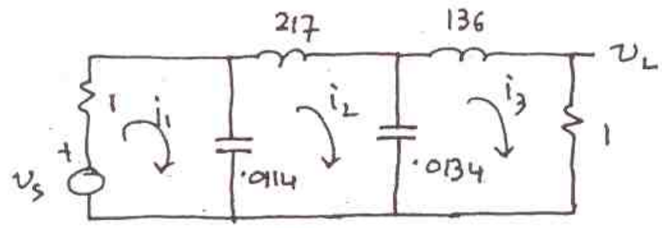
HINTS / ANSWERS , Problem set Ch. 2

2.1 The network:



The loop matrix eqn:

$$\begin{bmatrix} 1+\dfrac{1}{s(\cdot 0114)} & -\dfrac{1}{s(\cdot 0114)} & 0 \\ -\dfrac{1}{s(\cdot 0114)} & s(217)+\dfrac{1}{s(\cdot 0114)}+\dfrac{1}{s(\cdot 0134)} & -\dfrac{1}{s(\cdot 0134)} \\ 0 & -\dfrac{1}{s(\cdot 0134)} & 1+s(136)+\dfrac{1}{s(\cdot 0134)} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}$$

The determinant

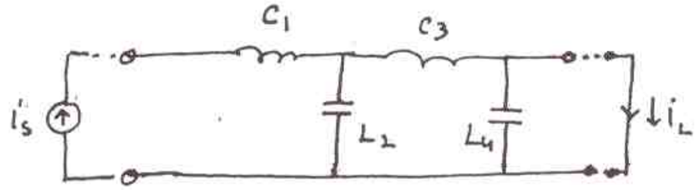
$$\Delta = \frac{1}{s^2}\left[2588988\cdot 93\, s^3 + 29512\, s^4 + 57308\cdot 2\, s^2 + 2310976\cdot 7\, s + 13092\cdot 4\right]$$

$$i_3 = \frac{1}{\Delta}\cdot\frac{v_s}{s^2(\cdot 0114)(\cdot 0134)} \quad ; \quad v_L = i_3 \cdot 1.$$

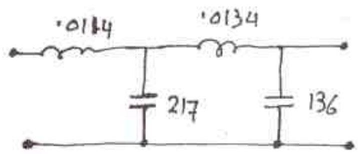
$$\frac{v_L}{v_s} = \frac{6\cdot 5462 \times 10^3}{29512\, s^4 + 2588988\cdot 93\, s^3 + 57308\cdot 2\, s^2 + 2310976.7\, s + 13092\cdot 4}$$

2-1 (Contd.)

N has to be the dual of the given network



i.e.



———— x ————

2.2 Use loop system

$$\begin{bmatrix} 1.5s + \frac{1}{1.33s} & -\frac{1}{1.33s} \\ -\frac{1}{1.33s} & 1 + .5s + \frac{1}{1.33s} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \end{bmatrix}$$

$$\Delta = \frac{1}{s} \cdot \left(1.5s^2 + .75s^3 + 1.504s + 0.7519 \right)$$

$$i_2 = \frac{1}{\Delta} \begin{vmatrix} 1.5s + \frac{1}{1.33s} & v_s \\ -\frac{1}{1.33s} & 0 \end{vmatrix} = \frac{1}{\Delta} \frac{v_s}{1.33s}$$

$$v_o = i_2 \cdot 1 \quad ; \quad \frac{v_o}{v_s} = \frac{1}{1.33} \cdot \frac{1}{.75s^3 + 1.5s^2 + 1.504s + 0.7519}$$

———— x ————

~~3.6.~~
2.3.

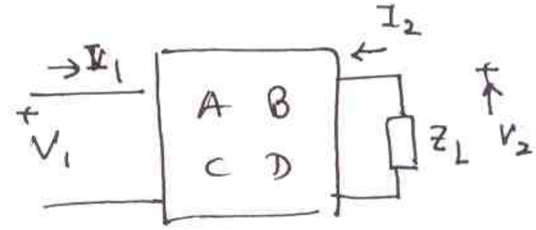
(a) See p. 211 for the derivation to show that the chain matrix of the network is

$$[a]_N = \begin{bmatrix} 1 & 0 \\ 0 & \frac{Z_2 Z_4}{Z_1 Z_3} \end{bmatrix}$$

Thus, this corresponds to a GIC with $K_1(s) = 1$ (See Table 2.2)

and $K_2(s) = \frac{Z_2 Z_4}{Z_1 Z_3}$

(b)



$$Z_{in} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$V_2 = -I_2 Z_L$

$$= \frac{A(-I_2 Z_L) - BI_2}{C(-I_2 Z_L) - DI_2}$$

$$= \frac{AZ_L + B}{CZ_L + D}$$

(c) $[a]_N$ with given values reduces to $\begin{bmatrix} 1 & 0 \\ 0 & \frac{G}{sC} \end{bmatrix}$

$$\therefore Z_{in} = \frac{R_L}{G/sC} = sCR\,R_L$$

$\therefore Z_{in} = s(CR)R_L$, an inductance of value $(CR)R_L$.

