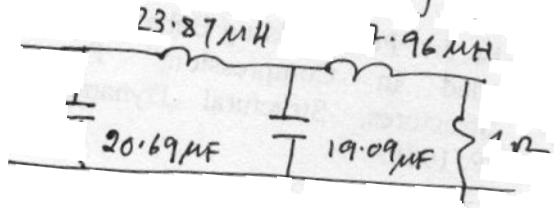
ELEC 441/6081, MT # 1 Soln

Winter 09-10 term

1/1

Q1: (a) Using frequency scaling by $a = 2\pi \times 10^9$ rad/s



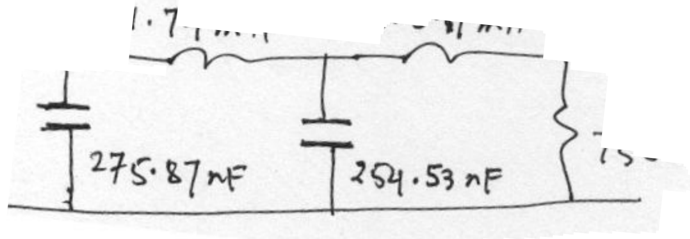
$1.3F \to \frac{1.3}{2\pi \times 10^9} F$

$1.5H \to \frac{1.5}{2\pi \times 10^9} H.$

$R \to$ does not change

Using impedance scaling by 75

$C \to C/75$, $L \to L \times 75$: $R \to R \times 75$



is the Final Circuit

(b) For hpf we apply component transformation first

Thus $L_i \to C_i$ $\quad C_i = \frac{1}{L_i \omega_{cn}}$; $\omega_{cn} = 2\pi \times 400$ rad/s

$C_j \to L_j$, $\quad L_j = \frac{1}{C_j \omega_{cn}}$

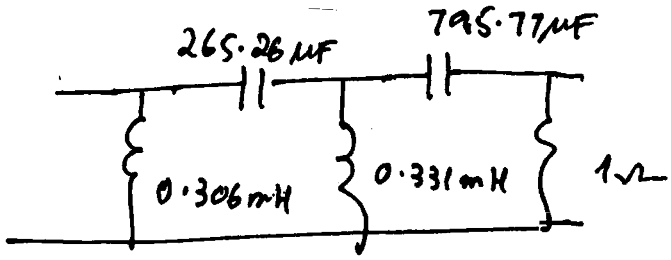
Thus: $1.3\,F \to$ inductance $\quad L_j = \frac{1}{1.3 \times 2\pi \times 400} = 0.306\ mH$

$1.2F \to 0.331\ mH$

$1.5H \to C_i = \frac{}{1.5 \times 2\pi \times 400} = 265.26\ \mu F$

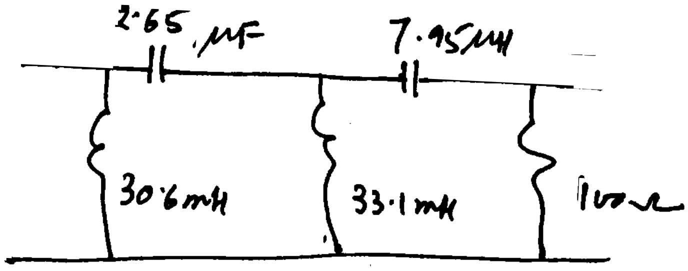
$0.5H \to \quad = 795.77\ \mu F$

Q1 (b) First cut



For 100 Ω termination, use impedance scaling

$L \to 100 \times L$, $C \to C/100$, $R \to R \times 100$.



Final answer

Q2:

At the passband edge, $\omega_n = 1$, $A_p = 1$ dB

so $\epsilon = \sqrt{10^{0.1A_p} - 1} = \sqrt{0.2589} = 0.5088$

a) $|H_N(j\omega_n)|^2 = \dfrac{1}{1 + .5088^2 \omega_n^{2m}} = \dfrac{1}{1 + .2589\, \omega_n^{2m}}$

At $f = 10$ kHz, $\omega_n = \frac{10}{1} = 10$

$|H_N(j\omega_n)|^2 = \dfrac{1}{1 + .2589 \times 10^{10}}$ $m = \text{order} = 5$

$\approx \dfrac{10^{-10}}{.2589}$

In dB → $10\log_{10} |\ \ |^2 = -100 \text{ dB} - 10\log_{10}(0.2589)$ -94.13

$= -100 + 5.87 =$

Attenuation = 94.13 dB Ans

(b) Now $10\log_{10}|H_N(j\omega_n)|^2 = -40$ at $\omega_n = 40 = \frac{40k}{1k}$

$10\log_{10}\left[\dfrac{1}{1 + .2589 \times (40)^{2m}}\right] = -40$

$-10\log_{10}[1 + .2589 \times (40)^{2m}] = -40$ $\log_{10} 1 = 0$

$10^4 = 1 + .2589 \times (40)^{2m}$; $0.2589 \times (40)^{2m} \approx 10^4$

$(40)^{2m} = \dfrac{10^4}{.2589} = 38.6249 \times 10^3$

$2m \log_{10}(40) = \log_{10}(38.6249 \times 10^3) = 4.5869$

$m = \dfrac{4.5869}{2 \times 1.6020} = 1.43 \rightarrow 2$ order → Ans

Q.3 Passband ripple implies a CHEB approximation

$A_p = 0.5\,dB$ leads to $\epsilon = 0.3493$

$$D = \frac{10^{.1A_a}-1}{10^{.1A_p}-1} = \frac{10^{3.5}-1}{\epsilon^2} = 25908.2$$

