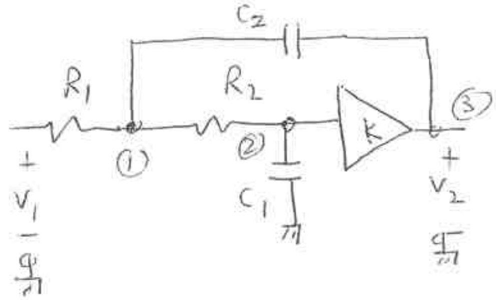
CHAPTER – 3 (Sol./hint to Problems)

3.1

(a)



$$\begin{bmatrix} G_1+G_2+sC_2 & -G_2 & -sC_2 \\ -G_2 & G_2+sC_1 & 0 \\ -sC_2 & 0 & sC_2 \end{bmatrix}\begin{bmatrix} V① \\ V② \\ V③ \end{bmatrix} = \begin{bmatrix} V_1G_1 \\ 0 \\ 0 \end{bmatrix}$$

$\because \; V③ = K\,V②$

$$\begin{bmatrix} G_1+G_2+sC_2 & -G_2-KsC_2 \\ -G_2 & G_2+sC_1 \end{bmatrix}\begin{bmatrix} V① \\ V② \end{bmatrix} = \begin{bmatrix} V_1G_1 \\ 0 \end{bmatrix}$$

output node of VCVS: $\cancel{-sC_2 \qquad 0+KsC_2}$

$$\Delta = (G_1+G_2+sC_2)(G_2+sC_1) - (G_2+sKC_2)G_2$$
$$= s^2C_1C_2 + \{C_1G_1 + C_1G_2 + (1-K)C_2G_2\}s + G_1G_2$$

$$V② = \frac{1}{\Delta}\cdot\begin{vmatrix} G_1+G_2+sC_2 & V_1G_1 \\ -G_2 & 0 \end{vmatrix} = +V_1\frac{G_1G_2}{\Delta}.$$

Using the values given

$$\frac{V②}{V_1} = \frac{1.3}{s^2\cdot 1 + \{3+1+(1-2)1\}s + 3} = \frac{3}{s^2+3s+3}$$

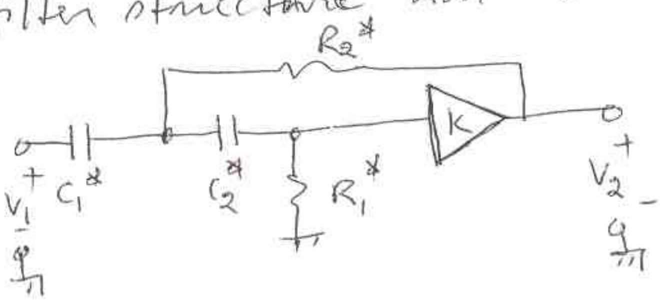
$\frac{V_2}{V_1} = K\frac{V②}{V_1}$

$\omega_p = \sqrt{3}, \quad \frac{\omega_p}{Q_p} = 3; \quad Q_p = \frac{\omega_p}{3} = \frac{1}{\sqrt{3}}$

3.1 (b) $\quad s \to \frac{1}{s}$, The HP function becomes

$$\frac{6}{s^2+3s+3} \xleftrightarrow[LP \leftrightarrow HP]{} \frac{6s^2}{3s^2+3s+1}, \quad \text{subst. } K = 2$$

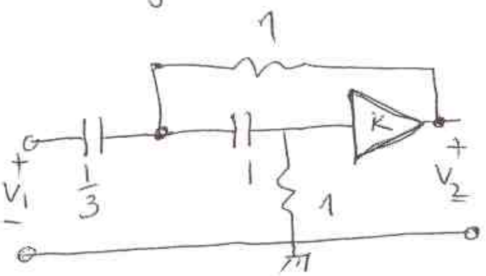
The filter structure will be

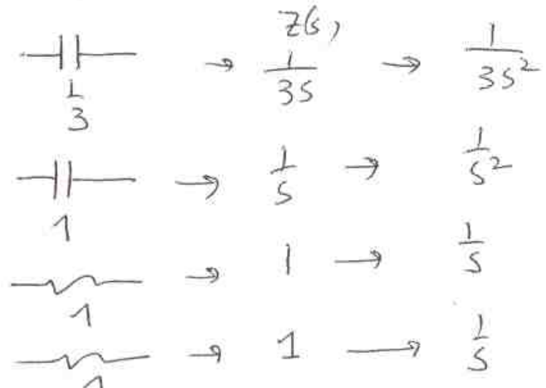


$C_1^* = \frac{1}{3}$
$C_2^* = 1$
$R_1^* = R_2^* = 1$

3.1 (c) By impedance transformation the T.F does not change. So it is:

$$\frac{6s^2}{3s^2+3s+1}$$





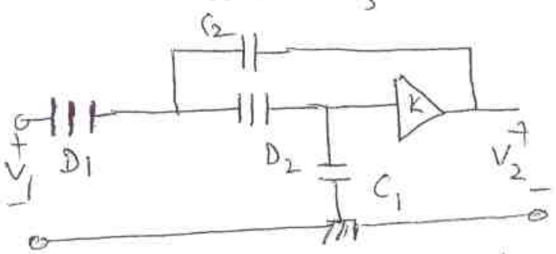
The capacitances change to frequency dependent reactances (negative resistance) and resistors change to capacitances.