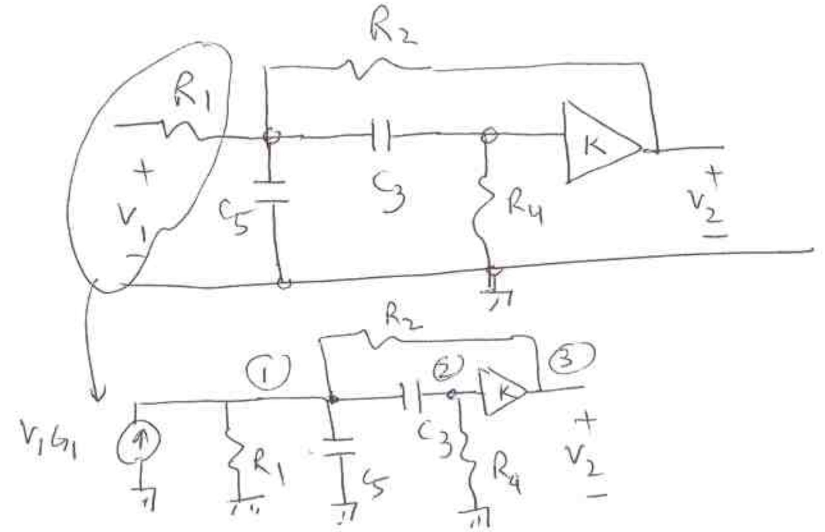
2.4. Consider the OA with R_3, R_4 resistances as a VCVS of gain $K = 1 + \frac{R_4}{R_3}$. Then apply nodal analysis with the principle of constraint. Convert V_i in series with R_1 to the Norton equivalent of $I_i = V_i / R_1$ in shunt with R_1 before embarking on nodal analysis.

x

2.5–2.8

~~2.6, 2.8~~ : Proceed using the method of constraints. Convert input series network to equivalent Norton network before embarking on nodal analysis.

2.5



$V_2 = V_{③}$

$$
\begin{array}{cccc}
 & ① & ② & ③ \\
\end{array}
$$

$$
\begin{bmatrix} G_1+G_2+sC_5+sC_3 & -sC_3 & G_2 \\ -sC_3 & sC_3+G_4 & 0 \\ -G_2 & 0 & G_2 \end{bmatrix} \begin{pmatrix} V_{①} \\ V_{②} \\ V_{③} \end{pmatrix} = \begin{pmatrix} I_1 \\ 0 \\ 0 \end{pmatrix} \quad ; I_1 = V_1 G_1
$$

$V_{③} = K V_{②}$

$$
\begin{pmatrix} G_1+G_2+sC_3+sC_5 & -sC_3-KG_2 \\ -sC_3 & sC_3+G_4 \\ \cancel{-G_2} & \cancel{KG_2} \end{pmatrix} \begin{pmatrix} V_{①} \\ V_{②} \end{pmatrix}
$$

discarded being the node corresponding to the output of a VCVS

$$
\begin{bmatrix} G_1+G_2+sC_3+sC_5 & -sC_3-KG_2 \\ -sC_3 & sC_3+G_4 \end{bmatrix} \begin{pmatrix} V_{①} \\ V_{②} \end{pmatrix} = \begin{bmatrix} I_1 \\ 0 \end{bmatrix}
$$

$$
\Delta = (G_1+G_2+sC_3+sC_5)(sC_3+G_4) - (sC_3+KG_2)sC_3
$$

$$
= (G_1+G_2)G_4 + (G_1+G_2)sC_3 + \cancel{s^2C_3^2} + sG_4C_3 + s^2C_3C_5 + sC_5G_4 - \cancel{s^2C_3^2} - KG_2sC_3
$$

