ELEC 441 / 6081 , MT Test #1 , Soln .

Winter 08-09

Q1: $0.707\,\Omega$ to $707\,\Omega$ involves a scaling by 1000.

So impedance scaling by 1000 has been done.

This takes $C = 1F$ to $1/1000 = 1mF$ [illegible]

But final C is $0.1\,\mu F = 10^{-7}$ F

So 'C' has been further scaled by $10^{-7}/10^{-3} = 10^{-4}$

Scaling on 'C' alone implies frequency scaling.

Frequency scaling 's' by 'a' ie $s \to s/a$

makes $s_{new} = a.s_{old}$

This scales C as $\overset{10^{-7}}{C_{new}} = \overset{10^{-3}}{C_{old}}/a \to C_{old}/\overset{a}{10^{4}}$

Thus $s_{new} = 10^4 . s_{old}$

Hence $T_1(s) \to T_1'(s)$ remains same due to impedance scaling

$T_1'(s) \to T_1''(s)\Big|_{s \to s/a}$ due to frequency scaling

$$T_1(s)\Big|_{new} = \frac{2}{s^2+\sqrt{2}s+1}\Bigg|_{s \to s/10^4} = \frac{2}{\frac{s^2}{10^8}+\sqrt{2}\frac{s}{10^4}+1}$$

$\downarrow$

$$T_2(s) = \frac{2\times10^8}{s^2+\sqrt{2}\,10^4 s+10^8}$$

Q2:
$$\Omega_{a_1} = 4300 \text{ r/s}, \quad \Omega_{a_2} = 6000 \text{ r/s}$$
$$\Omega_{p_1} = 3500 \text{ r/s}, \quad \Omega_{p_2} = 7000 \text{ r/s}$$

Band-stop filter, so $\omega_s = \dfrac{\Omega_{p_2} - \Omega_{p_1}}{\Omega_{a_2} - \Omega_{a_1}} = \dfrac{3500}{1700}$

$$\omega_s = 2.059$$

Pass-band has ripple → CHEB feature.

So we calculate $D = \dfrac{10^{.1 \times 40} - 1}{10^{.1 \times .5} - 1} = 81946.6 \approx 81947$

$$\sqrt{D} = 286.26$$

Then $\dfrac{\cosh^{-1} \sqrt{D}}{\cosh^{-1} \omega_s} = 4.702$ → order of assoc. normalized LP. filter

Take $n = 5$ for the normalized LPF.

We consullt table to get

$$H_N(s) = \frac{1}{2^{5-1} \cdot \epsilon} \cdot \frac{1}{D(s)} \qquad \text{where } \epsilon = \sqrt{10^{.1 \times .5} - 1} = 0.3493$$

$$H_N(s) = \frac{0.1789}{s^5 + 1.1725 s^4 + 1.9374 s^3 + 1.3096 s^2 + 0.7525 s + 0.1789}$$

To synthesize the frequency denormalized BSF, we need to use the frequency transformation

$$s \to \frac{B}{\Omega_0} \cdot \left[\frac{1}{\frac{s}{\Omega_0} + \frac{\Omega_0}{s}} \right]$$

where $B = \Omega_{a_2} - \Omega_{a_1} = 1700 \text{ r/s}$

$\Omega_0^2 = \Omega_{a_1} \Omega_{a_2} = 4300 \times 6000 \text{ rad}^2/\text{sec}^2$

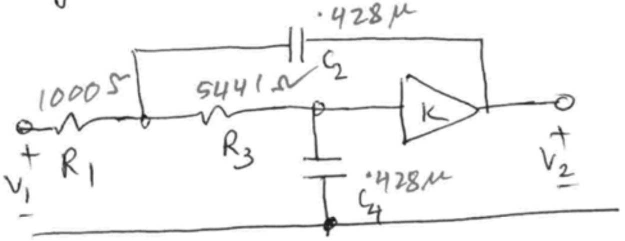
$\Omega_0 = 5079.37 \text{ rad/sec}.$

Q3:

Given $H_N(s) = T_N(s) = \dfrac{.9825}{s^2+1.098s+1.103}$

Finite Gain SAB (FGSAB – S&K) design

The given TF. is a LPF. So we consider the circuit



The network TF. is:

$$\frac{K / R_1R_3C_2C_4}{s^2+s\left(1/R_3C_4+1/R_1C_2+1/R_3C_2-K/R_3C_4\right)+1/R_1R_3C_2C_4}$$

The above TF. has a DC gain of K

We set K=5 as required in Q.4.

