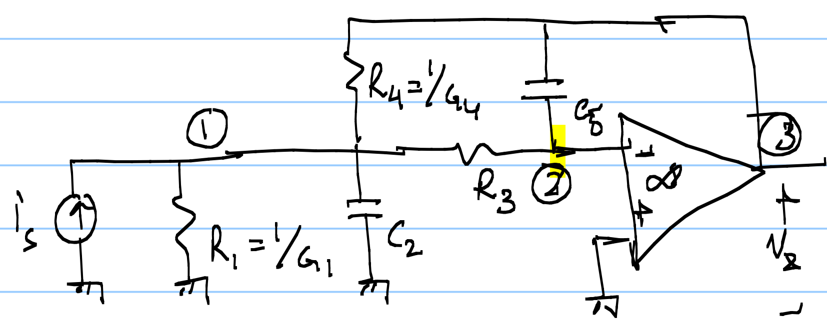
ELEC 441/6081 MT test #1, Feb 2013, Sol/hint

Note Title 2/13/2013

Q1: Using source transformation the circuit becomes



with $i_s' = V_i G_1$

The nodal admittance matrix is:

$$\begin{array}{cccc} ① & ② & ③ & \\ \end{array}$$

$$\begin{bmatrix} G_1+G_4+G_3+sC_2 & -G_3 & -G_4 \\ -G_3 & G_3+sC_5 & -sC_5 \\ -G_4 & -sC_5 & G_4+sC_5 \end{bmatrix} \begin{bmatrix} V_① \\ V_② \\ V_③ \end{bmatrix} = \begin{bmatrix} G_1 V_i \\ 0 \\ 0 \end{bmatrix}$$

$\because$ Gain of OA $= \infty$, $V_②$ is virtual short circuit. Also $V_③$ being the output of a voltage amplifier, we drop column ② & row # 3. So we get

Q1 (cont.)

$$\begin{bmatrix} G_1+G_4+G_3+sC_2 & -G_4 \\ -G_3 & -sC_5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_3 \end{bmatrix} = \begin{bmatrix} G_1 V_i \\ 0 \end{bmatrix}$$

Given output is $V_o = V_3 = V_2$

Then by using Krammer's rule,

$$V_3 = \frac{1}{\Delta}\begin{vmatrix} G_1+G_4+G_3+sC_2 & G_1 V_i \\ -G_3 & 0 \end{vmatrix}$$

where Δ = determinant

$$\begin{vmatrix} G_1+G_4+G_3+sC_2 & -G_4 \\ -G_3 & -sC_5 \end{vmatrix}$$

$$= -\left[s^2 C_2 C_5 + (G_1+G_4+G_3)sC_5 + G_3 G_4\right]$$

Finally,

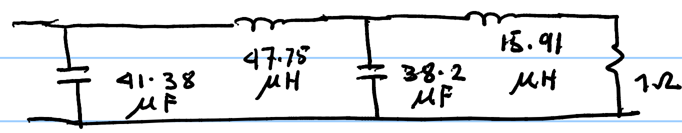
$$\frac{V_2}{V_i} = -\frac{1/(R_1 R_3 C_2 C_5)}{s^2 + \frac{s}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_4}\right) + \frac{1}{R_3 R_4 C_2 C_5}}$$

Q.2:

(a) Frequency is scaled by $a = 2\pi \times 5000$

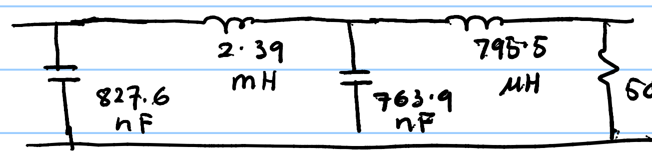
So $L \to L/a$, $C \to C/a$, R does not change.

The circuit becomes:



(b) $1\,\Omega$ to $50\,\Omega$, impedance scale factor $k_m = 50$

So $C \to C/50$, $L \to L \times 50$, $R \to R \times 50$. The new circuit will be



Q3:

Given $|H_N(j\omega)|^2 = \dfrac{1}{1+\epsilon^2\omega_n^{2m}}$; given $m=5$

at $\omega_n = 1$ i.e. $\omega = 2\pi \times 1000$ rad/sec, loss $= 1$ dB

i.e. $10\log_{10}\left[\dfrac{1}{1+\epsilon^2}\right] = -1$ (Loss brings in $-$ sign)

$\epsilon^2 = 10^{.1} - 1$; $\epsilon = \sqrt{0.25892} = 0.5088$

(a) At 5 KHz, $\omega_n = 5$; $|H_N(j\omega)|^2 = \dfrac{1}{1+\epsilon^2(5)^{10}}$

In dB, $10\log_{10}\left[|H_N(j\omega)|^2\right] = -64.03$; subst for ϵ^2

So loss is 64.03 dB

(b) loss of 40 dB implies $10\log_{10}\left[|H_N(j\omega)|^2\right] = -40$

i.e. $\dfrac{1}{1+\epsilon^2(30)^{2m}} = 10^{-4}$; $\omega_n = 30$ @ 30 KHz

$1 + 0.25892 \times (30)^{2m} = 10^4$; $m = 1.55 \to$ order 2

i.e. a second order filter

Q.4: Equiripple passband implies a CHEB response.

From given specs. $A_p = 0.5\,dB$, $A_a = 35\,dB$

$$\omega_c = 2\pi \times 12 \times 10^3 \;;\; \omega_a = 2\pi \times 3.5 \times 10^3 \;;\; \omega_s = \frac{12}{3.5} = 3.428$$

$$\eta = \frac{10^{.1A_a} - 1}{10^{.1A_p} - 1} = \frac{10^{3.5} - 1}{10^{.5} - 1} = 25908.19$$

$$\sqrt{\eta} = 160.96$$

The filter order $n \geq \dfrac{\cosh^{-1}\sqrt{\eta}}{\cosh^{-1}(\omega_s)} = 3.03$

Let $n = 4$

Looking up from Tables (for $A_p = 0.5\,dB$, i.e. $\epsilon = 0.3493$)

$$H_N(s)\Big|_{LP} = \frac{1}{2^{4-1}\epsilon} \cdot \frac{1}{s^4 + 1.197 s^3 + 1.717 s^2 + 1.025 s + 0.379}$$

For the frequency denormalized HPF, we scale

$$s \rightarrow \frac{2\pi \times 12000}{s} . \text{ Thus,}$$

$$H(s) = \frac{35785.86\, s^4}{37900 s^4 + 0.7728 \times 10^{10} s^3 + 0.9761 \times 10^{15} s^2 + 0.5131 \times 10^{20} s + 0.3232 \times 10^{25}}$$