Hints & Solutions to Problems in Ch 4

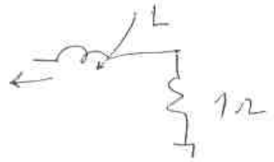
(4.1) Considering Table of BUT function

$$H_N(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} = \frac{1}{\underbrace{2s^2+1}_{M_2} + \underbrace{s^3+2s}_{M_1}}$$

$$Y_{22} = \frac{M_2}{M_1} = \frac{2s^2+1}{s^3+2s}$$, quotient will be of degree <1 in s

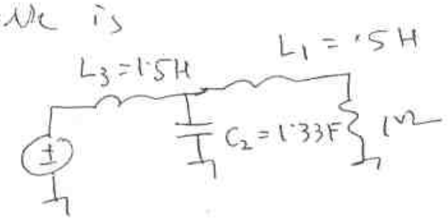
So consider $\frac{1}{Y_{22}}$ instead i.e.

$$\frac{M_1}{M_2} = \frac{s^3+2s}{2s^2+1}$$. As a result, we begin with a series inductance at the load end, i.e.



$$
\begin{array}{l}
2s^2+1 \,)\, s^3+2s \,(\, 0.5s \;\rightarrow \text{ inductance } L = 0.5\text{H} = L_1 \\
\qquad\qquad s^3 + .5s \\
\hline
\qquad 1.5s \,)\, 2s^2+1 \,(\, 1.33s \;\rightarrow \text{capacitance } C = 1.33\text{F} = C_2 \\
\qquad\qquad\quad 2s^2 + \\
\hline
\qquad\qquad 1 \,)\, 1.5s \,(\, 1.5s \;\rightarrow \text{ inductance } L = 1.5\text{H} = L_3 \\
\qquad\qquad\quad\;\; 1.5s \\
\hline
\qquad\qquad\quad\;\; \times
\end{array}
$$

So the network is



1Ω is the load. Excitation is an ideal voltage source i.e. with $R_s = 0$.

(4.2) The specs are $A_p = 3$ dB at $f_c = 1$ KHz
$A_a = 25$ dB at $f_c = 2$ KHz

Then $n_{BUT} = 5$.

$$H_N(s) = \frac{1}{s^5 + 3.2361 s^4 + 5.2361 s^3 + 5.2361 s^2 + 3.2361 s + 1}$$ from Tables

Then $M_2(s) = 3.2361 s^4 + 5.2361 s^2 + 1$

$M_1(s) = s^5 + 5.2361 s^3 + 3.2361 s$

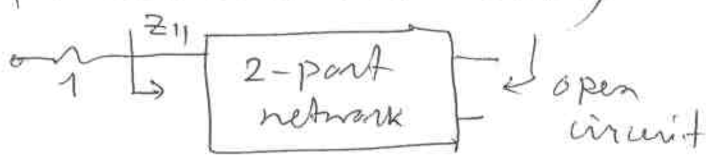
4.2 Using eq. 3.22 (Ch 3) with $R_s = 1$, $R_L = \infty$ (OA input impedance)

$$\frac{V_2}{V_s} = \frac{1}{A + C} = \frac{z_{21}}{1 + z_{11}}$$ using the equiv between A, B, C, D to $[z]$ parameters.

Following the discussion in section 4.3, we need to realize z_{11} as $\frac{m(s)}{n(s)}$ (if deg $m(s) >$ deg $n(s)$) or as $\frac{n(s)}{m(s)}$ (if deg. $n(s) >$ deg $m(s)$) where m & n are respectively the even and odd parts of $D(s)$, in

$$H(s) = \frac{V_2}{V_s} = \frac{N(s)}{D(s)}$$

For all-pole function, $N(s)$ will be a constant (ie. no finite transmission zeros)



