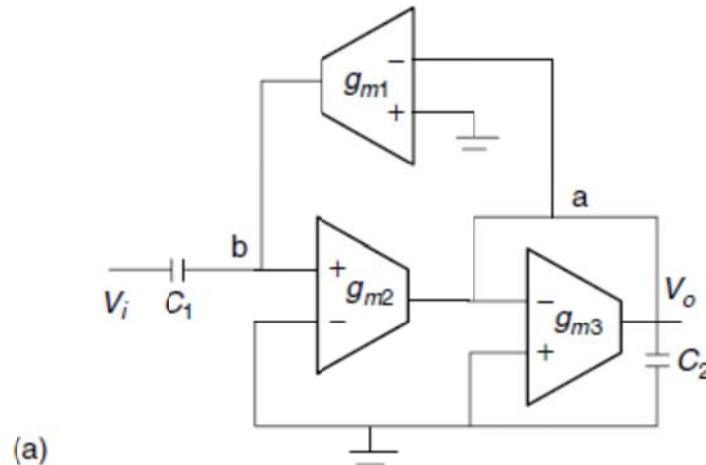


**Q.1:** Each function is a BPF. So we will take the BPF circuit using OTAs. One from the text book (p.139)is:



The design eqs. are:

$$\omega_p = \sqrt{\frac{g_{m1}g_{m2}}{C_1C_2}}, \frac{\omega_p}{Q_p} = \frac{g_{m3}}{C_2}, \text{ therefore } Q_p = \sqrt{\frac{C_2}{C_1} \frac{\sqrt{g_{m1}g_{m2}}}{g_{m3}}}$$

The TF is:

$$\frac{V_o(s)}{V_i(s)} = \frac{sg_{m2}C_1}{s^2C_1C_2 + sg_{m3}C_1 + g_{m1}g_{m2}}$$

Consider the 2<sup>nd</sup> order TF  $T_1(s) = \frac{21262s}{s^2 + 15056s + 0.28604 \times 10^{18}}$

Thus, comparing  $g_{m1}g_{m2}/(C_1C_2)=0.28604E18$ ,  $g_{m3}/C_2=15056$ ,  $g_{m2}/C_2=21262$

There are 3 eqs., but 5 components to design. We can have 2 free choices. Let us set  $C_1=C_2=9\text{pF}$  (i.e., less than 10pF).

Then solving  $g_{m1}=121.0779795$  mhos,  $g_{m2}= 1.91358 \cdot 10^{-7}$  mho,  $g_{m3}=1.35504 \cdot 10^{-7}$  mho

**Q.2:**

Using the Table and given data

>  $wn := \text{sqrt}(2.4212E5);$

492.0569073

$wp := \text{sqrt}(8.9833E5);$

947.8027221

$$\begin{aligned}
qp &:= \frac{wp}{1004.2}; && 0.9438386000 \\
hn &:= 1; && 1 \\
fs &:= 16E3; && 16000. \\
ts &:= \frac{1}{fs}; && 0.00006250000000 \\
ax &:= 2 \cdot fs; && 32000. \\
wph &:= ax \cdot \tan\left(\frac{wp}{ax}\right); && 948.0799808 \\
wnh &:= ax \cdot \tan\left(\frac{wn}{ax}\right); && 492.0956925 \\
F2 &:= ax^2 + \left(\frac{wph}{qp}\right) \cdot ax + wph^2; && 1.05704265610^9 \\
hd &:= \frac{hn \cdot (ax^2 + wnh^2)}{F2}; && 0.9689695607 \\
a1n &:= -\frac{2 \cdot (ax^2 - wnh^2)}{ax^2 + wnh^2}; && -1.999054294 \\
a2n &:= 1.; && 1. \\
a1d &:= \frac{2 \cdot (ax^2 - wph^2)}{F2}; && 1.935780242 \\
a2d &:= \frac{\left(ax^2 - \frac{wph \cdot ax}{qp} + wph^2\right)}{F2}; && 0.9391816402 \\
hz &:= \frac{hd \cdot (1 + a1n \cdot zx + a2n \cdot zx^2)}{1 - a1d \cdot zx + a2d \cdot zx^2}; && \frac{0.9689695607 (1 - 1.999054294zx + 1. zx^2)}{1 - 1.935780242zx + 0.9391816402zx^2}
\end{aligned}$$

In the above  $zx \rightarrow z^{-1}$ .

The above is the answer for part (a).

Part (b): At DC,  $z^x = z^{-1} = \exp(-j\Omega T) = 1$ , since  $\Omega=0$  (DC means *zero* frequency).

