m \frac{V_1}{sC}$ OR $\frac{sC - g_m}{sC} V_2 = -g_m \frac{V_1}{sC}$

$$\frac{V_2}{V_1} = -\frac{g_m}{sC - g_m} = \frac{g_m}{g_m - sC} = \frac{-g_m/C}{s - g_m/C}$$

(b)



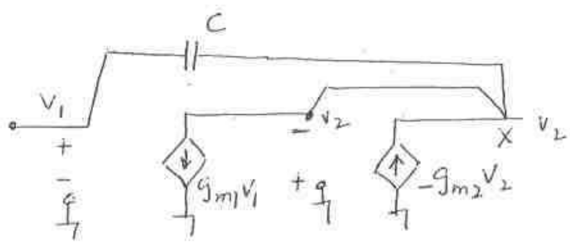
KCL gives

$-sC(V_1 - V_2) + g_m V_2 = 0$

$(g_m + sC) V_2 - sC V_1 = 0$

$$\frac{V_2}{V_1} = \frac{sC}{sC + g_m} = \frac{s}{s + g_m/C}$$

(c)



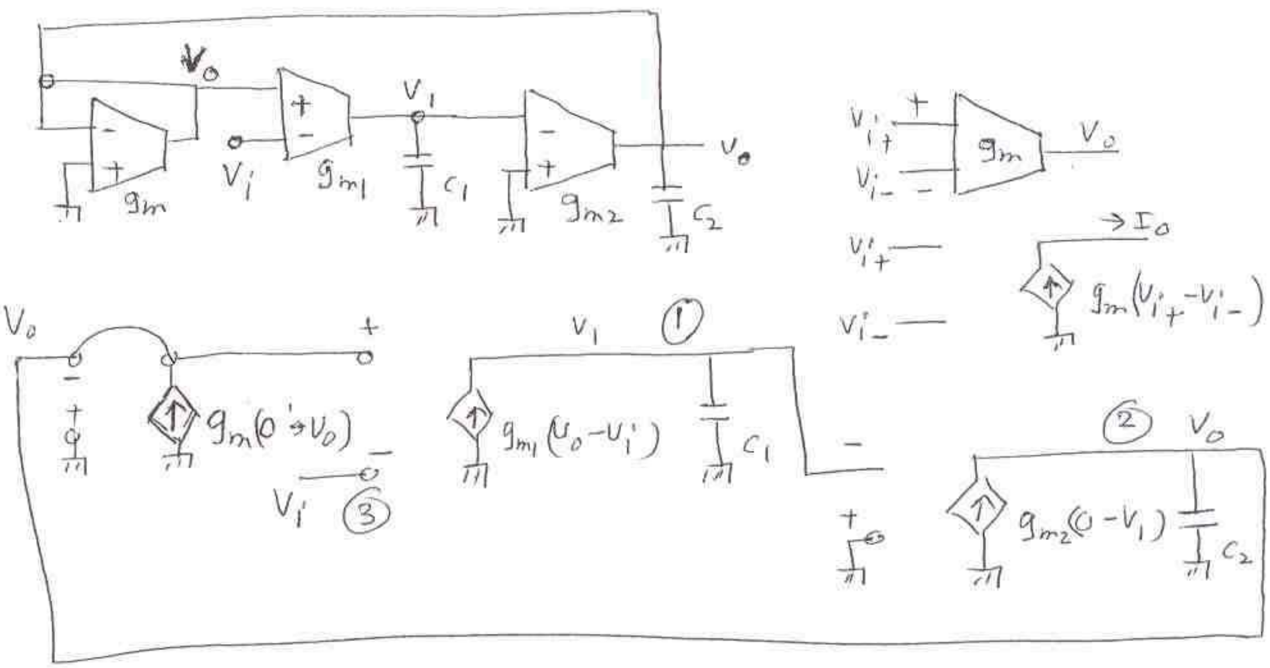
KCL at X

$-sC(V_1 - V_2) + g_{m1} V_1 + g_{m2} V_2 = 0$

$(g_{m1} - sC) V_1 + (g_{m2} + sC) V_2 = 0$

$$\frac{V_2}{V_1} = -\frac{g_{m1} - sC}{g_{m2} + sC} = -\frac{g_{m1}/C - s}{g_{m2}/C + s} = \frac{s - g_{m1}/C}{s + g_{m2}/C}$$

5.35



At ① $V_1 = \frac{1}{sC_1}\cdot g_{m1}(V_0 - V_i)$

At ② $V_0 = [g_{m2}(-V_1) + g_m(-V_0)]\frac{1}{sC_2}$

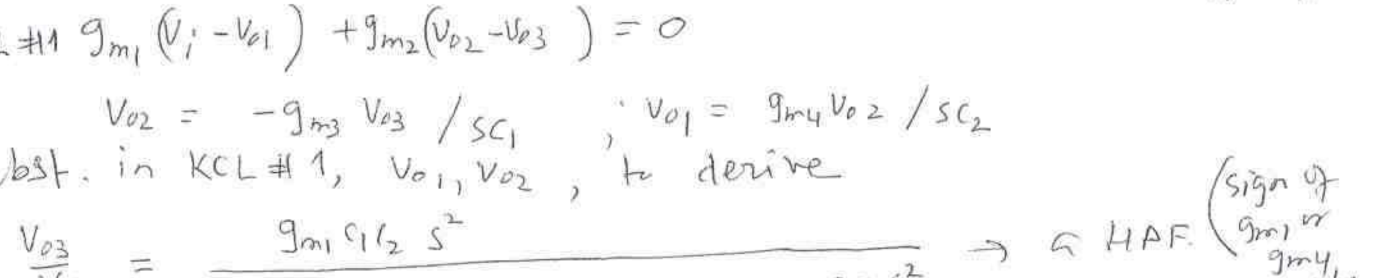
$= (-g_{m2}V_1 - g_m V_0)\frac{1}{sC_2}$