For a 3rd order BUT filter $D(s) = s^3 + 2s^2 + 2s + 1$

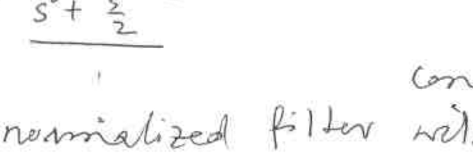
$m(s) = 2s^2 + 1$; $n(s) = s^3 + 2s$, deg $n(s) >$ deg $m(s)$

realize z_{11} as $\frac{n(s)}{D(s)} = \frac{s^3 + 2s}{2s^2 + 1}$

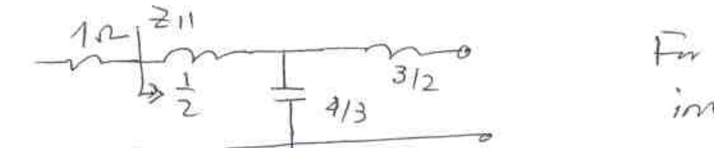
$$2s^2+1 \overline{\big)\, s^3 + 2s} \left(\frac{s}{2} \right. \rightarrow \text{inductor, first series element from } R_s \text{ side!}$$
$$\underline{s^3 + \frac{s}{2}}$$

continue .. as in Example 4.3

The normalized filter with $R_s = 1\,\Omega$ is then



For $R_s = 50\,\Omega$, use impedance scaling



4.3

Contd.

$Y_{22} = \frac{m_2}{m_1}$ of degree < 1. So start with $1/Y_{22}$ instead. The first element at the load end will be an inductance L.

$$3.2361s^4 + 5.2361s^2 + 1 \,)\; s^5 + 5.2361s^3 + 3.2361s \;(\, 0.309s \rightarrow \text{.309 H}$$
$$s^5 + 1.6180s^3 + .309s$$

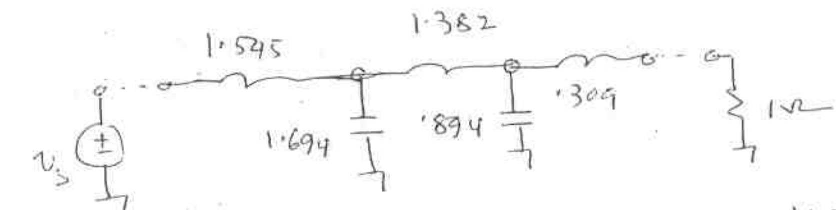
$$3.6181s^3 + 2.9271s \,)\; 3.2361s^4 + 5.2361s^2 + 1 \;(\, .894s \rightarrow \text{.894 F}$$
$$3.2361s^4 + 2.618s^2$$

$$2.6181s^2 + 1 \,)\; 3.6181s^3 + 2.9271s \;(\, 1.382s \rightarrow \text{1.382 H}$$
$$3.6181s^3 + 1.382s$$

$$1.5451s \,)\; 2.6181s^2 + 1 \;(\, 1.694s \rightarrow \text{1.694 F}$$
$$2.6181s^2 + 0$$

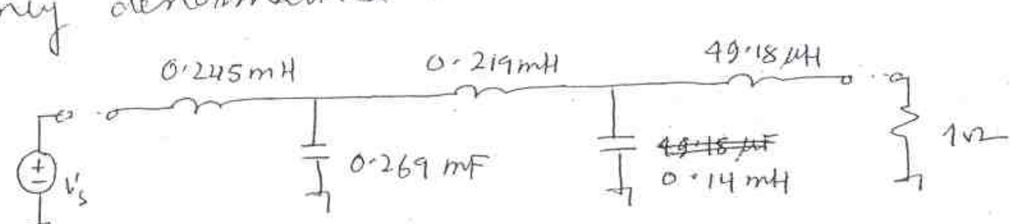
$$1 \,)\; 1.5451s \;(\, 1.5451s \rightarrow \text{1.5451 H}$$
$$1.5451s$$
$$\times$$

Normalized filter.

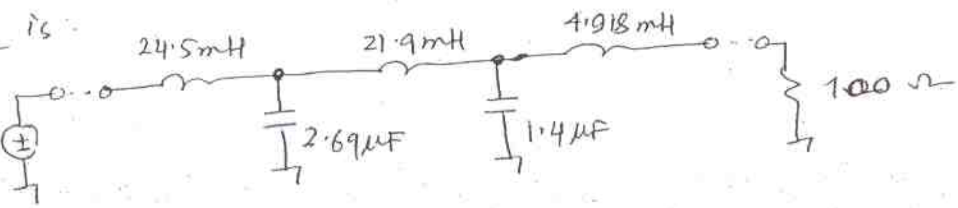


To have f_c = 1 KHz, ω_c = 6283.18 rad/sec. scale all components like $L \rightarrow L/6283.18$, $C \rightarrow C/6283.18$

Frequency denormalized filter is then.



