MT2, April '09, Grad

(60 min.)

For Graduate Students

Q.1 :

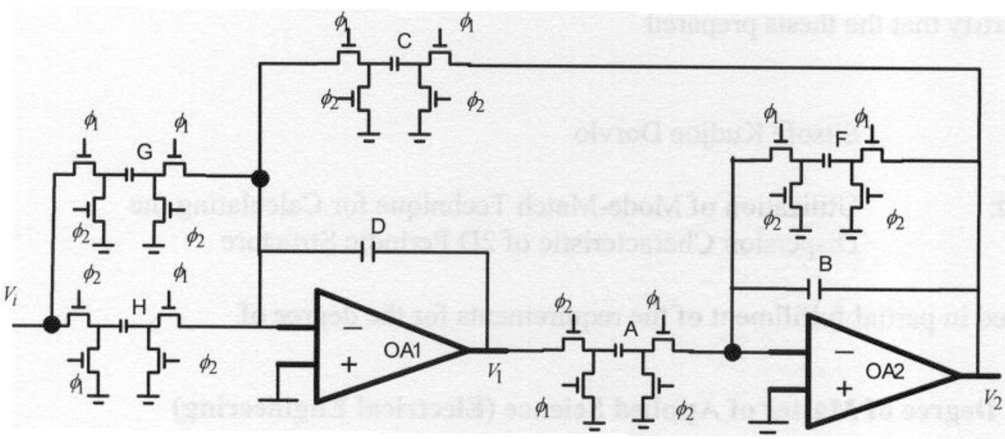


Figure 1:

The figure above shows a second order switched capacitor filter using parasitic insensitive switched capacitor integrators. Find an expression for the Z-domain transfer function $\frac{V_2^{(1)}}{V_i^{(1)}}$. *You can assume that the sample-and-hold property holds for the signals V_i, and V_1* i.e., $V_i^{(2)} = z^{-\frac{1}{2}} V_i^{(1)}$ etc.,.

Q.2: The schematic below represents a normalized low-pass CHEB filter of order 3 with equal terminating resistances.

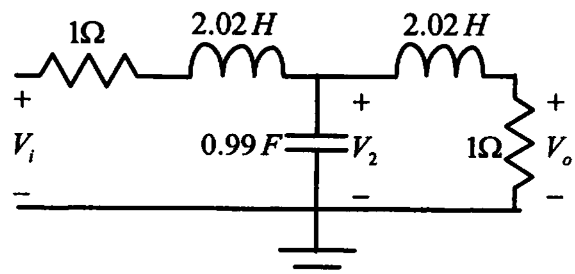


Figure 2(a):

Produce an active RC design for the above ladder filter with $100\,\Omega$ terminations, and a pass-band edge frequency of 1000 radians. Use *operational simulation* technique, according to the leap-frog interconnection as shown below (Fig.2(b)).

MT2, April'09, Grad

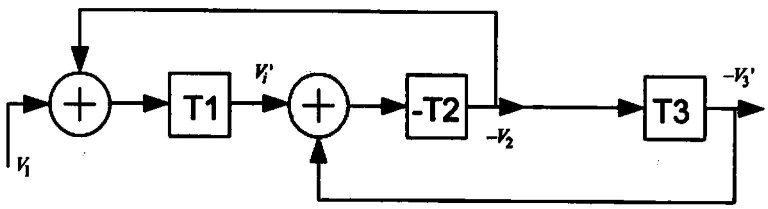


Figure 2(b):

In the above T1, T3 are the voltage transfer functions (VTF) associated with the series R,L segments and –T2 is the VTF associated with the shunt capacitance segment of the ladder filter.

Show your schematic and the designed element values clearly.

Q.3: Design, using OP-AMP or OTA, a second order notch filter which corresponds to a frequency normalized transfer function $T_N(s) = \dfrac{s^2+1}{s^2+1.5s+1}$. The stop-band frequencies of the notch filter are: $\Omega_l = 1000$ rad/sec, and $\Omega_h = 4000$ rad/sec respectively. *Note that the pole frequency of the filter is same as the (geometric) center frequency.*

Show the components of your designed circuit, and the schematic clearly.

MT2, April '09 UG

(60 min.)