Hence  $hz|_{\text{mag}}=0.269$

At 492 rad/sec,  $\Omega T = 0.0307$ ,  $z^x = 0.9995272560 - 0.03074515422I$ .

$hz|_{\text{mag}}=0.00007$  (approx.. zero, as it should happen since 492 rad/s is almost equal to the notch frequency)

**Q.3:**

By inspection

$$V_1^{(1)} = V_1^{(2)} \frac{\bar{z}^{-\frac{1}{2}}}{1-\bar{z}^{-1}} \cdot \frac{H}{D} - V_1^{(1)} \frac{1}{1-\bar{z}^{-1}} \frac{C}{D} - V_2^{(1)} \frac{1}{1-\bar{z}^{-1}} \frac{C}{D}$$

$$V_2^{(1)} = V_1^{(2)} \frac{\bar{z}^{-\frac{1}{2}}}{1-\bar{z}^{-1}} \cdot \frac{A}{B} - V_2^{(1)} \frac{1}{1-\bar{z}^{-1}} \cdot \frac{F}{B}$$

$$V_2^{(1)} = -V_2^{(1)} \frac{F/B}{1-\bar{z}^{-1}} + \frac{A}{B} \cdot \frac{\bar{z}^{-1}}{1-\bar{z}^{-1}} \cdot V_1^{(1)}$$

, using simple-and-fold property

$$V_2^{(1)} = -V_2^{(1)} \frac{F/B}{1-\bar{z}^{-1}} + \frac{A}{B} \cdot \frac{\bar{z}^{-1}}{1-\bar{z}^{-1}} \left[ V_1^{(1)} \frac{\bar{z}^{-1}}{1-\bar{z}^{-1}} \frac{H}{D} - V_1^{(1)} \frac{1}{1-\bar{z}^{-1}} \frac{C}{D} - V_2^{(1)} \frac{1}{1-\bar{z}^{-1}} \frac{C}{D} \right]$$

Thus:

$$V_2^{(1)} \left[ 1 + \frac{F/B}{1-\bar{z}^{-1}} + \frac{A}{B} \cdot \frac{\bar{z}^{-1}}{1-\bar{z}^{-1}} \cdot \frac{C}{D} \cdot \frac{1}{1-\bar{z}^{-1}} \right]$$

$$= V_1^{(1)} \left[ \frac{A}{B} \cdot \frac{\bar{z}^{-2}}{(1-\bar{z}^{-1})^2} \frac{H}{D} - \frac{A}{B} \cdot \frac{\bar{z}^{-1}}{(1-\bar{z}^{-1})^2} \frac{C}{D} \right]$$

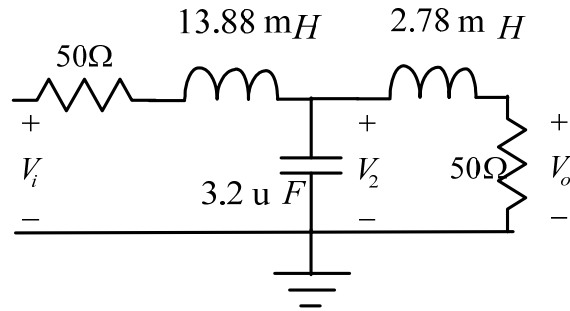
$$V_2^{(1)} \left[ (1-\bar{z}^{-1})^2 + (F/B)(1-\bar{z}^{-1}) + (AC/BD) \bar{z}^{-1} \right]$$

$$= V_1^{(1)} \left[ (AH/BD) \bar{z}^{-2} - (AC/BD) \bar{z}^{-1} \right]$$

$$\text{So } \frac{V_2^{(1)}}{V_1^{(1)}} = - \frac{\bar{z}^{-1} (AC/BD - (AH/BD) \bar{z}^{-1})}{\bar{z}^{-2} - \frac{2BD+FD-AC}{BD} \bar{z}^{-1} + \frac{BD+FD}{BD}}$$

**Q.4:**

After impedance scaling by 50 and frequency scaling by 3000, the LC ladder circuit becomes (multiply by 50/3000) all reactive elements, multiply by 50 all resistive elements)

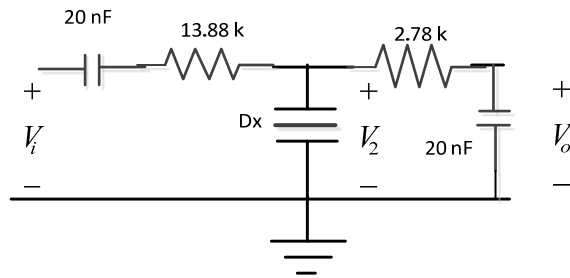


Active RC implementation by FDNR technique.