Then by comparing with the theoretical function

$$T_{LP}(s) = \frac{K\omega_p^2}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$

we can see $\dfrac{1}{R_1R_3C_2C_4} = \omega_p^2 = 1.103$

$$\frac{1}{R_3C_4}+\frac{1}{R_1C_2}+\frac{1}{R_3C_2}-5/C_4R_3 = \frac{\omega_p}{Q_p} = 1.098$$

Let $C_2 = C_4 = C = 1$ F

Then $\dfrac{1}{R_1R_3} = 1.103$; $\dfrac{1}{R_1} - \dfrac{3}{R_3} = 1.098$

Q3.

$R_3 - 3R_1 = 1.098 \quad R_1R_3 = \frac{1.098}{1.103} = 0.995$

$R_3 = 3R_1 + .995$

Subst. back in $\frac{1}{R_1R_3} = 1.103$

$R_1 \cdot (3R_1 + .995) = \frac{1}{1.103} = 0.9066$

$3R_1^2 + .995R_1 - .9066 = 0$

$$R_1 = \frac{-.995 \pm \sqrt{.995^2 + 4 \times .9066 \times 3}}{2 \times 3}$$

acceptable => positive value $\frac{-.995 + 3.445}{6} = 0.408$

Then $R_3 = ~~3 \times .74 + .995 = 3.215~~$ $3 \times .408 + .995 = 2.22$

First cut design values

$C_3 = C_4 = C = 1F$

$R_1 = 0.408\Omega$

$R_3 = ~~3.215~~\ 2.22\Omega$

satisfies $\frac{\omega_n}{Q_p} = 1.098$

$\omega_n^2 = 1.103$

Actual $w_p = 1000$ rad/sec.

So we need to scale frequency by $\frac{1000}{\sqrt{1.103}} = 952.165$

This will alter $C \to C/952.165 = 1.05$ mF.

Second cut design: $R_1 = .408\Omega$ $\quad C_2 = C_4 = C = 1.05$ mF

$R_3 = 2.22\Omega$

To make $R_1 \to 1000\Omega$, impedance scale by 2450.98

Then

QA^3 Then, the final design values come

$$R_1 = 1000\,\Omega \quad ; \quad R_3 = 5441\cdot 18\,\Omega$$

$$C_2 = C_4 = C = \frac{1\cdot 05\,mF}{2450\cdot 98} = 0\cdot 428\,\mu F.$$

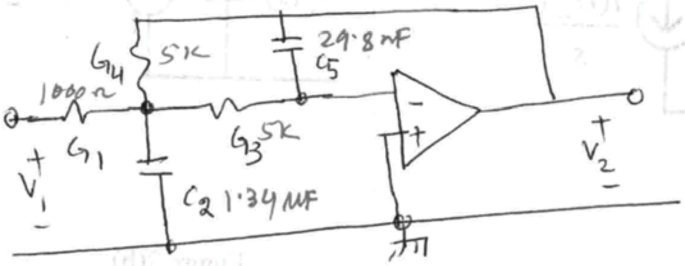
$K = 5$ is obtained using an OA as:

$$\frac{V_o}{V_i} = 1 + \frac{4K}{1K} = 5.$$

Q4^3:

IG-SAB design   IG → Infinite gain

Now consider the circuit (LPF)



$G_1 = \frac{1}{R_1}$

$G_4 = \frac{1}{R_4}$

$G_3 = 1/R_3$

The network TF. is:
$$\frac{-G_1G_3}{s^2 C_2C_5 + s\,C_5\,(G_1+G_3+G_4) + G_3G_4}$$

The associated theoretical TF is:
$$\frac{-H}{s^2 + \frac{\omega_p}{Q_p}\,s + \omega_p^2}$$

We re-write the network TF. as:
$$\frac{-\,G_1G_3\,/\,C_2C_5}{s^2 + \left(\frac{G_1}{C_2} + \frac{G_3}{C_2} + \frac{G_4}{C_2}\right) s + \frac{G_3G_4}{C_2C_5}}$$

Q3 (1G-SAB design contd.)