2.5

$$\Delta = (G_1+G_2)G_4 + s\left[C_3(G_1+G_2) + G_4C_3 + G_4C_5 - KG_2C_3\right] + s^2C_3C_5$$

$$V_{(2)} = \frac{1}{\Delta} \cdot \begin{vmatrix} G_1+G_2+sC_3+sC_5 & I_1 \\ -sC_3 & 0 \end{vmatrix} = \frac{I_1 sC_3}{\Delta}$$

$$V_{(3)} = KV_{(2)} = \frac{KsC_3}{\Delta} \cdot G_1V_1 = V_2$$

So
$$\frac{V_2}{V_1} = \frac{KsG_1C_3}{s^2C_3C_5 + s\left\{C_3(G_1+G_2) + C_3(G_4) + C_5G_4 - KC_3G_2\right\} + (G_1+G_2)G_4}$$

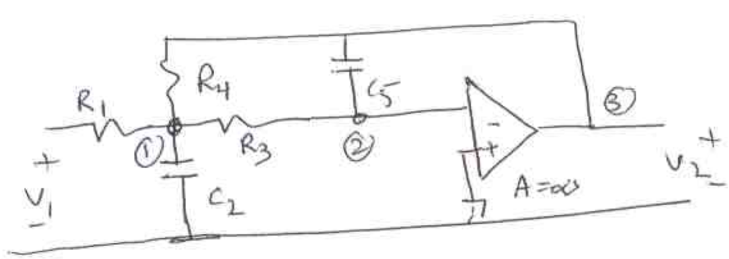
$$\frac{V_2}{V_1} = \frac{KG_1C_3\ s}{s^2C_3C_5 + s\left\{C_3(G_1+G_4) + C_3G_2(1-K) + C_5G_4\right\} + G_4(G_1+G_2)}$$

X

2.6 . Follow the method in P 2.5

X

2.7



$$\begin{array}{cccc} & (1) & (2) & (3) \\ & & & \end{array}$$

$$\begin{bmatrix} G_1+G_4+G_3+sC_2 & -G_3 & -G_4 \\ -G_3 & G_3+sC_5 & -sC_5 \\ -G_4 & -sC_5 & sC_5+G_4 \end{bmatrix} \begin{bmatrix} V_{(1)} \\ V_{(2)} \\ V_{(3)} \end{bmatrix} = \begin{bmatrix} \overset{I_s}{V_1G_1} \\ 0 \\ 0 \end{bmatrix}$$

2.7 Since OA gain $\to \infty$, $V_{②} \to 0$.

Discarding column ~~row~~ for $V_{②}$ and then row for $V_{③}$

$$\begin{pmatrix} G_1 + G_4 + G_3 + sC_2 & -G_4 \\ -G_3 & -sC_5 \end{pmatrix} \begin{pmatrix} V_{①} \\ V_{③} \end{pmatrix} = \begin{pmatrix} V_1 G_1 \\ 0 \end{pmatrix}$$

$$\Delta = -sC_5 \left(G_1 + G_3 + G_4 + sC_2 \right) - G_3 G_4$$
$$= -\left[s^2 C_2 C_5 + s\, C_5 (G_1 + G_3 + G_4) + G_3 G_4 \right]$$

$$V_{③} = \frac{1}{\Delta} \cdot \begin{vmatrix} G_1 + G_4 + G_3 + sC_2 & V_1 G_1 \\ -G_3 & 0 \end{vmatrix} = -V_1 \frac{G_1 G_3}{s^2 C_2 C_5 + s C_5 (G_1 + G_3 + G_4) + G_3 G_4}$$

So $$\frac{V_2}{V_1} = \frac{V_{③}}{V_1} = -\frac{G_1 G_3}{s^2 C_2 C_5 + s\, C_5 (G_1 + G_3 + G_4) + G_3 G_4}$$