For Undergraduate students

Q.1:

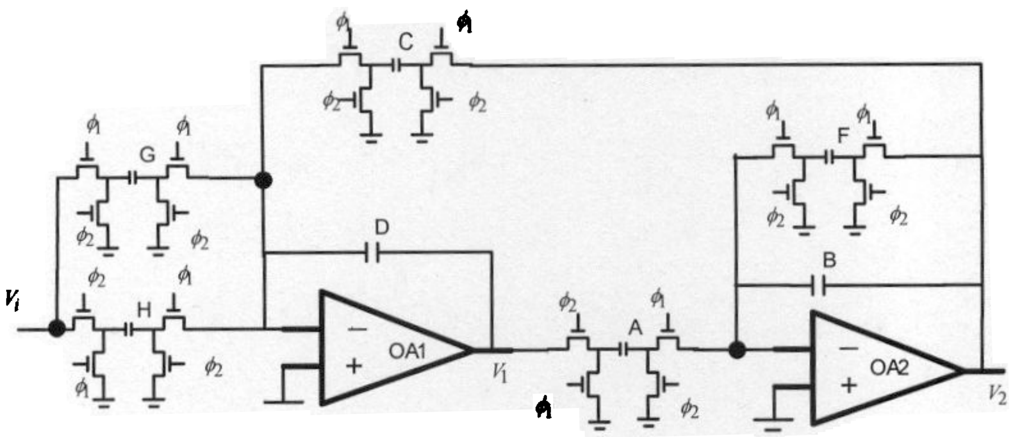


Figure 1:

The figure above shows a second order switched capacitor filter using parasitic insensitive switched capacitors. Find an expression for the Z-domain transfer function $\frac{V_2^{(1)}}{V_i^{(1)}}$. *You can assume that the sample-and-hold property holds for the signals V_i, and V_1 i.e.,* $V_i^{(2)} = z^{-\frac{1}{2}} V_i^{(1)}$ **etc.,.**

Q.2: The schematic below represents a normalized low-pass CHEB filter of order 3 with equal terminating resistances.

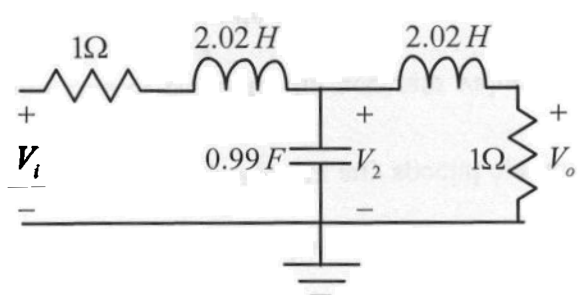


Figure 2:

Produce an active RC design for the above ladder filter with $100\,\Omega$ terminations, and a pass-band edge frequency of 1000 radians. Use *FDNR* (frequency dependent negative resistance) technique.

Show the components of your designed circuit, and the schematic clearly.

MT2, April '09, UG

Q.3: Design, using OP-AMP or OTA, a second order band-stop filter which corresponds to a frequency normalized transfer function $T_N(s) = \frac{s^2+1}{s^2+1.5s+1}$. The stop-band frequencies of the filter are: $\Omega_l = 1000$ rad/sec, and $\Omega_h = 4000$ rad/sec respectively. *Note that the pole frequency of the filter is same as the (geometric) center frequency.*

Show the components of your designed circuit, and the schematic clearly.

[redacted]

Q1: By inspection — MT2, Apr.'09 UG&G — } soln

ELEC 441/6081

p1/11

OA1 receives input at phase 1 so

$$V_1^{(1)} = -\frac{1}{D}\cdot V_i^{(1)} G \cdot \frac{1}{1-z^{-1}} + \frac{1}{D}\cdot V_i^{(2)}\cdot H \frac{z^{-1/2}}{1-z^{-1}} - \frac{1}{D}\cdot V_2^{(1)}\cdot C\cdot\frac{1}{1-z^{-1}}$$

$$= -\frac{G}{D}\frac{1}{1-z^{-1}} V_1^{(1)} + \frac{H}{D}\cdot\frac{z^{-1/2}}{1-z^{-1}} V_i^{(2)} - \frac{C}{D}\frac{1}{1-z^{-1}} V_2^{(1)}$$

Using $V_i^{(2)} = z^{-\frac{1}{2}} V_1^{(1)}$

$$V_1^{(1)} = -\frac{G}{D}\frac{1}{1-z^{-1}} V_i^{(1)} + \frac{H}{D}\frac{z^{-1}}{1-z^{-1}} V_i^{(1)} - \frac{C}{D}\frac{1}{1-z^{-1}} V_2^{(1)}$$

For OA 2, similarly,

$$V_2^{(1)} = -\frac{F}{B}\frac{1}{1-z^{-1}} V_2^{(1)} + \frac{A}{B}\frac{z^{-\frac{1}{2}}}{1-z^{-1}} V_1^{(2)}$$