Comparing with theoretical TF.

$$\frac{G_1 G_3}{C_2 C_5} = H$$

$$\frac{G_1}{C_2} + \frac{G_3}{C_2} + \frac{G_4}{C_2} = \frac{\omega_p}{Q_p}$$

$$\frac{G_3 G_4}{C_2 C_5} = \omega_p^2$$

DC gain $\frac{G_1 G_3}{G_3 G_4} = \frac{G_1}{G_4} = \frac{H}{\omega_p^2} = 5$ required

So $G_1 = 5G_4$ and $H = 5\omega_p^2$

Suggested design eqns are:

$$G_1 = \frac{H}{\omega_p} \quad ; \quad C_2 = \frac{Q_p}{\omega_p^2}\left(2\omega_p^2 + H\right) \quad , \quad G_3 = \omega_p$$

$$G_4 = G_3 \quad ; \quad C_5 = \frac{\omega_p^2}{Q_p\left(2\omega_p^2 + H\right)} = \frac{1}{C_2}$$

In the given TF.

$\omega_p^2 = 1.103$; So $H = 5 \times 1.103 = 5.515$

$\omega_p = \sqrt{1.103} = 1.05$

$G_1 = \frac{5\omega_p^2}{\omega_p} = 5\omega_p = 5 \times 1.05 = 5.25\ \mho$

$G_3 = 1.05\ \mho = G_4$; $C_2 = \frac{Q_p}{\omega_p}\left(2\omega_p + \frac{H}{\omega_p}\right)$

$C_2 = \frac{1}{1.098}\left(1.05 \times 2 + \frac{5.515}{1.05}\right)$ $\because \frac{\omega_p}{Q_p} = 1.098$ in the given TF. (coeff. of s term in D(s)).

$= 6.696$ F

$C_5 = 1/C_2 = 0.149$ F

Q3. (contd) 1G-SAB design.

First cut design values are:

$G_1 = 5.25\,\mho \rightarrow 0.1905\,\Omega$

$G_3 = G_4 = 1.05\,\mho \rightarrow$ ~~[illegible]~~ $0.9524\,\Omega$

$C_2 =$ ~~[illegible]~~ $6.696F$

$C_5 =$ ~~[illegible]~~ 0.149

verified $\left\{ \begin{array}{l} \omega_p^2 = 1.103 \\ \frac{\omega_p}{Q_p} = 1.098 \end{array} \right.$

R_1 is lowest valued resistance

To make it $1000\,\Omega$, impedance scale by $\frac{1000}{.1905} = 5249.34$

So

second cut design.
$$\left\{ \begin{array}{l} R_1 = .1905\,\Omega \rightarrow 1000\,\Omega \\ R_3 = R_4 = .9524\,\Omega \rightarrow 4999.47 \sim 5000\,\Omega \\ C_2 = 6.696F / 5249.34 = 1.2756\text{ mF} \\ C_5 = .149 / 5249.34 = 2.8384 \times 10^{-5} = 28.384\,\mu F. \end{array} \right.$$

The above is for $\omega_p = 1.05$ rad/sec

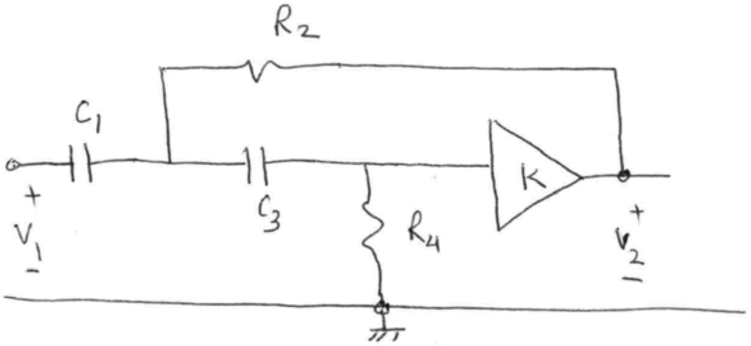
To achieve $\omega_p = 1000$ rad/sec, we frequency scale by $\frac{1000}{1.05} = 952.38$