——— × ———

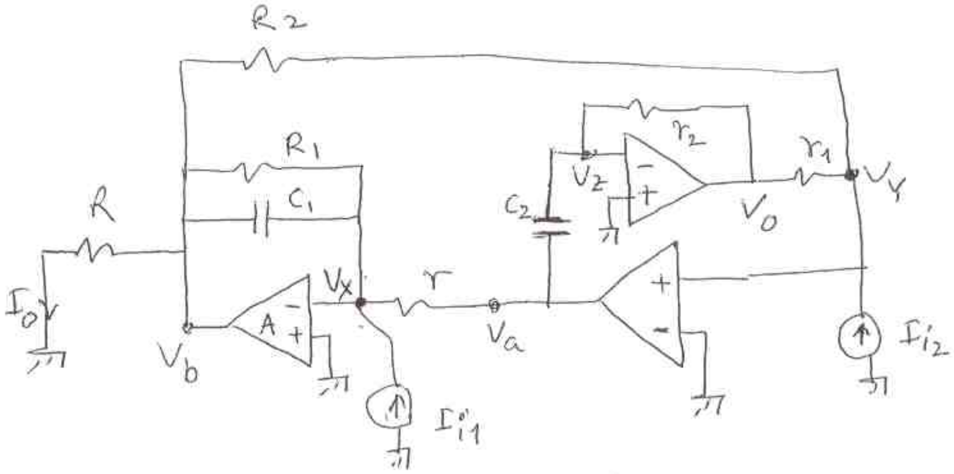
2·8 Follow as in P2.7

——— × ———

2·9 See Ch5 P5.29

——— × ———

2.10



At V_X node: $-I_{i1} - (V_a - V_x)g - (G_1+sC_1)(V_b - V_x) = 0$

or $I_{i1} = -V_a g + V_x g - (G_1+sC_1)\left(V_b + \frac{V_b}{A}\right)$

(1) $I_{i1} = -V_a g + V_x g - V_b\left(1+\frac{1}{A}\right)(G_1+sC_1)$; $V_x = -\frac{V_b}{A}$

$= -V_a g - V_b\left[\frac{g}{A} + \left(1+\frac{1}{A}\right)(G_1+sC_1)\right]$

At V_Y node: $-(V_b - V_Y)G_2 - (V_o - V_Y)g_1 - I_{i2} = 0$

(2) or $I_{i2} = -V_b G_2 + V_Y G_2 - V_o g_1 + V_Y g_1 = -V_b G_2 - V_o g_1 + \frac{V_a}{A}(G_2+g_1)$

At V_Z node: $+(V_a - V_Z)sC_2 + (V_o - V_Z)g_2 = 0$ But $V_Z = -\frac{V_o}{A}$

or $V_a sC_2 - V_Z sC_2 + V_o g_2 - V_Z g_2 = 0$

$V_a sC_2 + V_o g_2 + (g_2 + sC_2)\frac{V_o}{A} = 0.$

(3) or $V_a sC_2 + V_o\left(g_2 + \frac{g_2+sC_2}{A}\right) = 0.$

From (3) $V_o = -V_a \dfrac{sC_2}{g_2 + \frac{g_2+sC_2}{A}}$

From (1) with $I_{i1} = 0$ (assuming only one input source I_{i2})

$$V_a = -V_b\left[\frac{1}{A} + \frac{G_1+sC_1}{g}\left(1+\frac{1}{A}\right)\right]$$

So $V_o = +V_b\left[\frac{1}{A} + \frac{G_1+sC_1}{g}\left(1+\frac{1}{A}\right)\right] \dfrac{sC_2}{g_2 + \frac{g_2+sC_2}{A}}$

2.10 cont.

Subst. in (2)

$$I_{i2} = -V_b G_2 - V_b g_1\left[\frac{1}{A} + \frac{G_1+sC_1}{g}\left(1+\frac{1}{A}\right)\right]\left(\frac{sC_2}{g_2 + \frac{g_2+sC_2}{A}}\right)$$

$$- V_b \cdot \left[\frac{1}{A} + \frac{G_1+sC_1}{g}\left(1+\frac{1}{A}\right)\right]\frac{G_2+g_1}{A}$$

$$-I_{i2} = V_b\left\{G_2 + g_1\left[\frac{1}{A} + \frac{G_1+sC_1}{g}\left(1+\frac{1}{A}\right)\right]\left[\frac{sC_2}{g_2 + \frac{g_2+sC_2}{A}}\right] + \left(\frac{1}{A} + \frac{G_1+sC_1}{g}\left(1+\frac{1}{A}\right)\right)\frac{G_2+g_1}{A}\right\}$$

$$V_b = -\frac{I_{i2}}{D}$$

where $D = G_2 + g_1\left[\frac{1}{A} + \frac{G_1+sC_1}{g}\left(1+\frac{1}{A}\right)\right]\frac{sC_2}{g_2 + \frac{g_2+sC_2}{A}} + \left[\frac{1}{A} + \frac{G_1+sC_1}{g}\left(1+\frac{1}{A}\right)\right]\frac{G_2+g_1}{A}$