Using $V_1^{(2)} = z^{-\frac{1}{2}} V_1^{(1)}$

$$V_2^{(1)} = -\frac{F}{B}\cdot\frac{1}{1-z^{-1}} V_2^{(1)} + \frac{A}{B}\cdot\frac{z^{-1}}{1-z^{-1}} V_1^{(1)}$$

Changing sides

$$V_2^{(1)}\left[1+\frac{F}{B}\frac{1}{1-z^{-1}}\right] = \frac{A}{B}\cdot\frac{z^{-1}}{1-z^{-1}} V_1^{(1)}$$

$$V_2^{(1)}\left[1+\frac{F}{B}\frac{1}{1-z^{-1}}\right] = \frac{A}{B}\frac{z^{-1}}{1-z^{-1}}\left[\left(-\frac{G}{D}+\frac{H}{D}z^{-1}\right)\frac{1}{1-z^{-1}} V_i^{(1)} - \frac{C}{D}\frac{1}{(1-z^{-1})} V_2^{(1)}\right]$$

or $$V_2^{(1)}\left[1+\frac{F}{B}\frac{1}{1-z^{-1}} + \frac{AC}{BD}\cdot\frac{z^{-1}}{(1-z^{-1})^2}\right] = \frac{A}{B}\cdot\frac{z^{-1}}{(1-z^{-1})^2}\left(-\frac{G}{D}+\frac{H}{D}z^{-1}\right) V_i$$

$$V_2^{(1)}\frac{\left[BD(1-z^{-1})^2 + FD(1-z^{-1}) + ACz^{-1}\right]}{BD(1-z^{-1})^2} = \frac{A}{B}\frac{1}{(1-z^{-1})^2}\cdot\left(-\frac{G}{D}z^{-1}+\frac{H}{D}z^{-2}\right) V_i^{(1)}$$

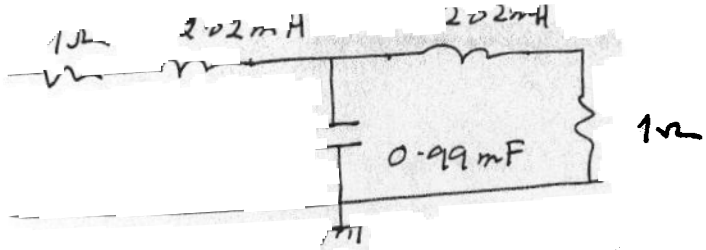
~~Q2:~~ Q1: $V_2^{(1)}\left[BD\left(1-z^{-1}\right)^2+ED\left(1-z^{-1}\right)+ACz^{-1}\right]$

$$= AD\left[-\frac{G}{D}z^{-1}+\frac{H}{D}z^{-2}\right]V_1^{(1)}$$

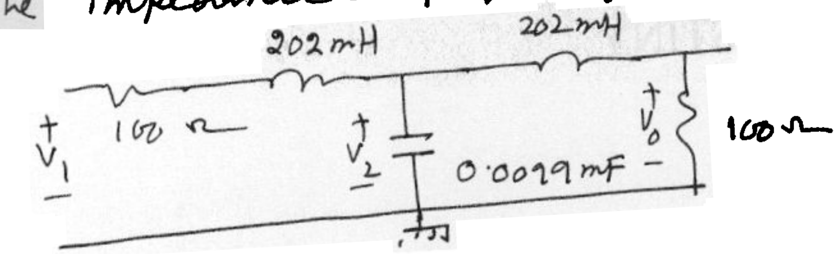
$$\frac{V_2^{(1)}}{V_1^{(1)}} = \frac{AHz^{-2}-GAz^{-1}}{BD\left(1-z^{-1}\right)^2+ED\left(1-z^{-1}\right)+ACz^{-1}}.$$

X

~~Q3~~. Q.2 The frequency denormalized filter is.



The impedence & frequency denormalized filter is



For operational simulation:

$$Y_1 \rightarrow \underset{100\Omega \quad 202mH}{\text{———}} \rightarrow \frac{1}{100 + s \times 202 \times 10^{-3}} = \frac{1}{202 \times 10^{-3}} \cdot \frac{1}{s + \frac{100}{202 \times 10^{-3}}}$$

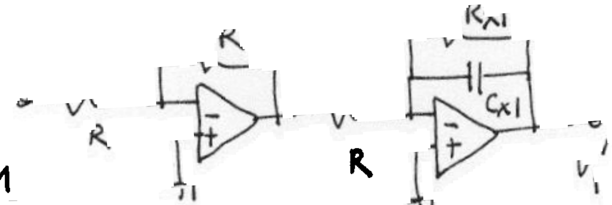
$$T_1 = RY_1 = \frac{R / 202 \times 10^{-3}}{s + \frac{100}{202 \times 10^{-3}}}$$

$$-Z_2 = -\frac{1}{s \times .0099\,mF} = -\frac{1}{s \times 99 \times 10^{-7}} = -\frac{1}{s \times 9.9 \times 10^{-6}}$$