For a load R_L = 100Ω, impedance scale $L \rightarrow L \times 100$, $C \rightarrow C/100$. Final filter network is



(4.4) The filter has CHEB magnitude response. The order is $n_{CHEB} = 4$ with $\epsilon = .3493$

Then $$H_N(s) = \frac{1}{2^3 \cdot \epsilon \, (s^4 + 1.197s^3 + 1.717s^2 + 1.025s + .379)}$$

Consider $D(s) = s^4 + 1.197s^3 + 1.717s^2 + 1.025s + .379$

$M_2(s) = s^4 + 1.717s^2 + .379$

$M_1(s) = 1.197s^3 + 1.025s$

$\because$ $\deg(M_2) > \deg(M_1)$, we start with $y_{22} = \frac{M_2}{M_1}$ as it is.

$1.197s^3 + 1.025s \,\big)\, s^4 + 1.717s^2 + .379 \,\big(\, 0.835s \rightarrow$ $.835$ F

$s^4 + .856s^2$

$.861s^2 + .379 \,\big)\, 1.197s^3 + 1.025s \,\big(\, 1.39s \rightarrow$ 1.39 H

$1.197s^3 + .527s$

$0.498s \,\big)\, .861s^2 + .379 \,\big(\, 1.729s \rightarrow$ 1.729F

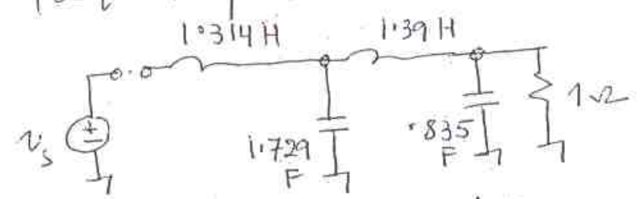
$.861s^2$

$.379 \,\big)\, .498s \,\big(\, 1.314s \rightarrow$ 1.314 H

$.498s$

x

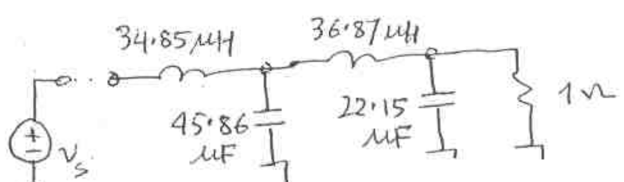
The frequency normalized filter is



Frequency denormalized filter is:

$L \rightarrow \frac{L}{2\pi \times 6000}$

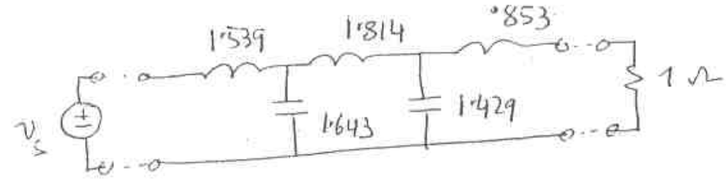
$C \rightarrow \frac{C}{2\pi \times 6000}$



x

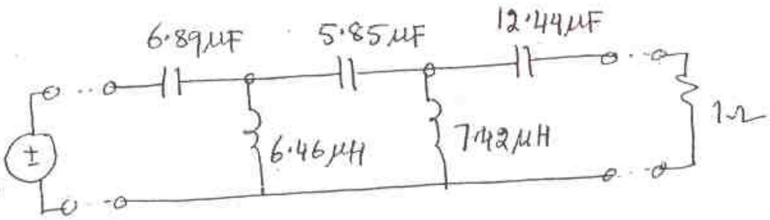
4.5 CHEB filter, $n_{CHEB} = 5$, $\epsilon = .3493$

4.5 (cont'd) contd. The normalized LP filter structure is:



The frequency denormalized HPF is (by component transformation formulae): $L_{HP} \rightarrow \frac{1}{\omega_{CH} C_{LP}}$; $C_{HP} \rightarrow \frac{1}{\omega_{CH} L_{LP}}$

$\omega_{CH} = 2\pi \times 15 \times 10^3$



4.6 With order $n = 3$ and $A_p = 0.5$ dB, $\epsilon = 0.3493$

Using eqn.(16) in Ch 6,

$$q = \frac{1}{2}\left[\left(\frac{1}{\epsilon} + \sqrt{1 + \frac{1}{\epsilon^2}}\right)^{1/n} - \left(\frac{1}{\epsilon} + \sqrt{1+\frac{1}{\epsilon^2}}\right)^{-1/n} \right] = 0.6265$$

$$R_m \lessgtr n/2 = 1$$

Then $C_1 = \frac{2}{q R_s} \cdot \sin\frac{\pi}{2n} = 1.596$ with $R_s = 1\,\Omega$

$$C_1 L_2 = \frac{4 \sin\left(\frac{4k-1}{2n}\pi\right) \sin\left(\frac{4k-3}{2n}\pi\right)}{q^2 + \sin^2\left(\frac{2k-1}{n}\pi\right)}$$

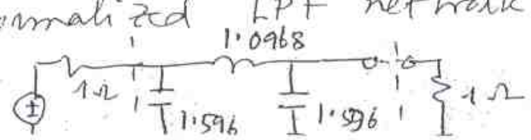
with $C_1 = 1.596$, $K = 1$

$$L_2 = \frac{1}{1.596} \cdot 1.7505 = 1.0968$$

$$C_3 L_2 = \frac{4\sin\left(\frac{4k-1}{2n}\pi\right)\sin\left(\frac{4k+1}{2n}\pi\right)}{q^2 + \sin^2\left(\frac{2k}{n}\pi\right)}$$ with $L_2 = 1.0968$

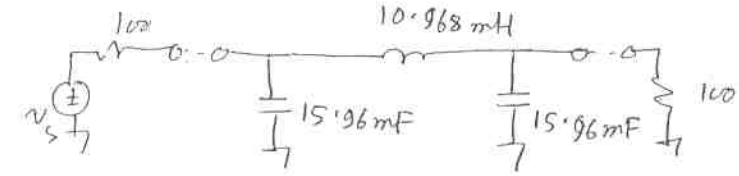
$C_3 = 1.596$

The normalized LPF network is then



(4.6) Contd.

For $R_L = R_S = 100\Omega$, the network will be



(4.7)

Use the formulae as above.

$n = 4$, $A_p = 1$ dB $\rightarrow \epsilon = 0.5088$, $q = .7293$

With $R_S = 1\Omega$, $R_L = \left[1 + 2\epsilon^2 \pm 2\epsilon\sqrt{1+\epsilon^2}\right] R_S$

$= 2.6595$ using the + sign before $2\epsilon\sqrt{1+\epsilon^2}$

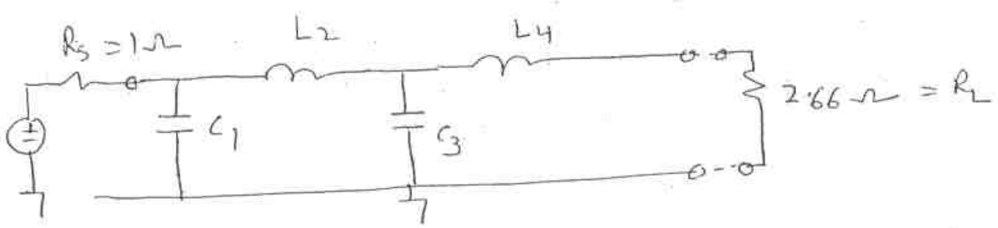
$k_m < n/2 = 2$. So $k_m = 1$

So $k = 1$ to be taken. Let $R_S = 1\Omega$ as suggested.