$$I_o = V_b G$$

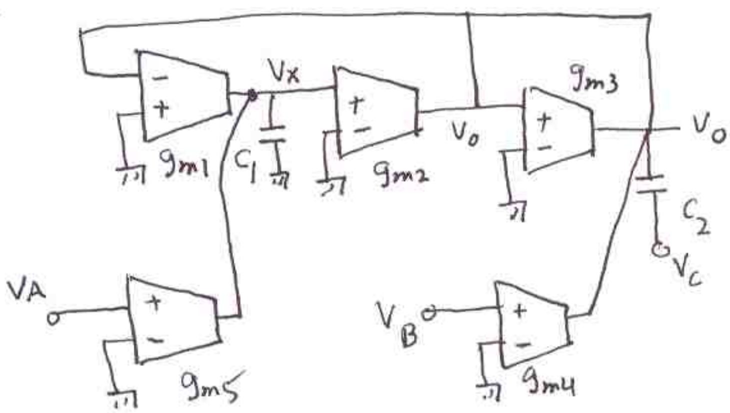
So $$\frac{I_o}{I_{i2}} = -\frac{G}{D}.$$

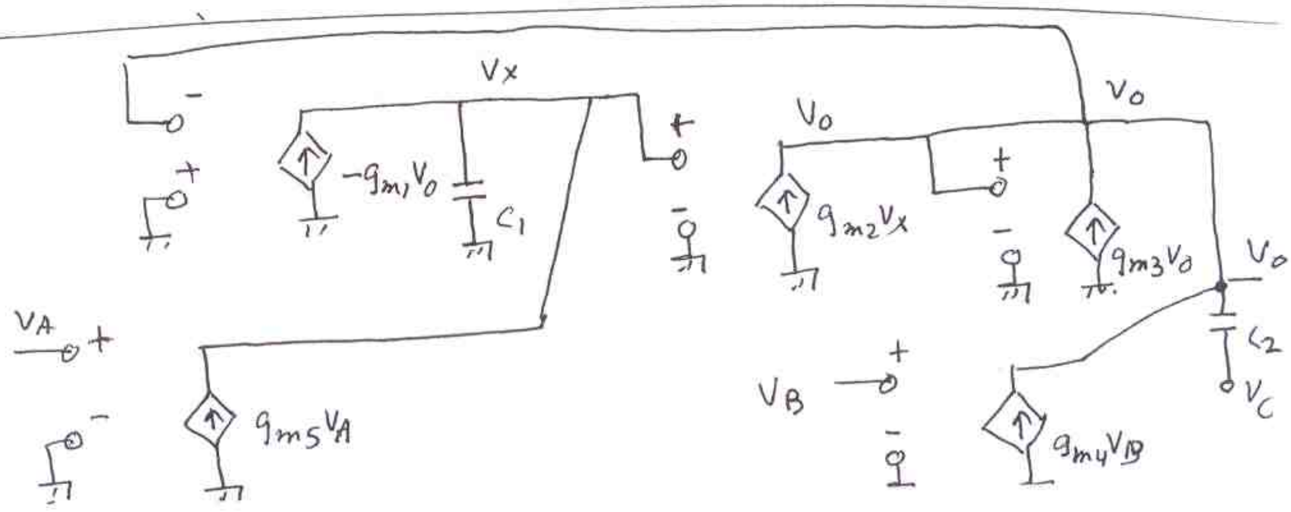
$$= -\frac{1/R}{\frac{1}{R_2} + \frac{1}{r_1}\left[\frac{1}{A} + \frac{\frac{1}{R_1}+sC_1}{\frac{1}{r}}\left(1+\frac{1}{A}\right)\right]\frac{sC_2}{\frac{1}{r_2} + \frac{\frac{1}{r_2}+sC_2}{A}} + \left(\frac{1}{A} + \frac{\frac{1}{R_1}+sC_1}{\frac{1}{r}}\left(1+\frac{1}{A}\right)\right]\frac{1/R_2+1/r_1}{A}}$$

$$\frac{I_o}{I_{i2}} = -\frac{1/R}{\frac{1}{R_2} + \left[\frac{1}{A} + \frac{\frac{1}{R_1}+sC_1}{\frac{1}{r}}\left(1+\frac{1}{A}\right)\right]\left[\frac{sC_2/r_1}{\frac{1}{r_2} + \frac{\frac{1}{r_2}+sC_2}{A}} + \frac{1/R_2+1/r_1}{A}\right]}$$

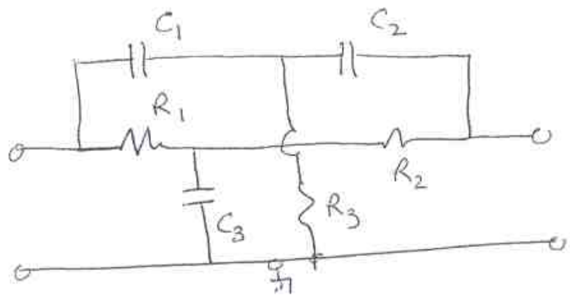
Multiplying num. & denom. by $\frac{1}{r}$, we get the result shown in the book.