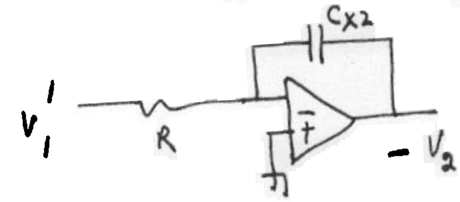
$$-T_2 = -\frac{Z_2}{R} = -\frac{1}{s \times R \times 9.9 \times 10^{-6}}$$

$$Y_3 \quad \underset{202mH \quad 100\Omega}{\text{———}} \rightarrow T_3 = \frac{R / 202 \times 10^{-3}}{s + \frac{100}{202 \times 10^{-3}}}$$

Then $T_1 \rightarrow$

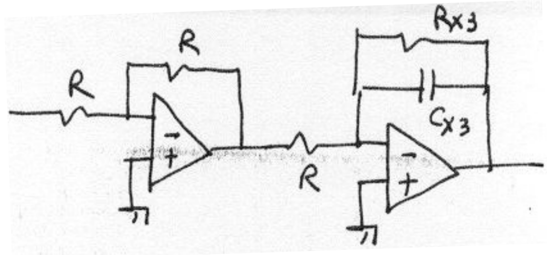


For T_2



Q2 contd.

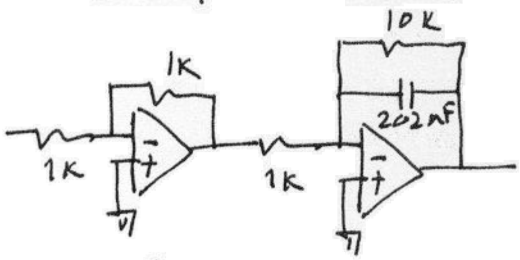
For T_3



Let $R = 1\,k\Omega$

The design equations are: $R_x = \frac{R^2}{R_s}$ & $C_x = \frac{L}{R^2}$

Thus for T_1 we have



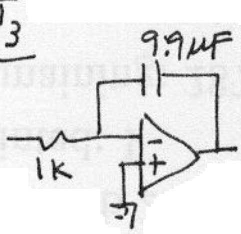
$R_x = \frac{10^6}{100} = 10^4 = 10K$

$C_x = \frac{L}{R^2} = \frac{202\,mH}{10^6}$

$= 202\ \cancel{mH}\ nF$

So also for T_3

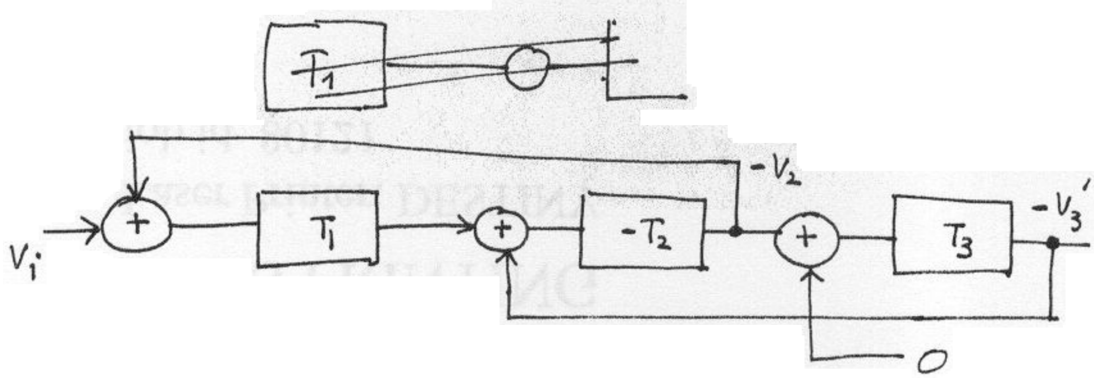
For $-T_2 \Rightarrow$



$\frac{1}{RC_{x2}} \rightarrow \frac{1}{RC}$

$C_{x2} = C = 9.9\,\mu F$

The interconnection is:

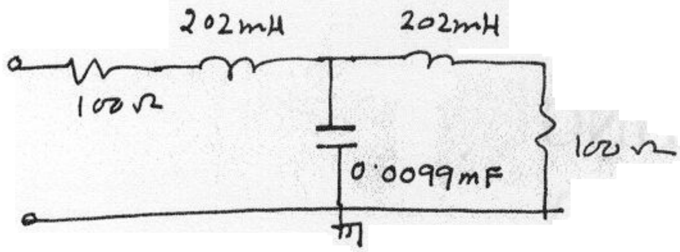


Q2 (Alt.) – ~~UG class only~~

ELEC 441, MT: W0809

p5 / 11

The frequency denormalized and impedance scaled ~~LC to~~ L, C, R ladder filter is



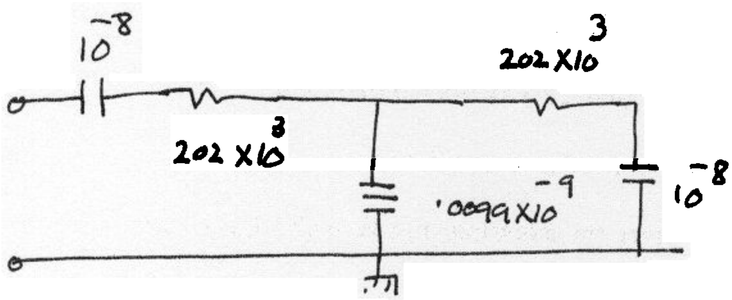
For FDNR technique we consider an impedance scaling by $\frac{1}{ks}$. So

$R \to \frac{R}{ks} = \frac{1}{s\left(\frac{k}{R}\right)} = \frac{1}{s\bar{C}}$; $\bar{C} = \frac{k}{R}$

$L \to sL / ks = \frac{L}{k} = \bar{R}$; $\bar{R} = \frac{L}{k}$

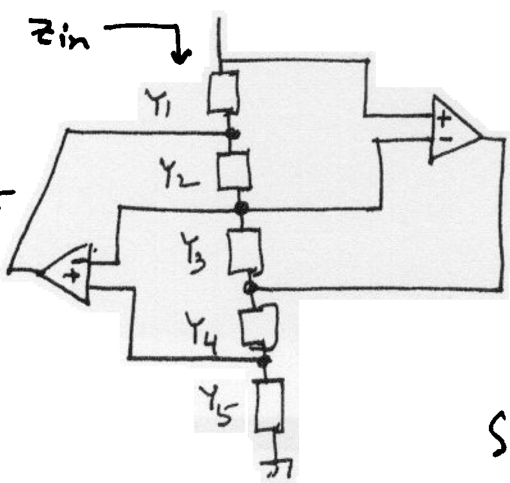
$C \to \frac{1}{sC} \frac{1}{ks} = \frac{1}{s^2} \frac{1}{kC} = -\frac{1}{\omega^2} \cdot \frac{1}{D}$, $D = kC$

The new filter will then look like. Let $k = 10^{-6}$



For the super capacitor D, we use the GIC circuit

$Z_{in} = \frac{Y_2 Y_4}{Y_1 Y_3 Y_5}$



With $Y_2, Y_4, Y_3 \to$ ~~sCx~~ G_x each

& Y_1, ~~Y_2~~, $Y_5 = $ ~~Gx~~ sC_x each

$Z_{in} = \frac{1}{s^2 C_x^2} \frac{G_x^3}{G_x}$ G_x

$= -\frac{1}{\omega^2} \frac{1}{C_x} \cdot G_x$

So $D = kC = \frac{C_x}{G_x}$

Q2 (alt.)

Sub class only

~~But here $D = .0099 \times 10^{-9} = \frac{1}{C_x G_x^3}$~~

~~Let $G_x \to \frac{1}{R_x} = 10^{-2}$ i.e. $R_x = 100\,\Omega$~~

~~Then $\frac{1}{C_x} = .0099 \times 10^{-9} \times 10^{-6} = .0099 \times 10^{-15}$~~

p6 | 11 / 11

~~But here $D = .0099 \times 10^{-9} = \frac{G_x}{G_x} \frac{C_x}{G_x}$~~

~~$C_x = G_x \times 99 \times 10^{-13}$~~

~~If $G_x = 10^{-2}$, i.e. $R_2, R_3, R_4 = 100\,\Omega$ each~~

~~$C_x = 99 \times 10^{-15} = 99\,f = .099\,pF$~~

Q2: Alt.