Subst. for V_1 $\quad V_0 = V_i \dfrac{g_{m1}g_{m2}}{s^2C_1C_2 + s g_m C_1 + g_{m1}g_{m2}}$

Which is a LPF

X

5.36



KCL #1 $g_{m1}(V_i - V_{o1}) + g_{m2}(V_{o2} - V_{o3}) = 0$

$V_{o2} = -g_{m3}V_{o3}/sC_1$; $V_{o1} = g_{m4}V_{o2}/sC_2$

Subst. in KCL #1, V_{o1}, V_{o2}, to derive

$\dfrac{V_{o3}}{V_i} = \dfrac{g_{m1}C_1C_2 s^2}{-g_{m1}g_{m4}g_{m3} + g_{m2}g_{m3}C_2 s + g_{m2}C_1C_2 s^2} \to$ a HAF (sign of g_{m1} or g_{m4} need be reversed!)

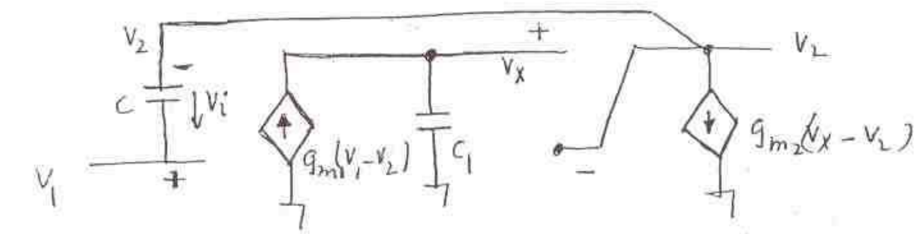
Then $\dfrac{V_{o2}}{V_i} = \dfrac{g_{m1}C_1C_2 s}{D(s)}$; $\dfrac{V_{o1}}{V_i} = \dfrac{g_{m1}C_1C_2}{D(s)}$

$D(s) = -g_{m1}g_{m4}g_{m3} + (g_{m2}g_{m3}C_2)s + (g_{m2}C_1C_2)s^2$

Note: Sign of g_{m1} or g_{m4} need be reversed !!

( ) Ideal OTA case $(C_i, C_o \to 0, \ R_o \to \infty)$

5.37



$$-sC_2(V_1 - V_2) + g_{m2}(V_x - V_2) = 0 \quad ; \quad V_x = g_{m1}(V_1 - V_2)\frac{1}{sC_1}$$

$$g_{m2} \cdot g_{m1} \cdot \frac{1}{sC_1}(V_1 - V_2) = sC_2(V_1 - V_2) + g_{m2}V_2$$

$$\left(-\frac{g_{m1}g_{m2}}{sC_1} + sC_2 - g_{m2}\right)V_2 = \left(sC_2 - \frac{g_{m1}g_{m2}}{sC_1}\right)V_1$$

5.37 contd.

non ideal OTA, low frequency

$$\Delta = sC_2 \left[\left[G_{o2} - g_{m2} + s(C_2 + C_{o2})\right]\left[G_{o1} + s(C_1 + C_{o1})\right] - g_{m1} g_{m2} \right]$$

$$+ sC_2 \left[-sC_2 \left(G_{o1} + s(C_1 + C_{o1}) \right) \right]$$

$$- g_{m1} \left[-sC_2 \cdot g_{m2} \right]$$

$$=$$

$$V_2 = \frac{1}{\Delta} \cdot \begin{vmatrix} sC_2 & I_1 & 0 \\ -sC_2 & 0 & g_{m2} \\ -g_{m1} & 0 & sC_{o1} + sC_1 + G_{o1} \end{vmatrix} ; \quad V_1 = \frac{1}{\Delta} \cdot \begin{vmatrix} I_1 & -sC_2 & 0 \\ 0 & G_{o2} + sC_{o2} + sC_2 - g_{m2} & g_{m2} \\ 0 & g_{m1} & sC_{o1} + sC_1 + G_{o1} \end{vmatrix}$$

$$\frac{V_2}{V_1} = \frac{\Delta_1}{\Delta_2}$$

$$\Delta_1 = sC_2 \cdot 0 + sC_2 \cdot I_1 \cdot (sC_{o1} + sC_1 + G_{o1}) - g_{m1} I_1 \cdot g_{m2}$$

$$= I_1 \left[-g_{m1} g_{m2} + sC_2 G_{o1} + s^2 (C_2 C_{o1} + C_2 C_1) \right]$$

$$\Delta_2 = I_1 \cdot \left[(G_{o2} + sC_{o2} + sC_2 - g_{m2})(sC_{o1} + sC_1 + G_{o1}) - g_{m1} g_{m2} \right]$$

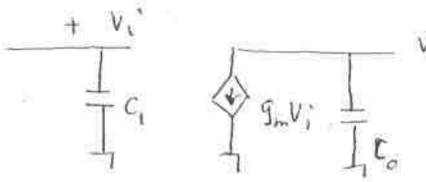
$$= I_1 \cdot \left[s^2 (C_{o1} C_{o2} + C_{o2} C_1 + C_2 C_{o1} + C_2 C_1) + s (G_{o2} C_{o1} + G_{o1} C_{o2} + C_2 G_{o1} - C_{o1} g_{m2} - C_1 g_{m2}) + G_{o1} G_{o2} - g_{m2} G_{o1} - g_{m1} g_{m2} \right]$$

Thus
$$\frac{V_2}{V_1} = \frac{s^2 (C_2 C_{o1} + C_2 C_1) + s\, C_2 G_{o1} - g_{m1} g_{m2}}{s^2 (C_{o1} C_{o2} + C_{o2} C_1 + C_2 C_{o1} + C_2 C_1) + s (G_{o2} C_{o1} + G_{o1} C_{o2} + C_2 G_{o1} - C_{o1} g_{m2} - C_1 g_{m2}) + G_{o1} G_{o2} - g_{m2} G_{o1} - g_{m1} g_{m2}}$$

which is of the form: $\dfrac{A_1 s^2 + B_1 s + C_1}{A_2 s^2 + B_2 s + C_2}$.