2.11

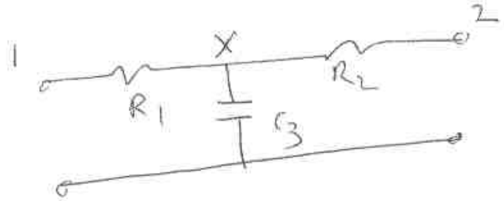

Vx
gm3
Vo
Vo
gm1
C1
gm2
C2
Vc
VA
VB
gm5
gm4


Vx
Vo
Vo
$-g_{m1}V_o$
C1
$g_{m2}V_x$
$g_{m3}V_o$
Vo
C2
VA
VB
Vc
$g_{m5}V_A$
$g_{m4}V_B$

2.12

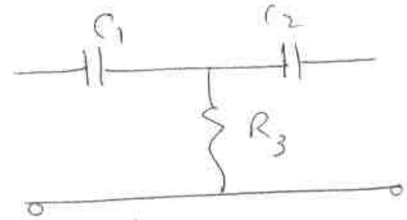


The subset of 2-port parameters will be the Y-parameters of



$[Y]_1$

and



$[Y]_2$

Then overall $[Y] = [Y]_1 + [Y]_2$

For $[Y]_1 \rightarrow$

$$\begin{array}{c} \text{X} \\ \begin{bmatrix} G_1 & -G_1 & 0 \\ -G_1 & G_1+G_2+sC_3 & -G_2 \\ 0 & -G_2 & G_2 \end{bmatrix} \end{array}$$

. Then suppress node X

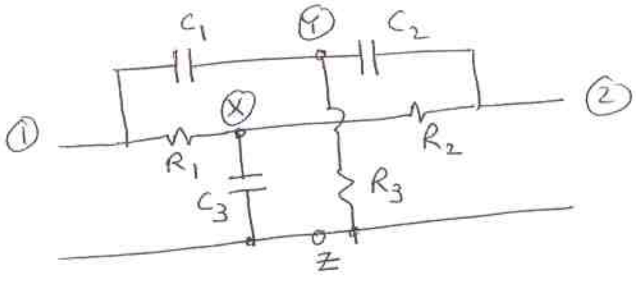
$$\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} - \frac{1}{G_1+G_2+sC_3} \begin{bmatrix} -G_1 \\ -G_2 \end{bmatrix} \begin{bmatrix} -G_1 & -G_2 \end{bmatrix}$$

$$[Y]_1 \rightarrow \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} - \frac{1}{G_1+G_2+sC_3} \begin{bmatrix} G_1^2 & G_1G_2 \\ G_1G_2 & G_2^2 \end{bmatrix}$$

continue with $[Y]_2$ in a similar way.

let $[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$, then VTF is $\left.\frac{V_2}{V_1}\right|_{I_2=0} = -\frac{Y_{21}}{Y_{22}}$

2.13



$$
\begin{array}{c}
\begin{array}{ccccc} \quad 1 \quad & \quad X \quad & \quad Y \quad & \quad Z \quad & \quad 2 \quad \end{array} \\
\begin{bmatrix}
G_1+sC_1 & -G_1 & -sC_1 & 0 & 0 \\
-G_1 & G_1+G_2+sC_3 & 0 & -sC_3 & -G_2 \\
-sC_1 & 0 & sC_1+sC_2+G_3 & -G_3 & -sC_2 \\
0 & -sC_3 & -G_3 & G_3+sC_3 & 0 \\
0 & -G_2 & -sC_2 & 0 & G_2+sC_2
\end{bmatrix}
\end{array}
$$