Letting $D_1(s) = \frac{1}{3s^2} \rightarrow$ D_1

$D_2(s) = \frac{1}{s^2} \rightarrow$ D_2 . .

$$H(s) = \frac{2s^2}{s^2+s+\frac{1}{3}}$$

$\frac{\omega_p}{Q_p}$ ↗ ω_p^2 ↗



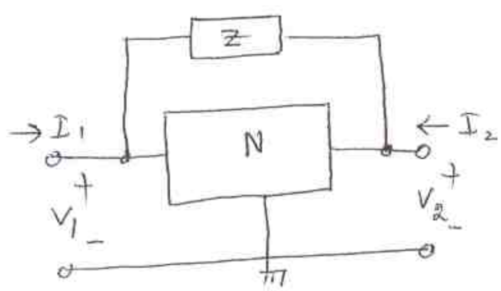
$\omega_p = \frac{1}{\sqrt{3}}$

$\frac{\omega_p}{Q_p} = 1$; $Q_p = \frac{\omega_p}{1} = \frac{1}{\sqrt{3}}$

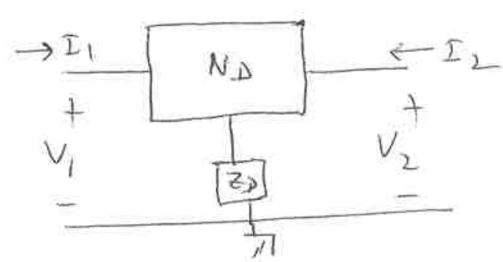
3 (d) $\omega_p = \frac{1}{\sqrt{3}}$; $Q_p = \frac{1}{\sqrt{3}}$

x

3.2



(a)



(b)

$Z \cdot Z_D = f(s)$; $Z = \frac{f(s)}{Z_D}$ & $Z_D = \frac{f(s)}{Z}$

$[N] \cdot [N_D] = f(s)$.

Then

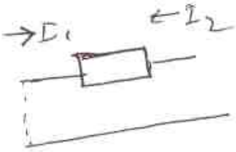
3.2/cont. $[a]_N = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$; $[a]_{N_D} = \begin{bmatrix} D & C\,f(s) \\ \frac{B}{f(s)} & A \end{bmatrix}$

In (a), [N] and Z are in parallel.

So $[Y]_{N \| Z} = [Y]_N + [Y]_Z = [Y]_N + \begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix}$

$$= \begin{bmatrix} \frac{D}{B} & -\frac{\Delta_a}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix} + \begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix} \qquad \underline{\Delta_a = AD - BC}$$

$$= \begin{bmatrix} \left(\frac{D}{B} + \frac{1}{Z}\right) & -\left(\frac{\Delta_a}{B} + \frac{1}{Z}\right) \\ -\left(\frac{1}{B} + \frac{1}{Z}\right) & \left(\frac{A}{B} + \frac{1}{Z}\right) \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$



Then $[a]_{N \| Z} = \begin{bmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta_y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{bmatrix}$, $\Delta_y = Y_{11} Y_{22} - Y_{12} Y_{21}$

For Fig(b), the two networks are in series

So $[Z]_{N_D + Z_D} = \begin{bmatrix} \frac{A_D}{C_D} & \frac{\Delta_{aD}}{C} \\ \frac{1}{C_D} & \frac{D_D}{C_D} \end{bmatrix} + \begin{bmatrix} Z_D & Z_D \\ Z_D & Z_D \end{bmatrix}$

note: $\Delta_{aD} \equiv \Delta_a$
$= AD - BC$

$$= \begin{bmatrix} \frac{D f(s)}{B} & \frac{\Delta_a . f(s)}{B} \\ \frac{f(s)}{B} & \frac{A f(s)}{B} \end{bmatrix} + \begin{bmatrix} Z_D & Z_D \\ Z_D & Z_D \end{bmatrix} \qquad Z_D = \frac{f}{Z}$$

$$= \begin{bmatrix} \frac{D}{B} f(s) + Z_D & \frac{\Delta_a}{B} f(s) + Z_D \\ \frac{f(s)}{B} + Z_D & \frac{A}{B} f(s) + Z_D \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

3.2 cont'd. Then

$$[a]_{N_D + Z_D} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta_Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

$\Delta_Z = Z_{11} Z_{22} - Z_{21} Z_{12}$

Using the original notation:

Let $AP_{ij} = (i,j)^{th}$ element of $N \| Z$

$AS_{ij} = (i,j)^{th}$ element of $N_D + Z_D$

$$AP_{11} = \frac{AZ + B}{Z + B}$$

$$AP_{12} = \frac{BZ}{Z + B}$$

$$AP_{21} = \frac{DZA + BD + AB - Z\Delta_a - B\Delta_a - B}{B(Z+B)}.$$

$$AP_{22} = \frac{DZ + B}{Z + B}$$

$$AS_{11} = \frac{DZ + B}{Z + B} \; ; \; AS_{12} = f \frac{(ZAD + BD + AB - Z\Delta_a - B\Delta_a - B)}{B(Z+B)}$$

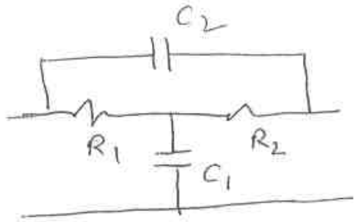
$$AS_{21} = \frac{BZ}{f(Z+B)} \; ; \; AS_{22} = \frac{AZ + B}{Z + B}.$$

Clearly, $AS_{11} = AP_{22}$; $AS_{12} = f\, AP_{21}$; $AS_{21} = \frac{1}{f} AP_{12}$

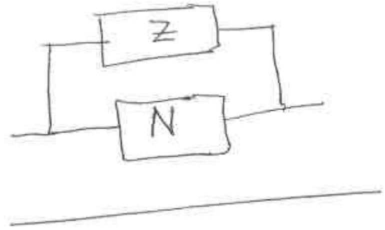
and $AS_{22} = AP_{11}$

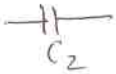
Hence $[N_D + Z_D]$ is dual of $[N \| Z]$

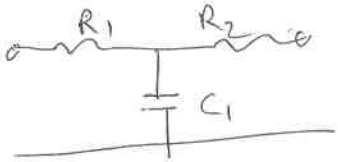
3.3

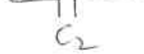


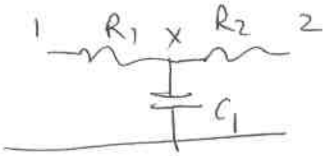
$\Rightarrow$



$Z \to$  ; in shunt with

$N \to$ 

For  , the Y-matrix is: $\begin{bmatrix} sC_2 & -sC_2 \\ -sC_2 & sC_2 \end{bmatrix}$