Apply impedance scaling by  $1/ks$  with  $k=10^{-6}$  (say). Then

$50\Omega \rightarrow k/50=20$  nF,  $13.88$  mH  $\rightarrow 13.88$  m/k = 13.88 k $\Omega$ , and  $3.2$   $\mu$ F  $\rightarrow 3.2\mu k = 3.2 \times 10^{-12}$  as super capacitor  $D_x$ .

In the GIC implementation of the super capacitor  $D_x = C_x^2 R_x$ . So the intermediate implementation is:



We now have to provide a DC path across the 20 nF capacitors so that the DC gain is held close to  $50/(50+50)=1/2$ . Then we need to derive the GIC circuit to realize  $D_x$  element.

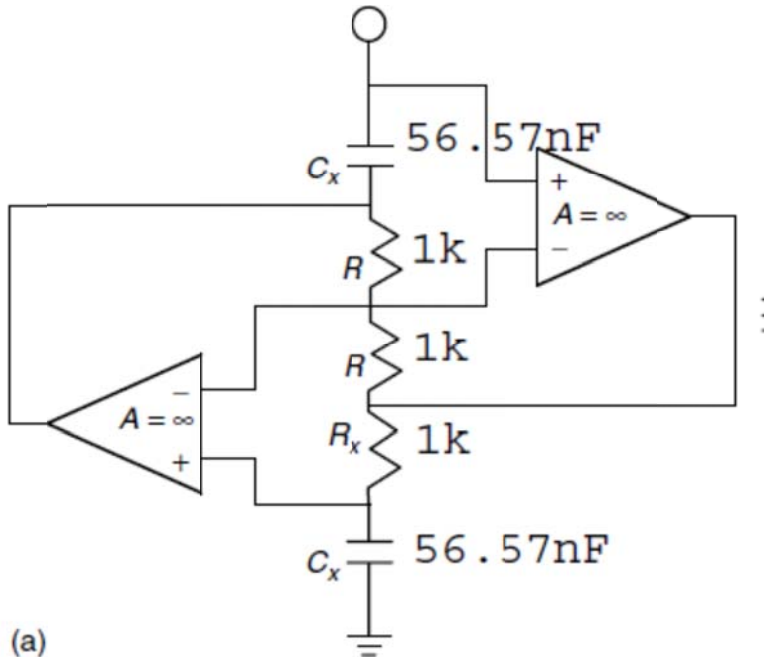
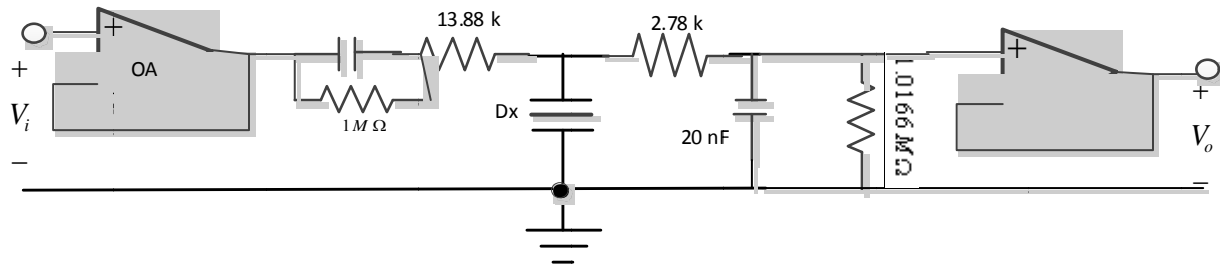
If  $R_a$  and  $R_b$  are the elements, we can design according to  $0.5 = \frac{R_b}{R_a + R_b + 16660}$ . Let  $R_a=1$  M $\Omega$ ,

then  $R_b=1.0166$  M $\Omega$

For the GIC circuit to realize the super capacitor  $D_x$  (text book p.216)

If  $R_x=1$  k $\Omega$ ,  $D_x = 3.2 \times 10^{-12}$  will need  $C_x = 56.57$  nF. We can let all  $R$  in the GIC circuit to be = 1 k $\Omega$ .