With $Y_2, Y_4, Y_3 = G_x$, $Y_1, Y_5 \to sC_x$ each

$$Z_{in} = \frac{G_x}{s^2 C_x^2} = -\frac{1}{\omega^2 C_x^2} G_x = -\frac{1}{\omega^2 D}, \quad \text{So } D = \frac{C_x^2}{G_x}$$

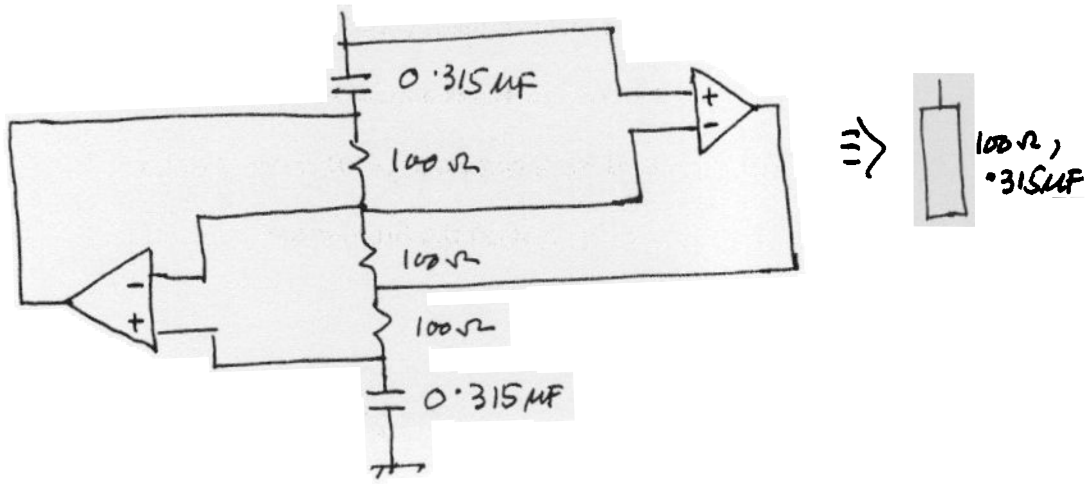
But here, $D = .0099 \times 10^{-9} = \frac{C_x^2}{G_x}$

Let $G_x = 10^{-2}$ i.e. $R_2 = R_4 = R_3 = 100\,\Omega$

So $C_x^2 = .0099 \times 10^{-11} = 99 \times 10^{-15}$

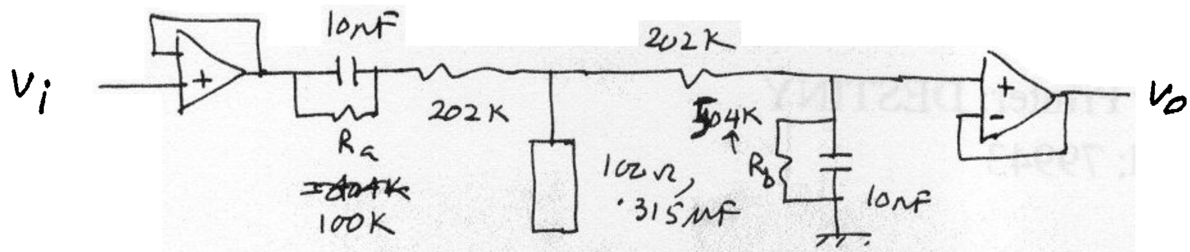
$C_x = \sqrt{99 \times 10^{-15}} = 0.315 \times 10^{-6}$

So design for D is



Q2 Alt. ← UG class only

The overall complete system will include buffers at input and output, together with resistive paths through the filter structure. Thus:



DC gain of the designed network is:

$$\frac{R_b}{R_b + R_a + 404K} = \text{DC gain of original ladder filter} = 0.5$$

$R_b = 0.5 R_a + 202K + 0.5 R_b$; If ~~$R_a = R_b = R$, $0.5R$~~
~~$0.5R = 202K$~~

~~$R = 404K$~~ $\quad R_b = R_a + 404K$; Let $R_a = 100K$, then $R_b = 504K$

This completes the system design.

$$\left\langle \frac{504K}{100^K + 404^K + 504^K} \rightarrow 0.5 \text{ DC gain} \right\rangle$$

Q3:
~~Q4~~:

$$T_N(s) = \frac{s^2+1}{s^2+1.5s+1}$$

One can frequency denormalize by $\Omega_0 = \sqrt{\Omega_L \Omega_h} = 2000$ rad/sec

OR, one can design the filter for

$$T_N = \frac{s^2+1}{s^2+1.5s+1} \quad \text{and}$$

then use frequency denormalization.