For  , the Y-matrix is calculated as:

$$\begin{array}{ccc} ① & ⓧ & ② \\ \end{array}$$

$$\left[\begin{array}{c|c|c} G_1 & -G_1 & 0 \\ \hline -G_1 & G_1+G_2+sC_1 & -G_2 \\ \hline 0 & -G_2 & G_2 \end{array}\right] \to \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} - \frac{1}{G_1+G_2+sC_1} \underset{2\times1}{\begin{bmatrix} -G_1 \\ -G_2 \end{bmatrix}} \underset{1\times2}{\begin{bmatrix} -G_1 & -G_2 \end{bmatrix}}$$

$$= \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} - \begin{bmatrix} G_1^2 & G_1G_2 \\ G_1G_2 & G_2^2 \end{bmatrix} \frac{1}{(G_1+G_2+sC_1)}$$

$$= \begin{bmatrix} G_1 - \dfrac{G_1^2}{G_1+G_2+sC_1} & -\dfrac{G_1G_2}{G_1+G_2+sC_1} \\ -\dfrac{G_1G_2}{G_1+G_2+sC_1} & G_2 - \dfrac{G_2^2}{G_1+G_2+sC_1} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{G_1(G_2+sC_1)}{G_1+G_2+sC_1} & -\dfrac{G_1G_2}{G_1+G_2+sC_1} \\ -\dfrac{G_1G_2}{G_1+G_2+sC_1} & \dfrac{G_2(G_1+sC_1)}{G_1+G_2+sC_1} \end{bmatrix}$$

3.3 (cont.) Let $Y_N = \begin{bmatrix} Y_{N11} & Y_{N12} \\ Y_{N21} & Y_{N22} \end{bmatrix}$

with $Y_{N11} = \dfrac{G_1(G_2+sC_1)}{G_1+G_2+sC_1}$; $Y_{N12} = -\dfrac{G_1G_2}{G_1+G_2+sC_1}$

$Y_{N21} = -\dfrac{G_1G_2}{G_1+G_2+sC_1}$; $Y_{N22} = \dfrac{G_2(G_1+sC_1)}{G_1+G_2+sC_1}$

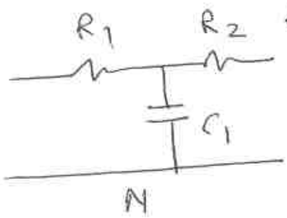
Then $[a]_N = \begin{bmatrix} -\dfrac{Y_{N22}}{Y_{N21}} & \dfrac{-1}{Y_{N21}} \\ -\dfrac{\Delta Y}{Y_{N21}} & \dfrac{-Y_{N11}}{Y_{N21}} \end{bmatrix}$

$[a]_N = \begin{bmatrix} \dfrac{G_1+sC_1}{G_1} & \dfrac{G_1+G_2+sC_1}{G_1G_2} \\ sC_1 & \dfrac{G_2+sC_1}{G_2} \end{bmatrix}$ $\quad [a]_{ND} \Rightarrow \begin{bmatrix} D & Cf(s) \\ \dfrac{B}{f(s)} & A \end{bmatrix}$

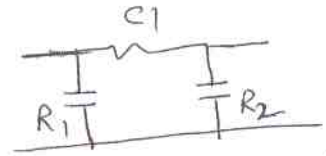
$[a]_{ND} = \begin{bmatrix} \dfrac{G_2+sC_1}{G_2} & C_1 \\ \dfrac{(G_1+G_2+sC_1)s}{G_1G_2} & \dfrac{G_1+sC_1}{G_1} \end{bmatrix}$ using $f(s) = \frac{1}{s}$.

$[z]_{ND} = \begin{bmatrix} \dfrac{(G_2+sC_1)G_1}{(G_1+G_2+sC_1)s} & \dfrac{G_1G_2}{(G_1+G_2+sC_1)s} \\ \dfrac{G_1G_2}{(G_1+G_2+sC_1)s} & \dfrac{(G_1+sC_1)G_2}{(G_1+G_2+sC_1)s} \end{bmatrix}$

3.3 contd. Adopting the method of element by element substitution for capacitive dual, we can infer that the capacitive dual of



will be



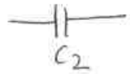
N_D ← capacitive dual

For N_D we can find $z_{11} = \frac{1}{sR_1} \| \left[C_1 + \frac{1}{sR_2}\right] = \frac{G_1}{s} \| \left[C_1 + \frac{G_2}{s}\right]$

$$z_{11} = \frac{G_1(G_2 + sC_1)}{(G_1+G_2+sC_1)s} \quad ; \quad z_{22} = \frac{G_2(G_1+sC_1)}{(G_1+G_2+sC_1)s}$$

$$z_{12} = z_{21} = \frac{G_1 G_2}{(G_1+G_2+sC_1)s}$$

These are same as the z-elements of $[z]_{N_D}$

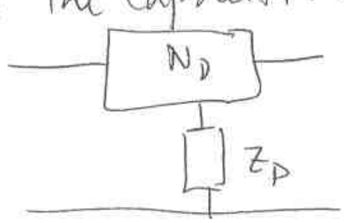
For  $\Rightarrow sC_2 = Y = \begin{bmatrix} sC_2 & -sC_2 \\ -sC_2 & sC_2 \end{bmatrix}$

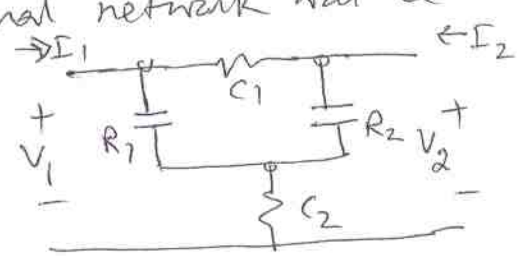
$$[a]_{z=\frac{1}{sC_2}} = \begin{bmatrix} -\frac{sC_2}{-sC_2} & \frac{1}{sC_2} \\ 0 & \frac{-sC_2}{-sC_2} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{sC_2} \\ 0 & 1 \end{bmatrix}$$

$$[a]_D = \begin{bmatrix} 1 & 0 \cdot f(s) \\ \frac{1}{sC_2} \cdot \frac{1}{f(s)} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{C_2} & 1 \end{bmatrix}, \text{ using } f(s) = \frac{1}{s}.$$

$$[z]_D = \begin{pmatrix} C_2 & C_2 \\ C_2 & C_2 \end{pmatrix} \rightarrow \text{a resistance of value } C_2$$

Hence the capacitive dual network will be





3.4.