It is no longer a notch filter function, since $B_1 \neq 0$

5.38



$V_o = -\frac{g_m V_i}{sC_o} = -\frac{g_m}{sC_o} V_i$

Unity gain @ $\omega C_o = g_m$, $\omega = \frac{g_m}{C_o}$

$\omega = \frac{250 \times 10^{-6}}{.19 \times 10^{-12}} \rightarrow 1315.789 \times 10^{6}$

$\rightarrow 209.414$ MHZ

For $f_u \rightarrow 9$ MHz, let C_L be the load cap.

Then $\omega = 2\pi \times 9 \times 10^{6} = \frac{g_m}{C_o + C_L}$; $C_o + C_L = \frac{250 \times 10^{-6}}{2\pi \times 9 \times 10^{6}} = \frac{250}{18\pi}$ pF

(a) $C_L = \frac{250}{18\pi} - 0.19$ pF $= 4.23$ pF.

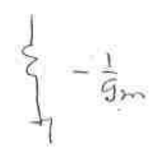
(b) For DC gain of 70 dB ? DC gain is $= -g_m R_L$ for a load R_L

So $|-g_m R_L| > 70\text{dB} \rightarrow 10^{3.5}$ $R_L > \frac{10^{3.5}}{250 \times 10^{-6}} \rightarrow 12649110.64 \rightarrow 12.65\ M\Omega$

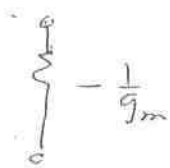
So $R_o \geq 12.65\ M\Omega$

Simulation ? → Left as an exercise.

5.39 In (a) $R_{sim} \rightarrow$



In (b) R_{sim} (floating) →



We need to arrange for a grounded inductor

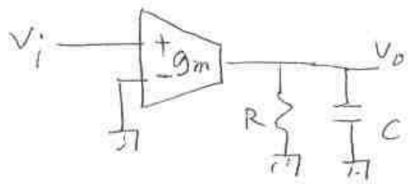
So to make $R_{tot} = 0$.

$R_p + R_{sim} = 0$;

$-R_{sim} = R_p = \frac{1}{g_m}$; $g_m = \frac{1}{R_p} = \frac{1}{250}\ \mho$

Required $g_m \rightarrow 4$ milli mho

5.40



$$V_o = g_m V_i \cdot \frac{\frac{1}{sC} R}{R + \frac{1}{sC}}$$

$$= g_m V_i \frac{R}{1+sCR} = \frac{g_m R}{CR} \cdot \frac{1}{s + 1/CR} V_i$$

So $T_1(s) = \frac{g_m}{C} \cdot \frac{1}{s + 1/CR}$; DC gain $= g_m R = 70\,dB$
$\Rightarrow 3162.28$

So $R = \frac{3162.28}{250 \times 10^{-6}} = 12649110.6\,\Omega \simeq 12.65\,M\Omega$

For unity gain frequency $|j\omega| \gg 1/CR$, so

$T_1(s) \approx \frac{g_m}{C} \frac{1}{s}$; $|T_1(s)| = 1 = \frac{g_m}{|jC\omega_u|}$; $\omega_u = \frac{g_m}{C} = 2\pi \times 10^7$

So $C = \frac{g_m}{2\pi \times 10^7} = \frac{250 \times 10^{-13}}{2\pi} = 3.978\,pF$.

$R = 12.65\,M\Omega$; $C = 3.978\,pF$.

x

5.41

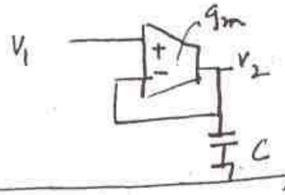
(a) MFM approximation, 3rd order: For $\omega_c = 1$ rad/sec, $A_p = 3$ dB

i.e. BUT filter

$$H_N(s) = \frac{K}{s^3 + 2s^2 + 2s + 1} = \frac{K_1}{(s+1)} \cdot \frac{K_2}{(s^2+s+1)}$$

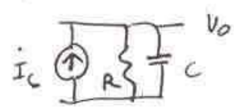
$K_1 K_2$ = dc gain = 30

The first order section can be obtained by a network similar to those shown in problem 4.27. Thus



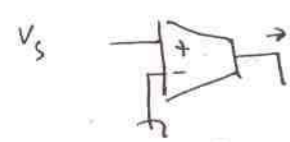
gives $\frac{V_2}{V_1} = -\frac{g_m/c}{s - g_m/c}$

To obtain a T.F. ~ $\frac{K_1}{s+1}$, consider the situation



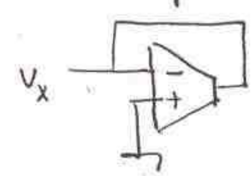
; $V_0 = I_s \cdot \frac{R \cdot 1/sc}{R + 1/sc} = I_s \cdot \frac{R}{scR+1}$