If Z is grounded, discard row & column for Z

$$
\begin{array}{c}
\begin{array}{cccc} \quad 1 \quad & \quad X \quad & \quad Y \quad & \quad 2 \quad \end{array} \\
\begin{bmatrix}
G_1+sC_1 & -G_1 & -sC_1 & 0 \\
-G_1 & G_1+G_2+sC_3 & 0 & -G_2 \\
-sC_1 & 0 & sC_1+sC_2+G_3 & -sC_2 \\
0 & -G_2 & -sC_2 & G_2+sC_2
\end{bmatrix}
\end{array}
$$

Suppress X

$$
\begin{bmatrix}
G_1+sC_1 & -sC_1 & 0 \\
-sC_1 & sC_1+sC_2+G_3 & -sC_2 \\
0 & -sC_2 & G_2+sC_2
\end{bmatrix}
- \frac{1}{G_1+G_2+sC_3}
\begin{bmatrix} -G_1 \\ 0 \\ -G_2 \end{bmatrix}
\begin{bmatrix} -G_1 & 0 & -G_2 \end{bmatrix}
$$

$$
= \begin{bmatrix}
G_1+sC_1 & -sC_1 & 0 \\
-sC_1 & sC_1+sC_2+G_3 & -sC_2 \\
0 & -sC_2 & G_2+sC_2
\end{bmatrix}
- \frac{1}{G_1+G_2+sC_3}
\begin{bmatrix}
G_1^2 & 0 & G_1G_2 \\
0 & 0 & 0 \\
G_1G_2 & 0 & G_2^2
\end{bmatrix}
$$

2,13 (contd.)

$[Y]$

$$\rightarrow \begin{array}{ccc} ① & ④ & ② \end{array}$$

$$\begin{bmatrix} G_1+sC_1-\dfrac{G_1^2}{G_1+G_2+sC_3} & -sC_1 & -\dfrac{G_1G_2}{G_1+G_2+sC_3} \\ -sC_1 & sC_1+sC_2+G_3 & -sC_2 \\ -\dfrac{G_1G_2}{G_1+G_2+sC_3} & -sC_2 & G_2+sC_2-\dfrac{G_2^2}{G_1+G_2+sC_3} \end{bmatrix}$$

Suppress ④

$$\begin{bmatrix} G_1+sC_1-\dfrac{G_1^2}{G_1+G_2+sC_3} & -\dfrac{G_1G_2}{G_1+G_2+sC_3} \\ -\dfrac{G_1G_2}{G_1+G_2+sC_3} & G_2+sC_2-\dfrac{G_2^2}{G_1+G_2+sC_3} \end{bmatrix} - \frac{1}{sC_1+sC_2+G_3}\begin{bmatrix} -sC_1 \\ -sC_2 \end{bmatrix}\begin{bmatrix} -sC_1 & -sC_2 \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}, \text{ where}$$

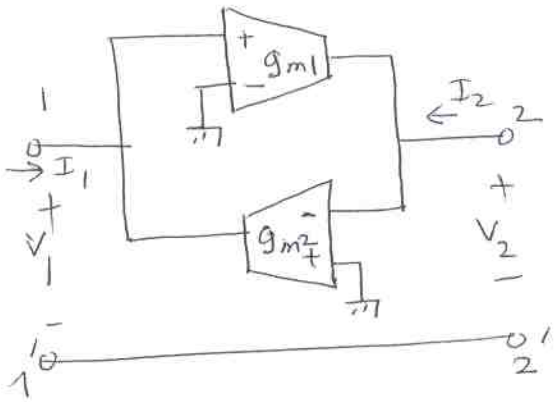
$$Y_{11} = \frac{G_1G_2 + s\left(G_1C_3+G_1C_1+G_2C_1\right)+s^2C_1C_3}{G_1+G_2+sC_3} - \frac{s^2C_1^2}{s(C_1+C_2)+G_3}$$

etc.

Voltage transfer function $-Y_{21}/Y_{22}$

$$= \frac{s^3C_1C_2C_3 + s^2\left(C_1C_2G_1+C_1C_2G_2\right)+\left(C_1G_1G_2+C_2G_1G_2\right)s + G_1G_2G_3}{s^3C_1C_2C_3 + s^2\left(G_2C_2C_3+G_1C_1C_2+G_2C_1C_3+G_2C_1C_2+G_3C_2C_3\right) + s\left(G_1G_2C_1+G_1G_2C_2+G_1G_3C_2+G_2G_3C_3+G_2G_3C_2\right)+G_1G_2G_3}$$

2.14



$I_2 = -g_{m1} V_1$

$I_1 = g_{m2} V_2$

Thus

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{g_{m2}} \\ -g_{m1} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