(a) $[a]_{VCVS} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $[a]_{r_m} = \begin{bmatrix} 1 & r_m \\ 0 & 1 \end{bmatrix}$

$[a]_{CCCS} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore$ The chain matrix of the over all network is,

$$[a]_N = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & r_m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & r_m \\ 0 & 0 \end{bmatrix}$$

This corresponds to a VCCS ~~of~~ with a transadmittance $g_m = 1/r_m$.

(b) Using Theorem 3.2, the chain matrix of dual of N is

$$[a]_{N_D} = \begin{bmatrix} 0 & 0 \\ \frac{r_m}{r_m r_n} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1/r_n & 0 \end{bmatrix}$$

$\therefore$ The dual of N, N_D is nothing but a CCVS of transresistance r_n.

Let us now see what the different subnetworks.

For The VCVS, the chain matrix of dual is $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ which is a unity gain CCCS.

3.4 (Contd.)

For the series resistor r_m, the dual element's chain matrix is

$$\begin{bmatrix} 1 & 0 \\ \frac{r_m}{r_m r_n} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/r_n & 1 \end{bmatrix}$$

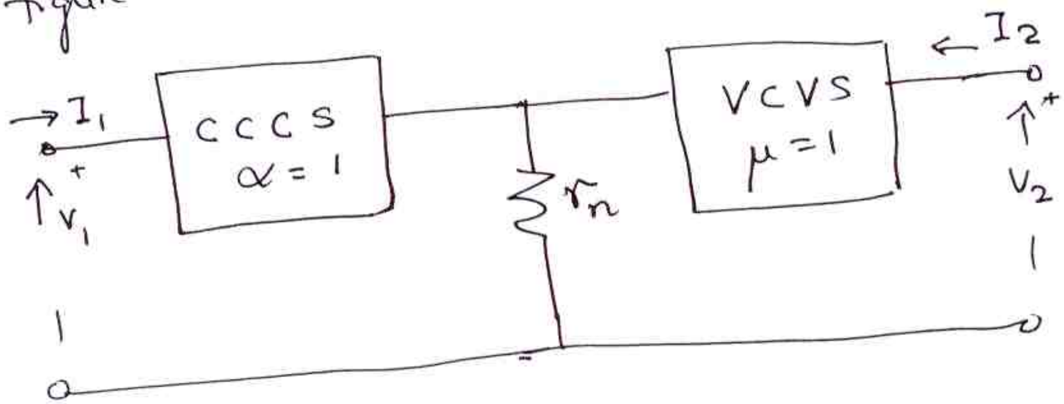
This corresponds to a shunt resistor of value r_n

Finally, the dual of the CCCS has the chain matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

which corresponds to a VCVS of unity gain.

Thus, from Theorem 3.1, we see that a CCVS of transresistance r_n can be realized as a cascade combination of a unity gain CCCS, a shunt resistor of value r_n and a VCCS of unity gain, as shown in the figure below.



3.4 (contd.)

(c) The chain matrix of the transpose reversed network is given by

$$[a]_{N_R^T} = \begin{bmatrix} D & B \\ C & A \end{bmatrix}$$

if $[a]_N = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Since N is a VCCS of transadmittance $1/r_m$ with a chain matrix $\begin{bmatrix} 0 & r_m \\ 0 & 0 \end{bmatrix}$, its transposed reverse has the chain matrix $\begin{bmatrix} 0 & r_m \\ 0 & 0 \end{bmatrix}$. This is nothing but a VCCS. Thus the ~~dual~~ transpose of a VCCS is itself with its input and output reverse

(d) $[a]_{N_D} = \begin{bmatrix} 0 & 0 \\ 1/r_n & 0 \end{bmatrix}$

$\therefore$ The chain matrix of $[N_D]_T^R$ is given by

$$\begin{bmatrix} 0 & 0 \\ 1/r_n & 0 \end{bmatrix}$$

which is a CCVS of transresistance r_n.

~~Hence, the transposed reverse network of a~~

Hence, the transpose of a CCVS is itself with its input and output ports reversed.

3.5

(a) $$[a]_{VCCS} = \begin{bmatrix} 0 & 1/g_m \\ 0 & 0 \end{bmatrix}$$

$$[a]_{CCVS} = \begin{bmatrix} 0 & 0 \\ 1/r_n & 0 \end{bmatrix}$$

$$\therefore [a]_N = \begin{bmatrix} 0 & 1/g_m \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1/r_n & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{g_m r_n} & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} r_m/r_n & 0 \\ 0 & 0 \end{bmatrix}$$

This is nothing but a VCVS of gain r_n/r_m.
(In the book there is printing error, where gain is given as r_m/r_n)

(b) If the positions of the VCCS and CCVS are reversed to form a network N', then

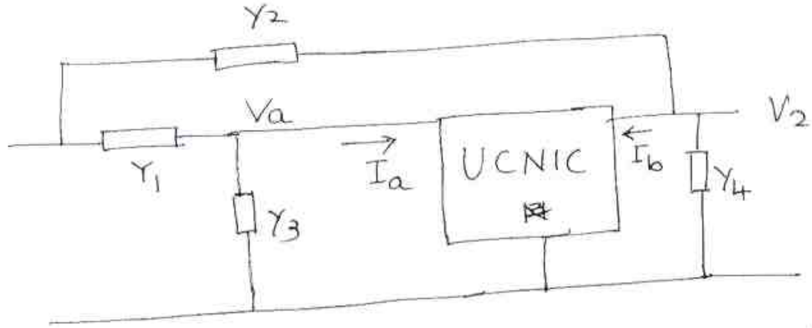
$$[a]_{N'} = \begin{bmatrix} 0 & 0 \\ 1/r_n & 0 \end{bmatrix} \begin{bmatrix} 0 & 1/g_m \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{r_m}{r_n} \end{bmatrix}$$

Thus N' corresponds to CCCS of gain r_n/r_m

(In the book there are 2 printing errors. In (b) VCVS should be VCCS and r_m/r_n should have been r_n/r_m.)