Let I_s be generated by an OTA like



$\rightarrow I_s = -g_m V_s$

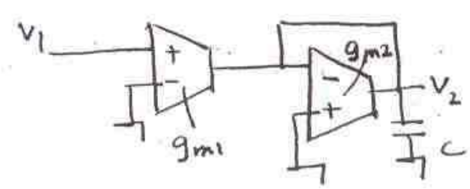
and the 'R' be generated by another OTA like:



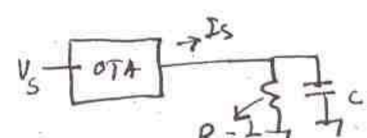
$\equiv$

$1/g_m = R$

Then



will be equivalent to



$$\frac{V_2}{V_1} = -\frac{g_{m1}}{g_{m2} + sC} \rightarrow -\frac{g_{m1}/c}{g_{m2}/c + s}$$

5.41

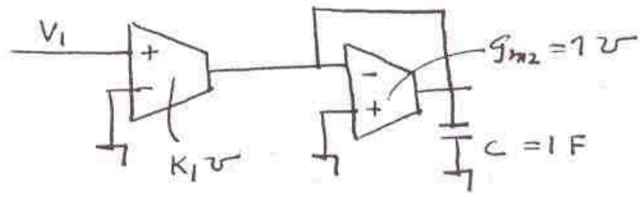
(a) contd.

Let $g_{m1}/C = K_1$ by assuming $C = 1$, $g_{m2} = 1$

$g_{m2}/C = 1$ $g_{m1} = K_1$.

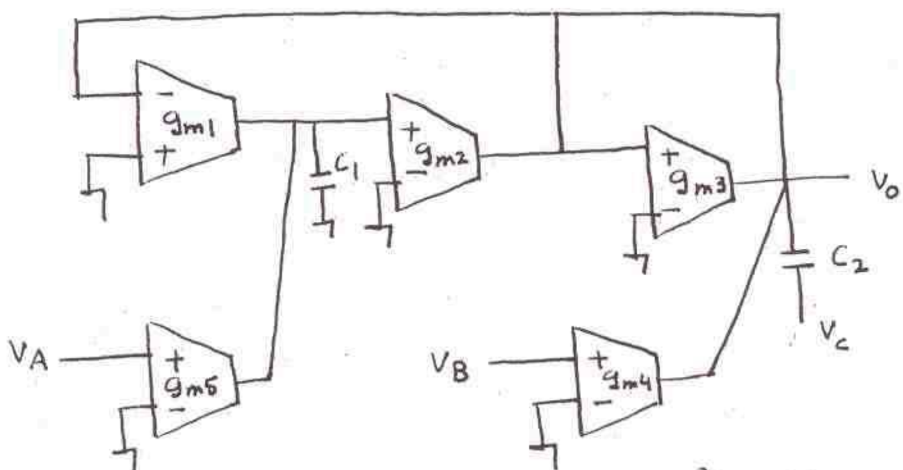
Then we get

$$\frac{V_2}{V_1} = -\frac{K_1}{s+1}$$



FIRST ORDER NETWORK

For the second order network with a TF. ~ $\dfrac{k_2}{s^2+s+1}$, we can take the universal form of LPF., ie. (see Fig. 21)



SECOND ORDER NETWORK

$$V_o = \frac{g_{m2}g_{m5}V_A - s\,g_{m4}C_1V_B + s^2C_1C_2V_c}{s^2C_1C_2 + s\,g_{m3}C_1 + g_{m1}g_{m2}}$$

With $V_B, V_C \to 0$ i.e. g_{m4} -OTA absent $\dfrac{V_o}{V_A}$ will have the form of a LPF.

$$\frac{g_{m2}g_{m5}/C_1C_2}{s^2 + s\dfrac{g_{m3}}{C_2} + \dfrac{g_{m1}g_{m2}}{C_1C_2}}$$

Let $g_{m1} = g_{m2} = 1$ ℧

$C_1 = C_2 = 1F$, and $g_{m5} = K_2$ ℧

$g_{m3} = 1$ ℧

That will produce $\dfrac{V_o}{V_A} = \dfrac{K_2}{s^2+s+1}$

Assume

$K_1 = K_2 = \sqrt{30}$

$= 5.48$

Put the FIRST ORDER & SECOND ORDER NETWOKS IN CASCADE

(b)

5.41

$n_{CHEB} = 3$, $\epsilon = .2674$

Roots of D(s) are

K	σ_K	ω_K
0	.3646	1.0719
1	0.7293	~0
2	.3646	−1.0719
3	−.3646	−1.0719
4	−.7293	~0
5	−.3646	1.0719