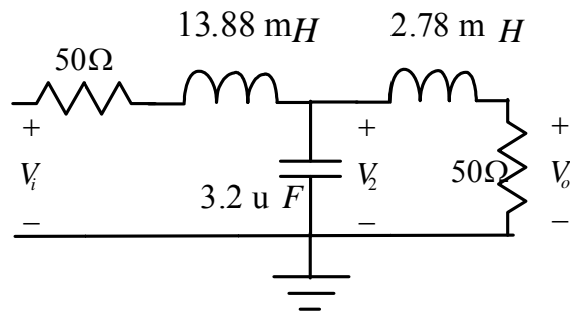
This completes the paper design. The overall system will appear as shown.



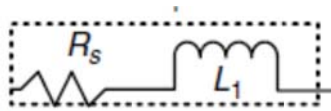
(a)

(Schematic of the  $D_x$  element)

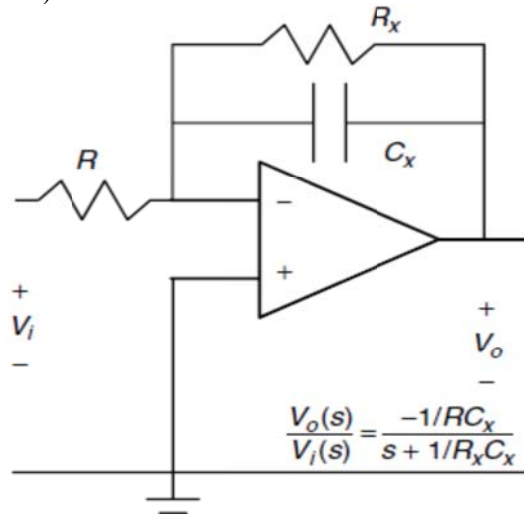
Active RC implementation by OPERATIONAL SIMULATION technique



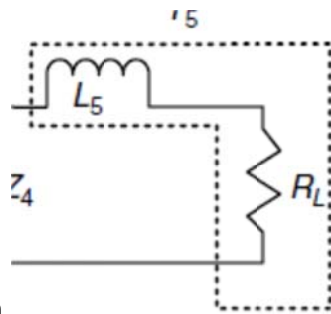
The series L,R branches will be simulated by lossy integrator circuits while the shunt capacitor will have to be simulated by an ideal integrator circuit. The integrators will be implemented by OP-AMP, R and C elements.



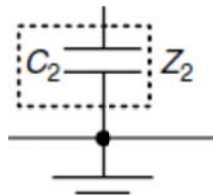
Consider  $\dagger$  with  $R_s = 50\Omega$ ,  $L_1 = 13.88$  mH. This branch can be simulated by a voltage TF  $T_1 = Y_1 R = \frac{R/L_1}{s + R_s/L_1}$ . This TF can be implemented (except for the minus sign) by the circuit (text book p.221)



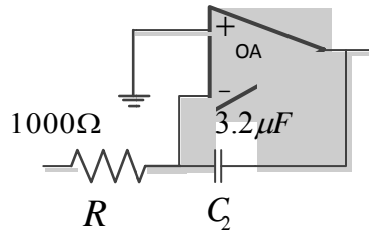
The design equations are: (from DC gain equality)  $R_x/R = R/R_s$ , and  $R_s/L_1 = 1/R_x C_x$ . Assuming  $R = 1000\Omega$ , and substituting given values of  $R_s$ ,  $L_1$ , we can get  $C_x = 13.88$  nF,  $R_x = 20000\Omega$



Similarly, for the branch  $\dagger$ , by substituting the values ( $L_5 = 2.78$  mH,  $R_L = 50\Omega$ , and assuming  $R = 1000\Omega$ , we'll get  $C_{x2} = 2.78$  nF,  $R_{x2} = 20000\Omega$ .



The branch  $\dagger$  can be implemented using a lossless RC integrator circuit. Thus,  $-Z_2/R = -1/(sC_2 R)$  can be implemented using the circuit



Now, using the sequence shown below (text book p.219),

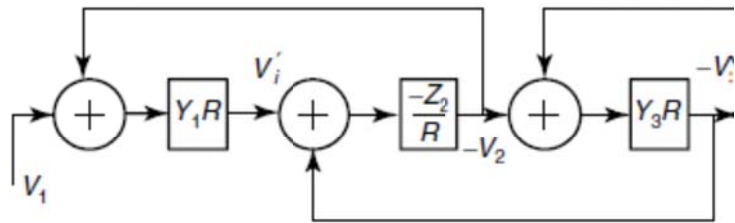


Figure 7.14 Implementation of the  $I - V$  relations in

we can derive the *leap-frog* structure as below