3.6

a)



Assume the UCNIC to be a 3-T device. Since its chain matrix is given by

$$[a] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

we have

$$V_a = V_2$$
$$I_a = I_b$$

Now,
$$(V_1 - V_a)\,Y_1 = V_a Y_3 + I_a$$
$$(V_1 - V_2)\,Y_2 = I_b + V_2 Y_4$$

Using $V_a = V_2$ and $I_a = I_b$, we get

$$\frac{V_2}{V_1} = \frac{Y_1 - Y_2}{(Y_1 - Y_2) + (Y_3 - Y_4)}$$

(b) The transposed network N_R^T corresponding to given network N is

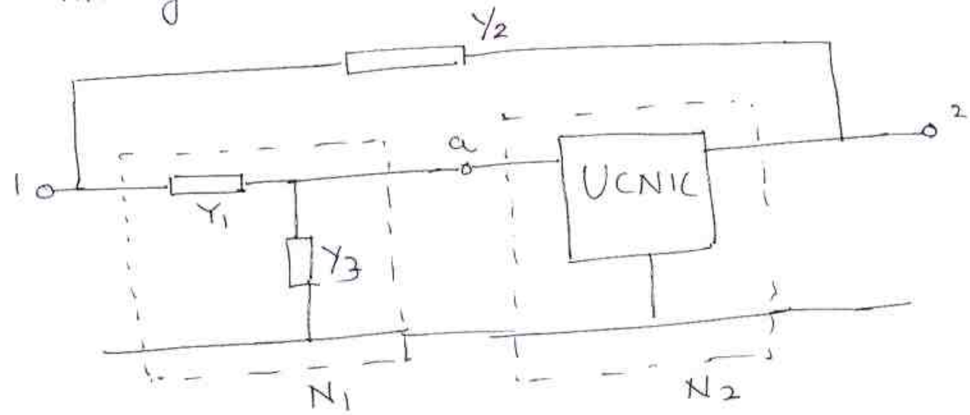
3.6 (contd.)

From Eq (3.21) we know that if $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the chain matrix of a network, then its transposed-reverse network has the chain matrix $\begin{bmatrix} D & B \\ C & A \end{bmatrix}$. Hence the [a] of the transposed-reverse element corresponding to UCNIC is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, which corresponds to a UVNIC. Using this, it can easily be shown that ~~the transpo~~ N_R^T is the ~~CTF~~ short-circuit CTF of

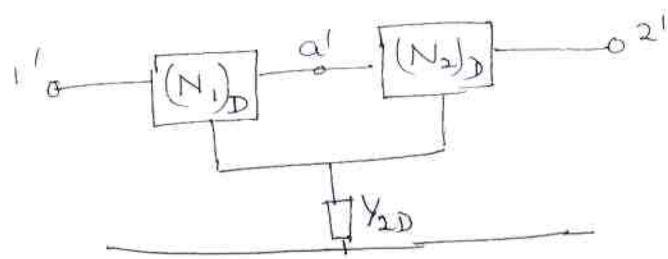
$$\frac{I_2}{I_1} = \frac{Y_1 - Y_2}{(Y_1 - Y_2) + (Y_3 - Y_4)}$$

the same as the VTF of the original network.

(c) The given network N can be rewritten as



Using Theorem 3.1 and the result of Problem P. 3.2, we see that the dual of the above is as shown below

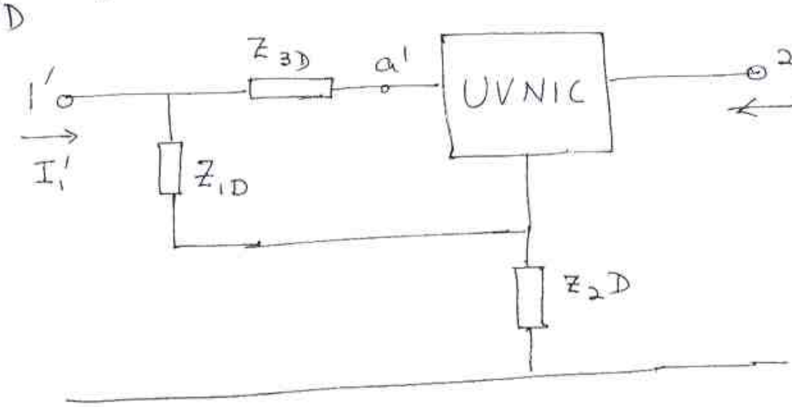


3.6 (contd.)

Assuming we are taking capacitive duals (page 45), the chain matrix of $(N_2)_D$ is given by

$$\begin{bmatrix} D & C\frac{1}{s} \\ Bs & A \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

which corresponds to UVNIC. Hence the network $(N)_D$ is as follows, where $Z_{iD} = \frac{1}{s} Y_i$



It can be shown using Kirchoff's laws that the shunt circuit CTF of the above network is the same as the open circuit VTF of the original networ

Just as N_R^T was obtained from N in section (b),

(d) we can find the structure for $(N_D)_R^T$ and show that its open circuit VTF is the same as that of the original.