$C_1 = \frac{2}{q R_S} \cdot \sin\frac{\pi}{2n} \rightarrow 1.0495$, $L_2 = 1.3059$

$C_3 = 1.7067$

$L_4 = R_L \frac{2}{q} \sin\frac{\pi}{2n} = 2.791$ with $R_L = 2.6595$



4.8

$n = 5$, $A_p = 0.5$ dB, $\epsilon = 0.3493$

$k_m = 2$ ∵ $n = 5$, $k_m < \frac{5}{2}$

$k = 1, 2$ can be taken. Assume $R_S = R_L = 1\Omega$

With $k = 1$, $C_1 = .8529$, $L_2 = 1.3468$

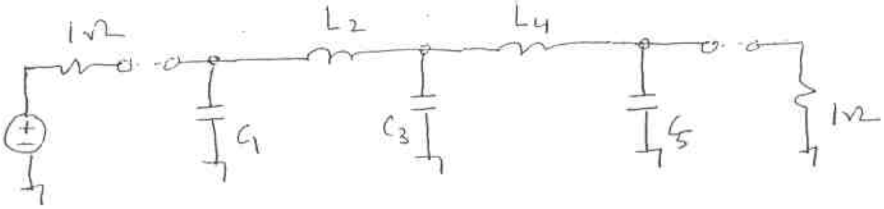
$C_3 = 1.6807$

4.8

Contd.

with $k = 2$

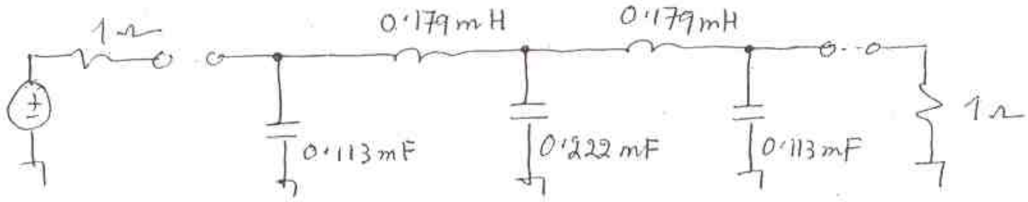
$$L_4 = \frac{1}{C_3} \cdot \frac{4 \sin\left(\frac{4k-1}{2n}\pi\right) \sin\left(\frac{4k-3}{2n}\pi\right)}{q^2 + \sin^2\left(\frac{2k-1}{n}\pi\right)} = 1.3468$$



and then $C_5 = \frac{1}{R_L} \cdot \frac{2}{q} \cdot \sin\frac{\pi}{2n} = C_1 = .8529 \quad \because R_L = R_s$

The frequency denormalized filter is $C \to \frac{C_N}{2\pi f_c}$; $L \to \frac{L_N}{2\pi f_c}$

where C_N, L_N are the values derived already. $f_c = 1.2 \times 10^3$



x

4.9

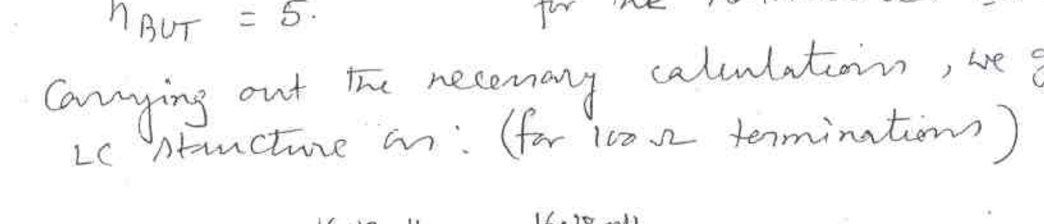
From the given specs, $\omega_c = 2\pi\,(3168 - 1578) = 2\pi\,(1590)$

$\omega_a = 2\pi\,(5000 - 1000) = 2\pi \times 4000$

This gives together with $A_p = 3$ dB, $A_a = 40$ dB,

$n_{BUT} = 5.$ for the normalized LPF.

Carrying out the necessary calculations, we get the LPF, LC structure as: (for 100 Ω terminations)



Use LP ⇔ BP component transformations to the L, C components to get the BPF. Note that:

$\Omega_0 = 2\pi\sqrt{3168 \cdot 1578} = 14048.37$ rad/sec.