For realizable filter, take σ_K with negative values

$$D(s) = (s+.7293)(s+.3646+j1.0719)(s+.3646-j1.0719)$$

$$= (s+.7293)\left([s+.3646]^2 + 1.0719^2\right)$$

$$= (s+.7293)(s^2+.7292s+1.2819)$$

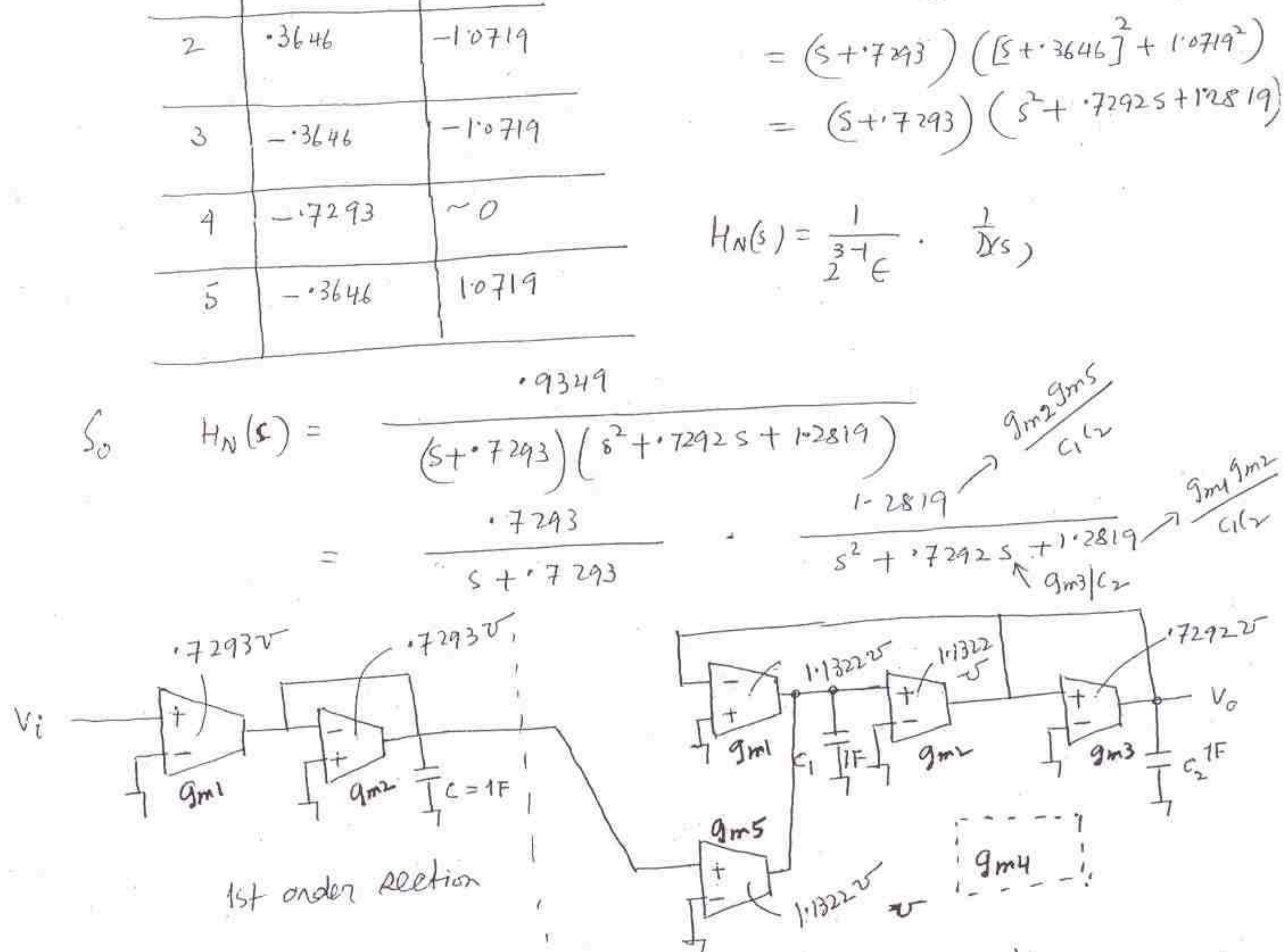
$$H_N(s) = \frac{1}{2^{3-1}\epsilon}\cdot\frac{1}{D(s)}$$

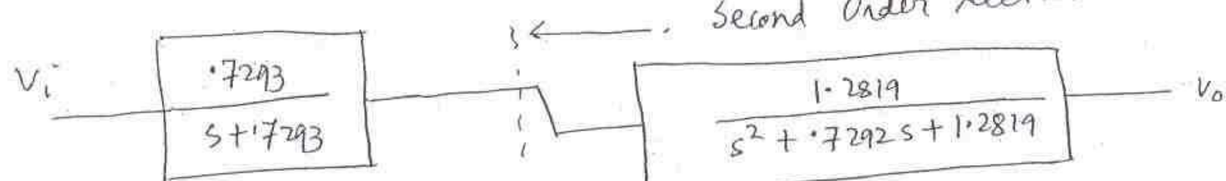
So

$$H_N(s) = \frac{.9349}{(s+.7293)(s^2+.7292s+1.2819)}$$

$$= \frac{.7293}{s+.7293}\cdot\frac{1.2819}{s^2+.7292s+1.2819}$$

$1.2819 \rightarrow \frac{g_{m2}g_{m5}}{C_1C_2}$, $1.2819 \rightarrow \frac{g_{m4}g_{m2}}{C_1C_2}$, $.7292 \leftarrow g_{m3}/C_2$





The above represents the design of the normalized 3rd order CHEB LPF.

To frequency denormalize, use the scaling of $10^7 \times 2\pi$

Make all $C = 10^{-7}F = 0.1\mu F/2\pi = .016\mu F$

5.41 (c)

For $A_p = 1$ dB, $\epsilon = .5089$, For $n = 7$, the normalized LPF transfer function is:

$$H_N(s) = \frac{1}{2^6 \epsilon} \cdot \frac{1}{(s+.2054)(s^2+.0914s+.9927)(s^2+.2562s+.6535)\cdot(s^2+.3702s+.2304)}$$

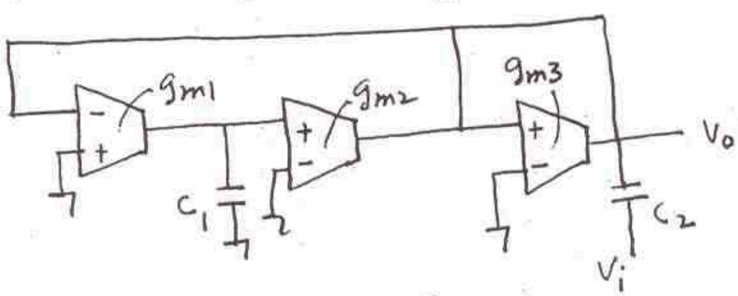
$$= \frac{.2054}{s+.2054} \cdot \frac{.9927}{s^2+.0914s+.9927} \cdot \frac{.6535}{s^2+.2562s+.6535} \cdot \frac{.2304}{s^2+.3702s+.2304}$$