3.7 $A_p = 3\,dB$, $A_s = 25\,dB$

$f_s / f_c = 4$ MFM approx. , BUT - response

$\epsilon = .997$ $\quad n = 3$

$$H_N(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$ (using Tables)

If $A_p = 1\,dB$, $\epsilon = .5088$, $n = 3$

HINTS / ANSWERS PROBLEM SET Ch.3

roots of $D(s)$:

		k
1.2526	0	0
.6263	1.0847	1
−.6263	1.0847	2
−1.253	0	3
−.6263	−1.0847	4
0.6263	−1.0847	5

$$H_N(s) = \frac{1}{\left((s+.6263)+j1.0847\right)\left((s+.6263)-j1.0847\right)(s+1.253)}$$

Chosen to make $|H(0)| = 1$.

$$= \frac{1.9657}{(s+1.253)(s^2 + 1.2526s + 1.5688)}$$

This is the normalized filter function

Alt: Use $H_N(s)$ for BUT function and let this be $H(\hat{s})$

$$H_N(\hat{s}) = \frac{1}{\hat{s}^3 + 2\hat{s}^2 + 2\hat{s} + 1}$$

Then use $s \to \hat{s}\,\epsilon^{-1/n}$ i.e. $\hat{s} = \epsilon^{\frac{1}{n}} s$

Now $\epsilon = .5088$; $(.5088)^{1/3} = .7983$

So
$$H_N(s) = \frac{1}{(.7983s)^3 + 2(.7983s)^2 + 2(.7983s) + 1}$$

$$= \frac{1.9658}{s^3 + 2.5056s^2 + 3.1386s + 1.9658}$$

which is same as:
$$\frac{1.9658}{(s+1.253)(s^2 + 1.2526s + 1.5688)}$$

3.8 BUT-LPF , so $A_p = 3$ dB

$f_c = 10^3$ Hz , $A_s = 28$ dB at $f = 2f_c$ (i.e. octave)

$\downarrow$ 3+25 dB per octave

So $A_p = 3$ dB at $f = 10^3$

$A_s = 28$ dB at $f = 2\times 10^3$ $\epsilon = .997 \approx 1.$

$n = 5$

$$\text{Normalized} \cdots H_N(s) = \frac{1}{s^5 + 3.2361 s^4 + 5.2361 s^3 + 5.2361 s^2 + 3.2361 s + 1}$$

(use Tables). Now de-normalize using $s \to s/(2\pi\times 10^3)$

x

MFM approximation $f_c = 8$ KHz , $A_p = 1$ dB

$f_s = 12$ KHz with $A_s = 15$ dB

Work like prob 3.1 $\epsilon = .5088$, $n = 6$

First use $H(\hat{s})$ for $n=6$ in BUT-function

$$H_N(\hat{s}) = \frac{1.}{\hat{s}^6 + 3.8637\hat{s}^5 + 7.4641\hat{s}^4 + 9.1416\hat{s}^3 + 7.4641\hat{s}^2 + 3.8637\hat{s} + 1}$$

(from Table)

Then use $s \to \hat{s}\,\epsilon^{-1/n}$ i.e. $\hat{s} = s\,\epsilon^{1/n} = (.5088)^{1/6}\cdot s$

$= .8935 s$

$$\text{So } H_N(s) = \frac{1}{(.8935 s)^6 + 3.8637(.8935 s)^5 + 7.4641(.8935 s)^4 + 9.1416(.8935 s)^3 + 7.4641(.8935 s)^2 + 3.8637(.8935 s) + 1}$$

$$H_N(s) = \frac{1.9653}{s^6 + 4.3242 s^5 + 9.3495 s^4 + 12.8156 s^3 + 11.7111 s^2 + 6.7847 s + 1.9653}$$

This is the normalized LPF. Use $s \to s/(2\pi\times 8\times 10^3)$ for de normalization to get the desired TF. $H(s)$

3.10 & 4.4 Equiripple passband → } CHEB - approximation
monotonic stop band → }

$A_p = 0.5$ dB, $f_c = 6$ KHz } $\epsilon = .3493$
$A_s \geq 30$ dB, $f_s \geq 15$ KHz } $n = 4$

Consulting Tables, for $n = 4$, $D(s)$ in CHEB for $\epsilon = .3493$
ie $A_p = 0.5$ dB

$$D(s) = s^4 + 1.197 s^3 + 1.717 s^2 + 1.025 s + 0.379$$

Then
$$H_N(s) = \frac{1/(2^{4-1} \cdot \epsilon)}{s^4 + 1.197 s^3 + 1.717 s^2 + 1.025 s + 0.379}$$

$$H_N(s) = \frac{0.3578}{s^4 + 1.197 s^3 + 1.717 s^2 + 1.025 s + 0.379}$$

Note: at $j\omega = 0$ ie d.c. $|H(0)| = \frac{0.3578}{0.379} = 0.944$ which is

$= \frac{1}{\sqrt{1+\epsilon^2}} = .944$ since the order is "even"

Now de normalize $s \to s/(2\pi \times 6000)$

3.11 & 4.5 Equiripple pass band } CHEB approximation
Monotonic stop band }

$\omega_{CH} = 2\pi \times 15 \times 10^3$; $\omega_{AH} = 7.8 \times 10^3 \times 2\pi$

$\omega_s = \frac{15}{7.8}$ $A_p = .5$ dB, $A_s \geq 35$ dB. Calculate $n = 5$
$\downarrow \epsilon = .3493$

From Tables, norm. (ie $\omega_c = 1$) LPF (use $A_p = 0.5$ dB in Tables)

$$H_{N\,LPF}(s) = \frac{0.1789}{s^5 + 1.17251 s^4 + 1.9374 s^3 + 1.3096 s^2 + .7525 s + 0.1789}$$

[note: for 'odd' order $|H(0)| = 1$]

together $s \to \frac{\omega_{CH}}{s}$

→ Frequency transformation $s \to \frac{1}{s}$ (LP → HP)

$$H_N(s)\Big|_{HPF} = \frac{.1789 s^5}{1 + 1.17251 s + 1.9374 s^2 + 1.3096 s^3 + .7525 s^4 + .1789 s^5}$$

→ Frequency scale $s \to \frac{s}{2\pi \times 15 \times 10^3}$ (ie $s \to \frac{s}{\omega_{CH}}$)

$$H(s)\Big|_{HPF} = \frac{0.1789 s^5}{(2\pi \times 15 \times 10^3)^5 + 1.17251 (2\pi \times 15 \times 10^3)^4 s + 1.9374 (2\pi \times 15 \times 10^3)^3 s^2 + 1.3096 (2\pi \times 15 \times 10^3)^2 s^3 + .7525 (2\pi \times 15 \times 10^3) s^4 + .1789 s^5}$$

is the desired TF. of the HPF.