4/6

$$\frac{\omega_a}{\omega_c} = \frac{15}{7.8} = \omega_s = 1.923$$

$$\sqrt{D} = 160.96$$

$$\cosh^{-1}(160.96) = 5.77 \quad ; \quad \cosh^{-1}(1.923) = 1.271$$

$$\text{Order of filter} \geq \frac{\cosh^{-1}\sqrt{D}}{\cosh^{-1}(\omega_s)} = 4.539 \rightarrow 5$$

The normalized LPF transfer function is

$$H_N(s) = \frac{1}{2^{5-1}\epsilon} \cdot \frac{1}{s^5 + 1.17251s^4 + 1.9374s^3 + 1.3096s^2 + 0.7525s + 0.1789}$$

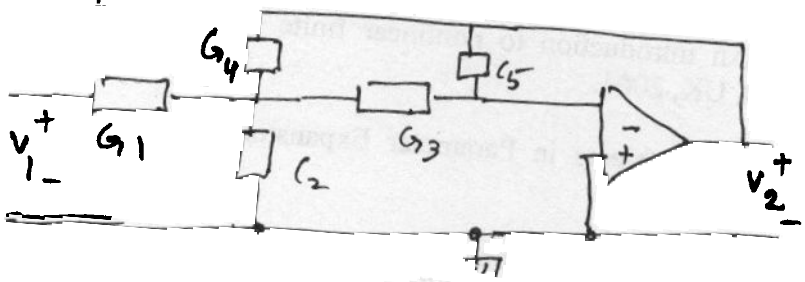
$$= \frac{0.1789}{s^5 + 1.1725s^4 + 1.9374s^3 + 1.3096s^2 + 0.7525s + 0.1789}$$

For the HPF with $\omega_{CH} = 15 \times 10^3 \times 2\pi$, the frequency de-normalized transfer function will be: $s \rightarrow \omega_{CH}/s$

$$H(s) = \frac{0.1789}{\left[\left(\frac{2\pi \times 15 \times 10^3}{s}\right)^5 + 1.1725\left(\frac{2\pi \times 15 \times 10^3}{s}\right)^4 + 1.9374\left(\frac{2\pi \times 15 \times 10^3}{s}\right)^3 + 1.3096\left(\frac{2\pi \times 15 \times 10^3}{s}\right)^2 + 0.7525\left(\frac{2\pi \times 15 \times 10^3}{s}\right) + 0.1789\right]}$$

Q 4

The given circuit is a LPF using infinite gain voltage amplifier. It can be compared with



Thus,

$R_1 \to G_1$
$C_1 \to C_2$
$R_2 \to G_4$
$R_3 \to G_3$
$C_2 \to C_5$

$\to$ Design guidelines for these are given in Table 4.3

$$\frac{E_2}{E_1} = \frac{G\omega_0^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$ can be compared with

$$\frac{V_2}{V_1} = \frac{H}{s^2 + \left(\frac{\omega_p}{Q_p}\right)s + \omega_p^2}$$

So $H = G\omega_0^2$. $\omega_0 \equiv \omega_p$, $Q \equiv Q_p$

From given data $H = 5\omega_0^2 = 5$, $\omega_0^2 = 1$ given $= b_0$

$$b_1 = \frac{\omega_0}{Q} = \frac{\omega_p}{Q_p} = 1.2 = \frac{1}{Q_p}$$

So $Q_p = \frac{1}{1.2} = 0.833$

Let's use the design guidelines

Q4: contd.

$$G_1 = \frac{H}{\omega_p} = \frac{5}{1} = 5 \qquad \text{So } R_1 = \frac{1}{5} = 0.2\,\Omega$$

$$C_2 = \frac{\alpha_p(2\omega_p^2+H)}{\omega_p^2} = \frac{0.833(2+5)}{1} = 5.831\text{ F}$$

Thus $C_1 = 5.831$ F in the given circuit

$G_3 = \omega_p = 1$ So $R_3 = G_3 = 1\,\Omega$ in the given circuit

$G_4 = G_3 = 1$, $R_2 = 1\,\Omega$ in the given circuit

$$C_5 = \frac{\omega_p^2}{\alpha_p(2\omega_p^2+H)} = \frac{1}{5.831} = 0.1715\text{ F}$$

$C_5 \to C_2 = 0.1277$ F in the given circuit

Hence:

$$\left.\begin{aligned} R_1 &= 0.2\,\Omega \\ R_2 &= 1\,\Omega \\ R_3 &= 1\,\Omega \\ C_1 &= 5.831\text{ F} \\ C_2 &= 0.1715 \end{aligned}\right\} \text{needed design values}$$

Proof:

$$\frac{1}{R_2R_3C_1C_2} = \frac{1}{1.1.\,5.831 \times .1715} \simeq 1 = b_0 = \omega_0^2$$

$$\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) = \frac{1}{5.831}(5+1+1) \simeq 1.2 = b_1$$

$$\frac{1}{R_1R_3C_1C_2} = \frac{1}{.2}\cdot\frac{1}{1}\cdot\frac{1}{1} = 5 = G\,b_0$$