$B = 2\pi \times 1590 = 9990.26$ rad/sec.

4.10. Follow the same procedure as shown in Example 4.7 of the book.

4.1)

$$\eta = \frac{10^{.1A_s}-1}{10^{.1A_p}-1} = \frac{10^{3.3}-1}{10^{.1}-1} = 7702.07$$

$$u(\eta) = \frac{1}{16\eta}\left(1+\frac{1}{2\eta}\right) \quad ; \quad v(\omega_s) = \frac{\sqrt{\omega_s}-1}{2(\sqrt{\omega_s}+1)} \quad ; \ \omega_s = \frac{2000}{1000} = 2$$

$$= 8.115 \times 10^{-6} \qquad\qquad = 0.1082$$

order $n = F(u)\,F(v)$, $F(x) = \frac{1}{\pi} \ln\left[x + 2x^5 + 15x^9\right]$

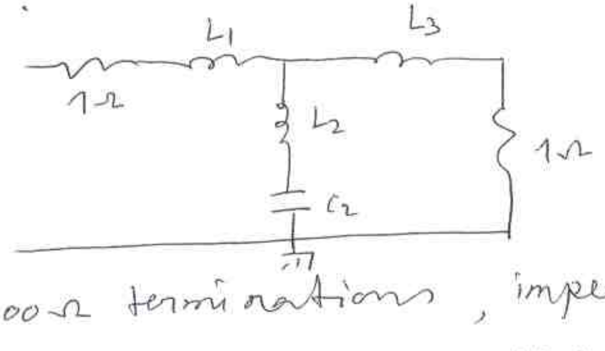
$$= \frac{1}{\pi} \ln\left[8.115\times10^{-6} + 2\times(8.115\times10^{-6})^5 + 15\times(8.115\times10^{-6})^9\right] \times$$

$$\frac{1}{\pi} \ln\left[0.1082 + 2\times(.1082)^5 + 15\times(.1082)^9\right]$$

$$\approx \frac{1}{\pi^2} \ln(8.115\times10^{-6}) \times \ln(0.1082) = \frac{1}{\pi^2}(-11.7218)(-2.229)$$

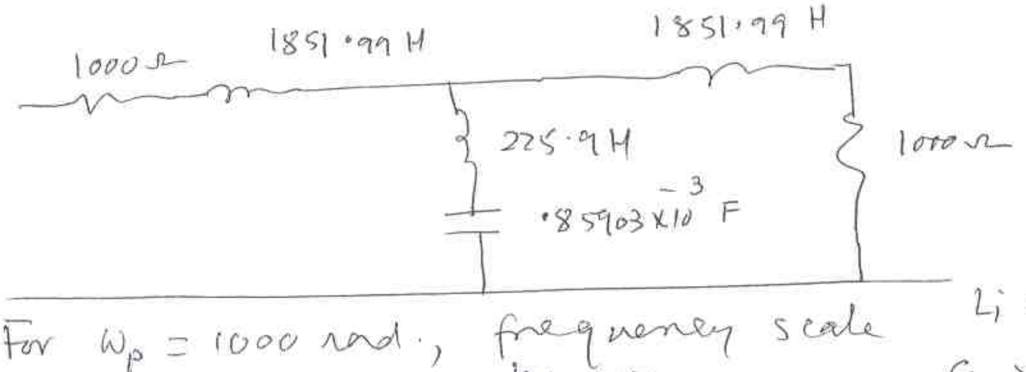
$$\cong \frac{1}{10} \times 26.128 = 2.61 \approx 3$$

Considering Table A-6 (Huelsman, 1993) for $A_p = 1\,dB$, $n=3$, $\omega_s=2$ the normalized low-pass elliptic filter components are:



$L_1 = 1.85199$ H
$C_2 = 0.85903$ F
$L_2 = 0.2259$ H
$L_3 = 1.85199$ H

For 1000 Ω terminations, impedance scale by 1000



For $\omega_p = 1000$ rad., frequency scale by 1000.

$L_i \Rightarrow \frac{L_i}{1000}$

$C_j \Rightarrow C_j/1000$