3.12 $A_p = 0.5$; $A_s = 60$ when $\omega_s = \frac{\omega_a}{\omega_c} = 10$

$\epsilon = .3493$, $n = 3$

Using formula and Table

$$H_N(s) = \frac{\frac{1}{2^{3-1}\cdot\epsilon} \rightarrow 0.716}{s^3 + 1.253s^2 + 1.535s + 0.716}$$ is the normalized LPF. transfer function

x

3.13 The equivalent normalized LPF has

$$\left.\begin{aligned} \omega_c &= 2\pi(20-15)\times 10^3 \\ \omega_a &= 2\pi(35-8.6)\times 10^3 \end{aligned}\right\} \text{ So } \omega_s = \frac{26.4}{5} = 5.28$$

$A_p = 1\,dB$, $A_s = 40\,dB$. The approximation is MFM

find $\epsilon = .5088$, $n = 4$

So $$H_N(\hat{s})\Big|_{LPF} = \frac{1}{\hat{s}^4 + 2.613\hat{s}^3 + 3.414\hat{s}^2 + 2.613\hat{s} + 1}$$ from Tables of BUT-function where $\epsilon = 1$

For $\epsilon = .5088$, use $\hat{s} = s\epsilon^{1/n} = .8446s$

$$H_N(s)\Big|_{LPF} = \frac{1}{\left[(.8446)^4 s^4 + 2.613(.8446)^3 s^3 + 3.414(.8446)^2 s^2 + 2.613(.8446)s + 1\right]}$$

$$= \frac{1.9651}{s^4 + 3.0938s^3 + 4.7859s^2 + 4.3369s + 1.9651}$$

Apply LP $\leftrightarrow$ BP transformation.

$B = 2\pi \cdot 5\times 10^3$; $\Omega_o = \left(\sqrt{20\times 15}\right) 2\pi\times 10^3$

Change: $$s \rightarrow \frac{2\pi\times 10^3\sqrt{20\times 15}}{2\pi\times 10^3\times 5}\left[\frac{s}{2\pi\times 10^3\sqrt{20\times 15}} + \frac{2\pi\times 10^3\sqrt{20\times 15}}{s}\right]$$

$$\rightarrow 3.183\times 10^{-5}s + \frac{3.7699\times 10^5}{s}$$

$$\rightarrow \frac{s^2 + 1.184352529\times 10^{10}}{31415.92654\,s}$$

3.13

note that

$$f(s) = \frac{b_0}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

under $s \to \dfrac{s^2 + \Omega_0^2}{Bs}$ transformation changes to $\bar{f}(s)$

$$\bar{f}(s) = \frac{b_0 \cdot B^4 s^4}{(s^2+\Omega_0^2)^4 + a_1 B s (s^2+\Omega_0^2)^3 + a_2 B^2 s^2 (s^2+\Omega_0^2)^2 + a_3 B^3 s^3 (s^2+\Omega_0^2) + a_4 B^4 s^4}$$

The denominator can be simplified to

$$s^8 + a_1 B s^7 + (a_2 B^2 + 4\Omega_0^2) s^6 + (3 a_1 B \Omega_0^2 + a_3 B^3) s^5 + (a_4 B^4 + 2a_2 B^2 \Omega_0^2 + 6\Omega_0^4) s^4 + (3 a_1 B \Omega_0^4 + a_3 B^3 \Omega_0^2) s^3 + (a_2 B^2 \Omega_0^4 + 4\Omega_0^6) s^2 + a_1 B \Omega_0^6 s + \Omega_0^8$$

In the present case:

$$H_N(s)\Big|_{LPF} \Rightarrow H(s)\Big|_{BPF} = \frac{1.9651 \cdot (2\pi \times 10^3 \times 5 s)^4}{(s^2 + 1.1843\times10^{10})^4 + 3.0938 \times 3.142\times10^4 s (s^2+1.1843\times10^{10})^3 + 4.7859\times(3.142\times10^4)^2 s^2 (s^2+1.1843\times10^{10})^2 + 4.3369\times(3.142\times10^4)^3 s^3 (s^2+1.1843\times10^{10}) + 1.9651\,(3.142\times10^4)^4 s^4}$$

$$\cong \frac{4.7879\times10^{20}\, s^4}{\left[250 s^8 + 2.4302\times10^7 s^7 + 1.3024\times10^{13} s^6 + 8.9705\times10^{17} s^5 + 2.3884\times10^{23} s^4 + 1.0624\times10^{28} s^3 + 1.8267\times10^{33} s^2 + 4.0367\times10^{37} s + 4.9179\times10^{42}\right]}$$

3.14 equiripple passband $\rightarrow$ CHEB filter

$A_p = 1.0\,dB$, $\epsilon = .5089$, $\Omega_{p1} = 2\pi 40k$, $\Omega_{p2} = 2\pi 100k$, $A_s = 15\,dB$

$\Omega_{a1} = 2\pi.50K$; $\Omega_{a2} = 2\pi.80K$

$$\text{So } \omega_s = \frac{100-40}{80-50} = \frac{60}{30} = 2, \quad n \geq 2.33 \rightarrow 3$$

$$\text{Then } H_N(s)\Big|_{LPF} = \frac{1/(2^{3-1} \times .5089)}{s^3 + .988 s^2 + 1.238 s + .491}$$

Now $\Omega_o = 2\pi\sqrt{50 \times 80} \times 10^3$ rad/sec. $= 397383.5306$ rad/sec.

$B = 2\pi(80-50) \times 10^3 = 2\pi \times 30 \times 10^3 = 188495.5592$ rad/sec.

Apply LP $\rightarrow$ BS transformation $s \leftrightarrow B . \dfrac{s}{s^2 + \Omega_o^2}$.

$$H(s)\Big|_{BSF} = \frac{.491}{B^3 \dfrac{s^3}{(s^2+\Omega_o^2)^3} + .988 . B^2 . \dfrac{s^2}{(s^2+\Omega_o^2)^2} + 1.238 B . \dfrac{s}{(s^2+\Omega_o^2)} + .491}$$

$$= \frac{.491(s^6 + 47374 \times 10^{11} s^4 + 7.481 \times 10^{22} s^2 + 3.9378 \times 10^{33})}{[.491 s^6 + 2.3336 \times 10^5 s^5 + 2.6771 \times 10^{11} s^4 + 8.6398 \times 10^{16} s^3 + 4.2275 \times 10^{22} s^2 + 5.8191 \times 10^{27} s + 1.9335 \times 10^{33}]}$$

3.15

(see prob 3.7)

Ripple in passband implies a CHEB response.