This changes only the capacitors

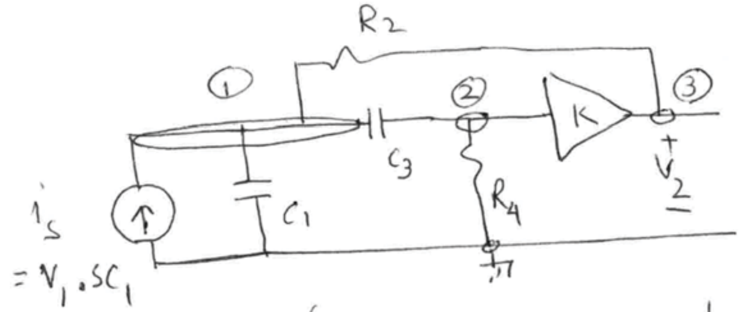
So final design

$$\left. \begin{array}{l} R_1 = 1000\,\Omega \\ R_3 = R_4 = 5000\,\Omega \\ C_2 = \frac{1.2756 \times 10^{-3}}{952.38} = 1.34\,\mu F \\ C_5 = 28.384\,\mu F / 952.38 = \cancel{[illegible]}\ 29.8\text{ nF}. \end{array} \right\}$$

Q.4:



For nodal matrix analysis, we re-configure the above circuit as:



All $G = \frac{1}{R}$

The NAM for the 3-node system is:

$$\begin{array}{c} \begin{array}{ccc} ① & ② & ③ \end{array} \\ \begin{pmatrix} sC_1 + sC_3 + G_2 & -sC_3 & -G_2 \\ -sC_3 & G_4 + sC_3 & 0 \\ -G_2 & 0 & G_2 \end{pmatrix} \end{array} \begin{pmatrix} V_{①} \\ V_{②} \\ V_{③} \end{pmatrix} = \begin{pmatrix} i_s \\ 0 \\ 0 \end{pmatrix}$$

Applying the constraint $V_{③} = K V_{②}$, we write

$$C_{②} = C_{②} + K C_{③}$$

$$\begin{pmatrix} sC_1 + sC_3 + G_2 & -sC_3 - KG_2 \\ -sC_3 & G_4 + sC_3 \\ -G_2 & KG_2 \end{pmatrix} \begin{pmatrix} V_{①} \\ V_{②} \\ V_{③} \end{pmatrix} = \begin{pmatrix} i_s \\ 0 \\ 0 \end{pmatrix}$$

Then, we discard row ③ corresponding to node ③

Q4

ie, the output node of a voltage amplifier

So:

$$\begin{pmatrix} sC_1+sC_3+G_2 & -sC_3-KG_2 \\ -sC_3 & G_4+sC_3 \end{pmatrix}\begin{pmatrix} V_{\textcircled{1}} \\ V_{(2)} \end{pmatrix} = \begin{pmatrix} i_s \\ 0 \end{pmatrix}$$

det of the matrix is

$$\left(sC_1+sC_3+G_2\right)\left(G_4+sC_3\right) - sC_3\left(sC_3+KG_2\right) = \Delta.$$

Simplifying: $\Delta = s^2 C_1C_3 + s\left(C_1G_4+C_3G_4+C_3G_2-KC_3G_2\right)+G_2G_4$

By applying Krammer's method:

$$V_{(2)} = \frac{1}{\Delta}\cdot\begin{vmatrix} sC_1+sC_3+G_2 & i_s \\ -sC_3 & 0 \end{vmatrix}$$

$$= \frac{sC_3\ i_s}{\Delta}$$

So $V_{(2)} = \frac{sC_3}{\Delta}\cdot V_1\cdot sC_1 \quad \because\ i_s = V_1 sC_1$

$$\frac{V_{(2)}}{V_1} = \frac{s^2C_1C_3}{\Delta}$$

But $V_2 = KV_{\textcircled{2}}$; So

$$\frac{V_2}{V_1} = K\frac{s^2C_1C_3}{\Delta} = \frac{s^2\ KC_1C_3}{s^2C_1C_3+s\left(C_1G_4+C_3G_4+C_3G_2-KC_3G_2\right)+G_2G_4}$$

Proved $= \dfrac{Ks^2}{s^2+s\left(\frac{1}{C_3R_4}+\frac{1}{C_1R_4}+\frac{1}{C_1R_2}-K\frac{1}{C_1R_2}\right)+1/R_2R_4C_1C_3}$