The denormalized HPF transfer function will be $(s \to \omega_p/s)$, $\omega_p = 2\pi \times 15 \times 10^6$

$$H(s)\Big|_{HP} = \frac{s}{s + 458.84995\times10^6} \cdot \frac{s^2}{s^2 + 8.6776\times10^6 s + 8.94796\times10^{15}} \cdot \frac{s^2}{s^2 + 36.94917\times10^6 s + 13.59242\times10^{15}} \cdot \frac{s^2}{s^2 + 151.43458\times10^6 s + 38.55314\times10^{15}}$$

Compare the second order coefficients with the network (figure 21 of notes on Ch 4).

$$\frac{V_o}{V_i} = \frac{s^2}{s^2 + \frac{g_{m3}}{C_2}s + \frac{g_{m1}g_{m2}}{C_1C_2}}$$ with $V_A = 0$, $V_B = 0$, $V_C = V_i$



Let $C_1 = C_2 = 1$ nF each.

Then $8.6776\times10^6 \to \frac{g_{m3}}{C_2}$ gives $g_{m3} = 8.6776$ m℧

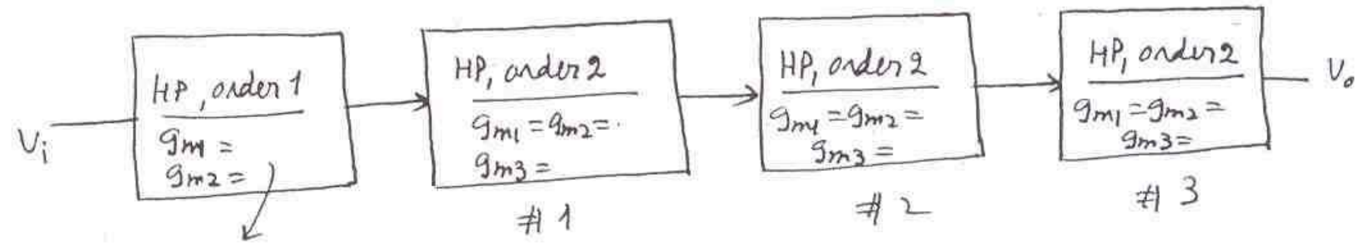
$8.94796\times10^{15} \to \frac{g_{m1}g_{m2}}{C_1C_2}$ gives $g_{m1}g_{m2} = 8.94796\times10^{-3}$

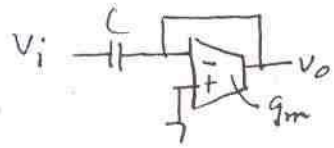
i.e. $g_{m1} = g_{m2}$ (assume) $= 94.594$ m℧

5.41

(c)

C = 1nF in each section

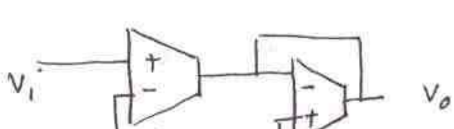




; $\frac{V_o}{V_i} = \frac{s}{s + g_m/C}$

Section	g_{m1}	g_{m2}	g_{m3}	g_{m4}	g_{m5}
first order HP	458.85 m℧	–	–	–	–
HP Second order #1	94.594 m℧	94.594 m℧	8.6776 m℧	–	–
HP Second order #2	116.59 m℧	116.59 m℧	36.9492 m℧	–	–
H.P. Second order #3	196.35 m℧	196.35 m℧	151.4346 m℧	–	–

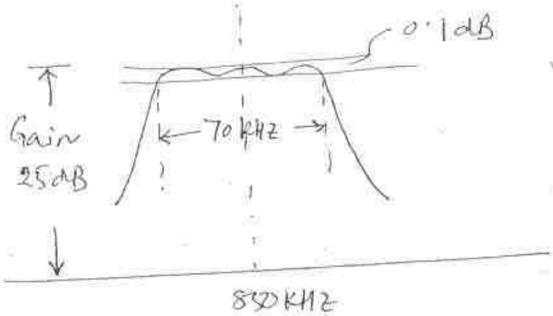
An additional gain stage can be added to make up the required flat gain. Thus a network



will provide a voltage gain (or attenuation) of $\left| g_{m1}/g_{m2} \right|$.

5.41 Fifth order BPF implies an associated normalized LPF of order 5. (BPF is an even function characteristic, it can never be 5, 7, 9 .. order !)

So we have to start-off from a 5th order LPF transfer function.



$\sqrt{\omega_{c1}\omega_{c2}} = 2\pi \times 850 \times 10^3$

$\omega_c \approx 35 \times 10^3 \times 2\pi$

$B = 70 \times 2\pi \times 10^3$

$\Omega_0 = 2\pi \times 850 \times 10^3$

$H_N(s) = \frac{1}{2^4 . \epsilon} \cdot \frac{1}{D(s)}$, where $\epsilon = 0.1526$ and roots of $D(s) = 0$ are as: only the ones with neg. real roots are shown in the table on left.