$A_p \leq 1$ dB ; $\epsilon = 0.5089$

$$D = \frac{10^{.1A_s} - 1}{10^{.1A_p} - 1} = \frac{10^{2.5} - 1}{10^{.1} - 1} = \frac{315.23}{.259} = 1217.45$$

$\sqrt{D} = 34.89$; Order $n \geq \dfrac{\cosh^{-1}\sqrt{D}}{\cosh^{-1} \omega_s} = \dfrac{\cosh^{-1}(34.89)}{\cosh^{-1}(4)}$

$\omega_s \to f_s/f_c = 4$

order $n \geq 2.057$. Take $n = 3$

$$\text{So } H_N(s) = \frac{1}{2^2 \epsilon} \cdot \frac{1}{s^3 + 0.988s^2 + 1.238s + .491} \text{ (from Table)}$$

$$= \frac{0.491}{s^3 + .988s^2 + 1.238s + 0.491}.$$

which is a CHEB-LP filter of order 3

3.16 Elliptic filter function

$\omega_c = 1$ rad/sec , $\omega_s/\omega_c = 1.2$, $A_p = 0.5$ dB , $A_s = 45$ dB

For MFM $n \geq \dfrac{\log D}{2\log(\omega_s/\omega_c)}$, $D = \dfrac{10^{.1A_s} - 1}{10^{.1A_p} - 1} = \dfrac{10^{4.5} - 1}{10^{.05} - 1}$

$n_{MFM} \geq 34.18$ $n_{MFM} = 35$ $= 259155.69$

For CHEB $n \geq \dfrac{\cosh^{-1}\sqrt{D}}{\cosh^{-1}(1.2)} \Rightarrow \dfrac{\cosh^{-1}(509.073)}{\cosh^{-1}(1.2)} = 11.13$

So $n_{CHEB} = 12$

For elliptic $u(D) = \dfrac{1}{16D}\left(1 + \dfrac{1}{2D}\right) = 0.24117 \times 10^{-6}$

$$v(\omega_s) = \frac{\sqrt{\omega_s} - 1}{2(\sqrt{\omega_s} + 1)} = \frac{\sqrt{1.2} - 1}{2[\sqrt{1.2} + 1]} = 0.02277$$

$$F(u) = \frac{1}{\pi} \ln[u + 2u^5 + 15u^9] = -4.8497$$

$$F(v) = \frac{1}{\pi} \ln[v + 2v^5 + 15v^9] = -1.2037$$

order $n \geq F(u)/F(v) = 5.83$; So $n_{ELLIP} = 6$

~~3.18~~ 3.17

An MFM filter of order 'n' has

$$|H_n(j\omega)|^2 = \frac{1}{\epsilon^2 \omega^{2n} + 1}$$

, $\omega \rightarrow$ normalized frequency w.r.t. the pass-band edge frequency.

In the case of $A_p = 1$ dB

$$10 \log |H_n(j\omega)|^2 = -1 = 10 \log_{10} \left[\frac{1}{\epsilon^2 + 1} \right]$$

$$\epsilon = 10^{.1 A_p} - 1 = 0.2589$$

For a sixth order MFM, $n = 6$

$$|H_6(j\omega)|^2 = \frac{1}{\epsilon^2 \omega^{12} + 1} = \frac{1}{(.2589)^2 \omega^{12} + 1}$$

with $\omega = \frac{\omega_a}{\omega_c} = 10$; $|H_6(j\omega)|^2 = \frac{1}{(0.2589)^2 \times 10^{12} + 1}$

$$\approx \frac{10^{-12}}{\quad ^{12}}$$

3.18 Consulting Table B-2 (Huelsman, L.P.)
For $A_p = 1$ dB, $A_s > 46$ dB, $n = 8$ with $A_p = 1$ dB, $A_s = 49.33$ dB

$$T(s) = H.\ \frac{(s^2 + 1.1139)}{s^2 + .5482s + .1915} \cdot \frac{(s^2 + 1.2431)}{s^2 + .2607s + .6404} \cdot \frac{(s^2 + 1.9061)}{s^2 + .0845s + .9118} \cdot \frac{(s^2 + 11.0467)}{s^2 + .0182s + 0.9994}$$

$H \to$ choose suitably

3.19 normalized frequency is $700 \times 2.5 \times 10^{-3} = 1.54$
Delay error < 3%
magnitude error < 3 dB for up to 1000 rad/sec.
We will try to look for a magnitude error of < 2 or 2.5 dB until $\omega = 700$ rad/sec which has been used for calculating the normalized frequency.
A more ~~consid~~ conservative case will be to look for the BT filter function using $\omega = 1000$ rad/sec,
normalized frequency $y = 1000 \times 2.5 \times 10^{-3} = 2.5$, all the specs.
Then surely for $\omega = 700$ rad/sec, will be satisfied.
From Fig 3.16 (a), (b) $n = 6$ for magn. error < 3 dB

$n = 5$ for delay error < 3%.

So we need a BT filter of order 6.
Using 700 rad/sec to calculate the normalized frequency could lead to $n = 4$. But $n = 6$ is a better choice.

$$T(s) = \frac{10395}{s^6 + 21s^5 + 210s^4 + 1260s^3 + 4725s^2 + 10395s + 10395}$$

Using Table A-5 for $n = 6$

3.20 Consider a BT filter as an AP filter.

Then $y = \omega_0 \tau = 4\times10^{3} \times 2\pi \times 300\times10^{-6} = 7.54$

From Fig 3.16(b), for delay error $< 1.5\%$, $n = 11$

The AP function is an 11^{th} order BT filter.

Use eqn. (3.83), (3.84) to derive the expression for the function.