After frequency denormalization, one gets

$$T(s) = \frac{s^2/\Omega_0^2+1}{\frac{s^2}{\Omega_0^2}+1.5\frac{s}{\Omega_0}+1} = \frac{s^2+\Omega_0^2}{s^2+1.5\,s\Omega_0+\Omega_0^2}$$

Example design using state variable technique {Possible Design #1}

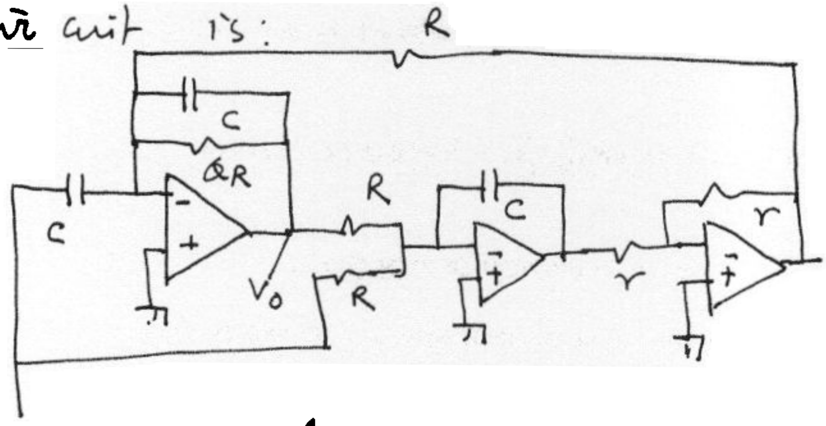
Tow-Thomas biquad

For $T_N(s)$, we see $T_N(s)\big|_{s\to\infty} = 1$, $\because \Omega_n^2 = \Omega_p^2 = 1$

So $C_1 = C$; R_1 = open (does not exist)

$R_2 = R \cdot \left(\frac{\omega_p}{\omega_n}\right)^2 / \text{hf gain} = R$ $\because$ $\omega_p = \omega_n$ here. & hf. gain = 1

The circuit is:

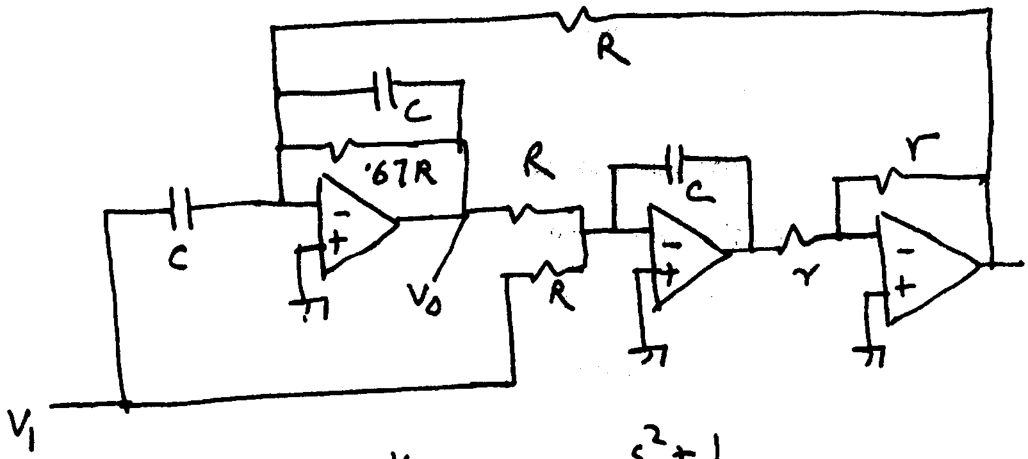


$\frac{\omega_p}{Q}$ = coeff. of s in denom = 1.5, while $\omega_p^2 = 1$

$$Q = \frac{\omega_p}{1.5} = \frac{1}{1.5} = 0.67$$

Q3: ~~Q1~~ (contd.)

Thus the circuit is:



$$T_N(s) = \frac{V_o}{V_1} = \frac{s^2+1}{s^2+1.5s+1}$$

For normalized filter we take $R=1,\ r=1,\ C=1$

For frequency denormalized filter, $\because \Omega_p = 2000$ rad/sec

we can take $R = \frac{1}{\sqrt{2000}}$ & $C = \frac{1}{\sqrt{2000}}$, so

that $\frac{1}{RC}$ becomes 2000. ~~we can keep~~

Thus $R = 2.236 \times 10^{-2}$; $C = 2.236 \times 10^{-2}$

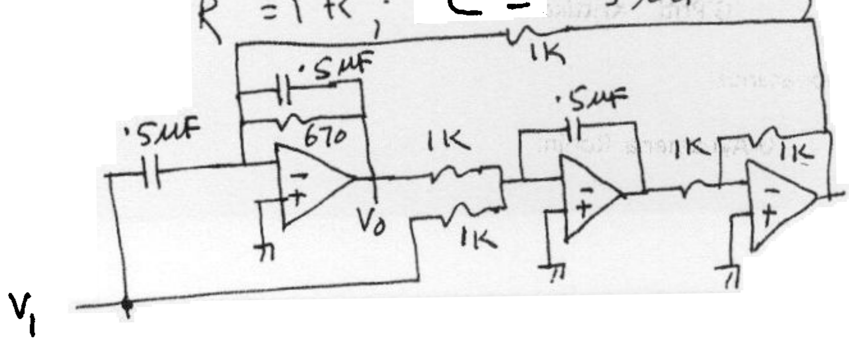
For practical $R = 1\ K$, we use impedance

scaling by $\frac{1000}{2.236 \times 10^{-2}} = 44721.36$

Then $R = 1\ K$, $C = \frac{2.236 \times 10^{-2}}{44721.36} = 0.499\ \mu F$

Let $r = 1\ K$ also. So final design is:

$R = 1\ K$; $C \cong .5\ \mu F$ $r = 1K$



~~Q1~~ Q3: (Contd.) Using Fleischer-Tow circuit

{Possible Design #2}

p10/11

$$\frac{V_o}{V_1} = -\frac{(R_8/R_6)s^2 + \frac{1}{R_1C_1}\left[\frac{R_8}{R_6} - \frac{R_1R_8}{R_4R_7}\right]s + \frac{R_8}{R_3R_5R_7C_1C_2}}{s^2 + \frac{1}{R_1C_1}s + \frac{R_8}{R_2R_3C_1C_2R_7}}$$

To match with

$$|T_N| = \frac{s^2+1}{s^2+1.5s+1}$$

We shall make $R_8/R_6 = 1$; $\frac{R_8}{R_3R_5R_7C_1C_2} = 1$

$$\frac{R_8}{R_6} = \frac{R_1R_8}{R_4R_7}, \quad \& \quad \frac{1}{C_1R_1} = 1.5, \quad \frac{R_8}{R_2R_3C_1C_2R_7} = 1$$

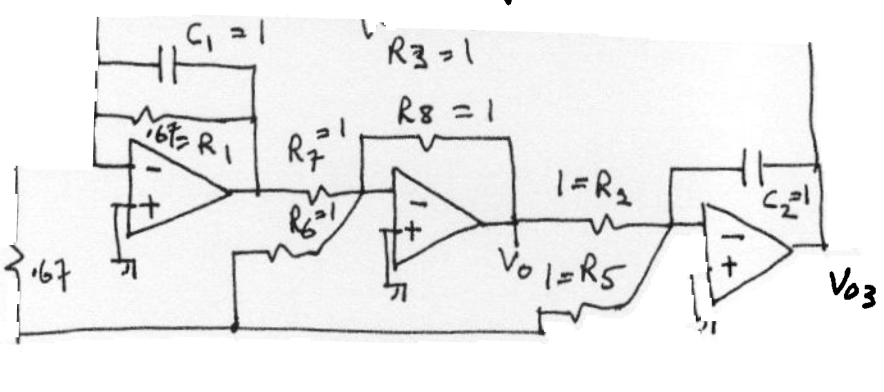
Let $R_8 = 1 = R_6 = R_3 = R_5 = R_7 = C_1 = C_2 \neq R_2$

Then $\frac{1}{C_1R_1} \to \frac{1}{R_1} = 1.5$; $R_1 = 0.67$

$\frac{1}{R_2} = 1$, $R_2 = 1$

$$\frac{R_1R_8}{R_4R_7} = \frac{0.67 \cdot 1}{R_4 \cdot 1} = \frac{R_8}{R_6} = 1 \quad \text{So } R_4 = 0.67 = R_1$$

Thus, the normalized design circuit is:



For frequency denormalized filter, since

Q3 contd.

Q3:

$$\frac{R_8}{R_3 R_5 R_7 C_1 C_2} = \Omega_0^2 = \frac{R_8}{R_2 R_3 C_1 C_2 R_7}$$

We can make $C_1 = C_2 = \frac{1}{\Omega_0} = \frac{1}{2000}$

Second ckt design is:

$C_1 = \quad = 0{\cdot}0005$ F, all others as before

4 now we want $R_8 = R_8 = R_3 = R_5 = R_7 = 1K\Omega$

we do impedance scaling Then

$C_1 = \quad = {\cdot}0005/1000 = 5\times10^{-7} = \cancel{005}\ 0{\cdot}5\mu F$

$R_1 = R_4 = 670\Omega$, other resistances $= 1K\Omega$

Final circuit is