(a) From Table 2.2, the [a] matrix is

$$\begin{bmatrix} 0 & -\frac{1}{g_{m2}} \\ -g_{m1} & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & \frac{1}{G_1} \\ G_2 & 0 \end{bmatrix}; \text{ Then } \frac{G_2}{G_1} = \frac{g_{m1}}{g_{m2}} > 0.$$

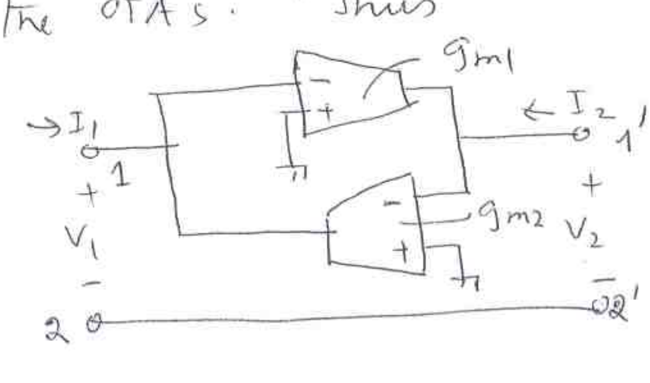
So it is a PII (positive impedance inverter).

(b) With Z_L connected at ②, $V_2 = -I_2 Z_L = +g_{m1} V_1 Z_L$

But $V_2 = \frac{I_1}{g_{m2}} = +g_{m1} V_1 Z_L$. Then $\frac{V_1}{I_1} = \frac{Y_L}{g_{m1} g_{m2}} = Z_1$

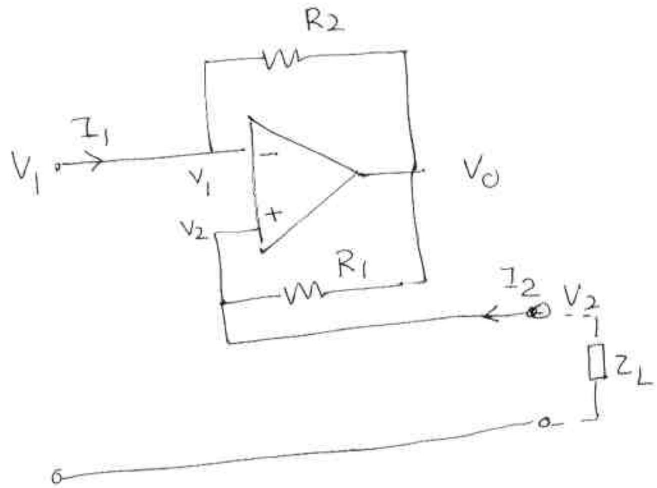
(c) If $Z_L = \frac{1}{sC}$; $Z_1 = Z_{in} = \frac{sC}{g_{m1} g_{m2}} \rightarrow$ inductor $L = \frac{C}{g_{m1} g_{m2}}$

2.15 In Table 2.2, we see $G_2/G_1 < 0$ produces an NII. Thus in problem P2.14, g_{m1} and g_{m2} should be such that $-g_{m1}$ and $-\frac{1}{g_{m2}}$ be of opposite sign. This can be easily achieved by reversing the input terminals of one of the OTAs. Thus



is a possible NII system.

2.16.



$$V_1 - V_2 \simeq 0 \quad \therefore \quad V_1 = V_2$$

$$V_1 - V_o = I_1 R_2 \quad , \quad V_2 - V_o = I_2 R_1$$

$$\therefore \quad V_1 - V_o = I_1 R_2 = I_2 R_1$$

$$I_2 = + \frac{R_2}{R_1} I_1$$

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{R_2}{R_1} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\therefore [a] = \begin{bmatrix} 1 & 0 \\ 0 & -R_2/R_1 \end{bmatrix}$$, which is the $[a]$ of a CNIC (page 17)

Hence The given network represents a CNIC

If Z_L is connected at port 2, $V_2 = -I_2 Z_L$

$$\therefore \quad V_2 = -\frac{R_2}{R_1} Z_L \cdot I_1$$

Since $V_1 = V_2$, we get $V_1 = -\left(\frac{R_2}{R_1}\right) Z_L I_1$

$$\text{or} \quad Z_{in} = \frac{V_1}{I_1} = -\left(\frac{R_2}{R_1}\right) Z_L$$

Hence Z_{in} corresponds to a negative impedance.

———— x ————

(2.17): Assume $A \to \infty$ and work out.