K	σ_K	ω_K
0	—	—
1	—	—
2	—	—
3	—	—
4	—	—
5	−.1665	−1.0804
6	−.4359	−.6677
7	−.5389	0
8	−.4359	.6677
9	−.1665	1.0804

$$D(s) = (s + .5389)\left[(s+.1665)^2 + 1.0804^2\right]\left[(s+.4359)^2 + .6677^2\right]$$

$$= (s + .5389)(s^2 + .333s + 1.19499)(s^2 + .8718s + .6358321)$$

Then

$$H_N(s)\Big|_{LP} = \underbrace{\frac{.5389}{s + .5389}}_{T_1} \cdot \underbrace{\frac{1.19499}{s^2 + .333s + 1.19499}}_{T_2} \cdot \underbrace{\frac{.63583}{s^2 + .8718s + .63583}}_{T_3}$$

On using the algorithm and MATLAB program pertaining to LP $\leftrightarrow$ BP transformation we shall have

$T_1 \rightarrow$ a second order TF.

$T_2 \rightarrow$ two second order TF in cascade

$T_3 \rightarrow$ two second order TF in cascade

(1.) 5.41/ Thus

(d)

$$T_1 \rightarrow \frac{2.3702\times10^5 \cdot s}{s^2+2.3702\times10^5 s+2.8523\times10^{13}} = T_{1B}$$

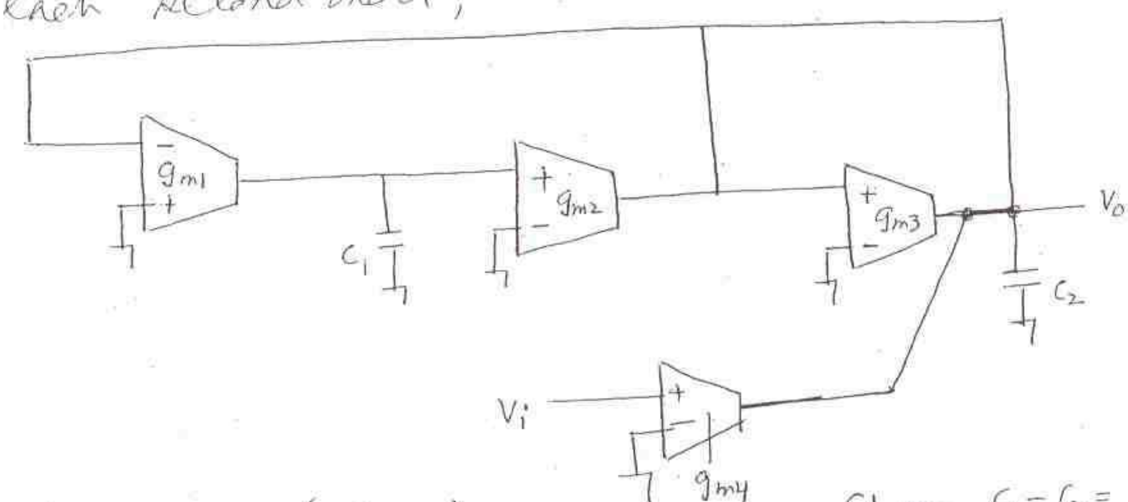
$$T_2 \rightarrow \underbrace{\frac{4.808\times10^5\, s}{s^2+6.9976\times10^4 s+2.6096\times10^{13}}}_{T_{21}} \cdot \underbrace{\frac{4.808\times10^5\, s}{s^2+7.6483\times10^4 s+3.1176\times10^{13}}}_{T_{22}}$$

$q_P = 73.0022$

$$T_3 \rightarrow \underbrace{\frac{3.5071\times10^5\, s}{s^2+1.8645\times10^5 s+2.6997\times10^{13}}}_{T_{31}} \cdot \underbrace{\frac{3.5071\times10^5\, s}{s^2+1.9699\times10^5 s+3.0136\times10^{13}}}_{T_{32}}$$

$q_P = 27.8675$

For each second order, use the network:



$$\frac{V_o}{V_i} = -\frac{(g_{m4}/C_2)\, s}{s^2+\frac{g_{m3}}{C_2}s+\frac{g_{m1}g_{m2}}{C_1C_2}}$$

Choose $C_1 = C_2 = 1\,nF$ (say)

Then $g_{m3} = 10^{-9}\times\left(\frac{\omega_p}{q_p}\right)$

Let $g_{m1} = g_{m2}$

Then $g_{m1} = g_{m2} = \omega_p\times10^{-9}$

Further $\frac{g_{m4}}{C_2}s \rightarrow H_o s$ in the numerator.

So $g_{m4} = H_o\times10^{-9}$

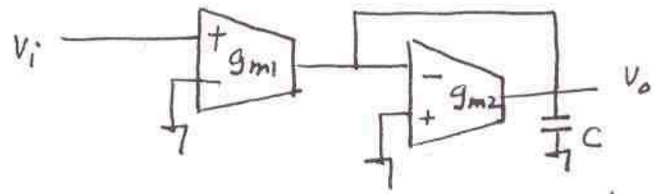
Design the FIVE BPF sections and put in cascade using a VCVS an interface between adjacent biquad sections.

5.41 (e)

Since 'k' is not specified, assume $k = 16.9$.
Then the three sections to cascade is:

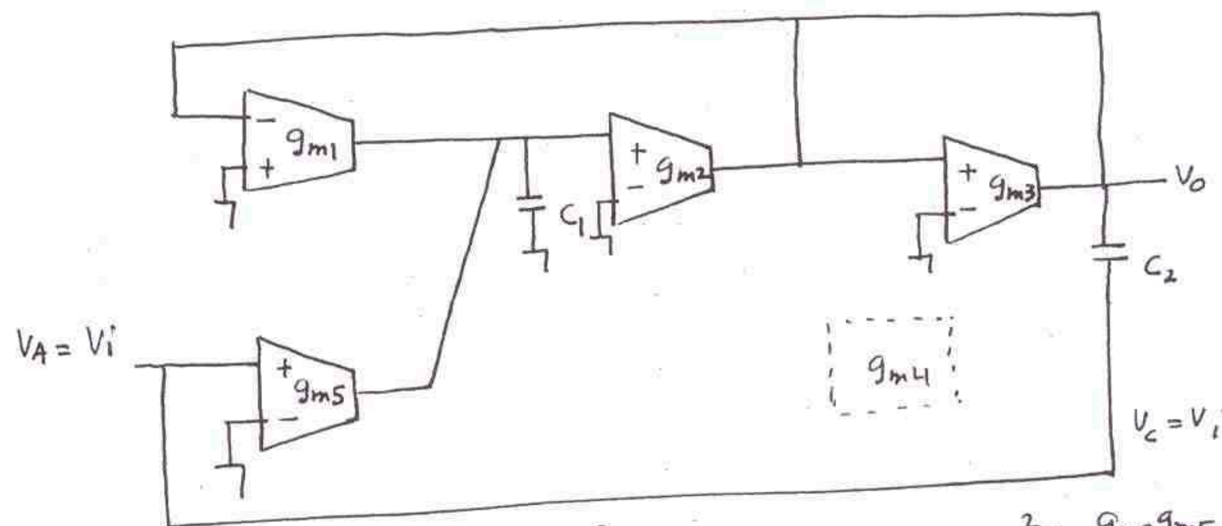
$$\frac{16.9}{s+16.9} \cdot \frac{s^2 + 29.2^2}{s^2 + 19.4s + 20.01^2} \cdot \frac{s^2 + 43.2^2}{s^2 + 4.72s + 22.52^2} \qquad s \text{ in Mrad/sec.}$$

For the first order LP section, use the network:



With $C = 1F$
$g_{m2} = 16.9\,℧ \leftarrow \frac{g_{m2}}{C}$
$g_{m1} = 16.9\,℧ \leftarrow g_{m1}/C$

For the second order notch section use:



$$\frac{V_o}{V_i} = \frac{g_{m2}g_{m5} + s^2 C_1 C_2}{s^2 C_1 C_2 + g_{m3} C_1 s + g_{m1} g_{m2}} = \frac{s^2 + g_{m2}g_{m5}/C_1C_2}{s^2 + s\, g_{m3}/C_2 + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

Let $C_1 = C_2 = C = 1F$; $g_{m1} = g_{m2} = g_m$; $g_{m5} = \frac{\omega_z^2 \cdot C^2}{g_{m2}} \rightarrow \frac{\omega_z^2}{g_{m2}}$

$g_m = \omega_p C$; $g_{m3} = \left(\frac{\omega_p}{Q_p}\right) C. \rightarrow \frac{\omega_p}{Q_p}$. ($\because C = 1$ assumed)
$\rightarrow \omega_p$

network	g_{m1}	g_{m2}	g_{m3}	g_{m4}	g_{m5}
LP	16.9 ℧	16.9 ℧	—	—	—
Notch # 1	20.1 ℧	20.1 ℧	19.4 ℧	—	42.42 ℧
Notch # 2	22.52 ℧	22.52 ℧	4.72 ℧	—	82.87 ℧

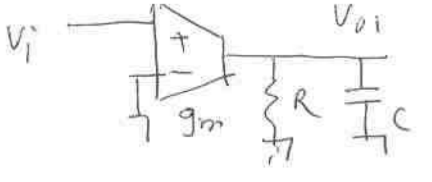
Follow methods as in previous problems

5.42

$$H(s) = H_1(s)\, H_2(s)\, H_3(s)$$

$$= \frac{K}{s+16.9} \cdot \frac{s^2 + 29.2^2}{s^2 + 19.4s + 20.01^2} \cdot \frac{s^2 + 43.2^2}{s^2 + 4.72s + 22.52^2}$$

For the first order section we can use (as in P5.40)



with $1/RC = 16.9$; $\frac{g_m}{C} = K$

$$\frac{V_{01}}{V_i} = \frac{g_m}{C} \frac{1}{s + 1/RC}$$

Choose suitable pair of R and C.

For the second order sections we can use the universal structure as in Fig 5.19 (a). Then

$$V_0 = \frac{1}{D(s)} \cdot \left[s^2 V_C + (g_{m4}/C_2)\, s V_B + \frac{g_{m2} g_{m5}}{C_1 C_2} V_A \right]$$

with $D(s) = s^2 + (g_{m3}/C_2)\, s + g_{m1} g_{m2}/C_1 C_2$

Let $V_B = 0$ and $V_A = V_C$.

Then $H_2(s) = \frac{1}{D(s)} \left[s^2 + g_{m2} g_{m5}/C_1 C_2 \right] = \frac{V_o}{V_C}$.

Thus

H_2	H_3
$\frac{g_{m2} g_{m5}}{C_1 C_2} = 29.2^2$	$\frac{g_{m2} g_{m5}}{C_1 C_2} = 43.2^2$
$\frac{g_{m3}}{C_2} = 19.4$	$\frac{g_{m3}}{C_2} = 4.72$
$\frac{g_{m1} g_{m2}}{C_1 C_2} = 20.01^2$	$\frac{g_{m1} g_{m2}}{C_1 C_2} = 22.52^2$

Choose $g_{m1}, g_{m2}, \ldots g_{m5}, C_1, C_2$ suitably.

note:
* While simulating the filter, insert a unit gain voltage buffer circuit as an interstage between two OTA-stages. This is to ensure that each OTA-block receives the input signal from a voltage source.