Solutions/Hints for Problems in Ch 6

6.1 $\tau = \frac{1}{\omega} = \frac{1}{2\pi \times 125} = CR$, $R = \frac{1}{2\pi \times 125 \times 15.10^{-12}}$

84.883 MΩ

$$C_R = \frac{1}{fR} = \frac{1}{100 \times 125 \times 84.883 \times 10^6} = 0.942 \text{ pF.}$$

6.2 The given T.F. is that of a notch filter with

$\omega_z^2 = 1.4212 \times 10^5$, $\omega_p^2 = 6.9833 \times 10^5$, $\frac{\omega_p}{Q_p} = 1004.2$

(a) For $f_s = 8$ KHZ, $\hat{\omega}_p = 2f_s . \tan(\omega_p/2f_s)$, $\hat{\omega}_n = 2f_s \tan(\omega_n/2f_s)$

and so on (see Table 5.4) and

$$H(z) = h_D . (1 + a_{1N} z^{-1} + a_{2N} z^{-2}) / (1 - a_{1D} z^{-1} + a_{2D} z^{-2})$$

with $h_D = H_N . (a^2 + \hat{\omega}_n^2) / F_2$ where

H_N is the flat gain in the analog filter and

$F_2 = a^2 + (\hat{\omega}_p/Q_p) a + \hat{\omega}_p^2$, $a = 2f_s$.

In the given function $H_N = 1$. Calculations will give

$h_D = 0.939$ $a_{1D} = 1.8718$

$\frac{\hat{\omega}_n}{a} = .0236$; $a_{1N} = -1.9978$ $a_{2D} = .8821$ } For $f_s = 8$ KHZ.

$a_{2N} = 1$

$$H(z) = 0.939 \frac{1 - 1.9978 z^{-1} + z^{-2}}{1 - 1.8718 z^{-1} + .8821 z^{-2}}$$

(b) For $f_s = 128$ KHZ, similarly,

$\frac{\hat{\omega}_r}{a} = 1.473\text{E-}3$, $H(z) = .9961 \frac{1 - 1.9999 z^{-1} + z^{-2}}{1 - 1.9921 z^{-1} + .9922 z^{-2}}$ } For $f_s = 128$ KHZ

The attached plots show that the response for $f_s = 128$ KHZ is way off from the desired response H(s).

6.2 contd.

(b) The reason seems to be in the loss of numerical accuracy when $f = 128$ KHz is used. Since $T = 1/128\times10^3$ is much smaller now, the coefficients a_{1N}, a_{1D}, a_{2D} are to be calculated with more accuracy and used in the calculation of $|H(z)|$. Thus, if we recalculate with more accuracy, we find, for $f_s = 128$ KHz

$$h_0 = 0.9960842445 \quad a_{1N} = -1.999991326, \quad a_{2N} = 1$$

$$a_{1D} = 1.992142941 \quad a_{2D} = 0.992185397 \qquad \frac{\hat{\omega}_r}{a} = 1.472610688\times10^{-3}$$

On using these values, the calculated $H(z)$ shows excellent matching with $H(s)$.

The MATLAB program for computing $H(z)$, $H(s)$ is attached with the plots of $|H(s)|$, $|H(z)|$. The plot in - - - line is for $H(z)$ with $f_s = 128$ KHz but for a_{1N}, a_{1D} .. retained with less accuracy.

6.3

$$H(s) = \frac{.891975\, s^2 + 1.140926\times10^8}{s^2 + 356.047\, s + 1.140926\times10^8} = .891975\,\frac{s^2 + 1.27910087\times10^8}{s^2+356.047 s + 1.140926\times10^8}$$

transforms to:

$$H(z) = 0.8909255285\,\frac{1 - 1.992198066\, z^{-1} + z^{-2}}{1 - 1.990275499\, z^{-1} + 0.9972254644}$$

$$\frac{\hat{\omega}_r}{a} = 4.42074135E-2$$

From fig. 6.20, ignoring L, K, F and assuming $B = D = 1 = A$, we have

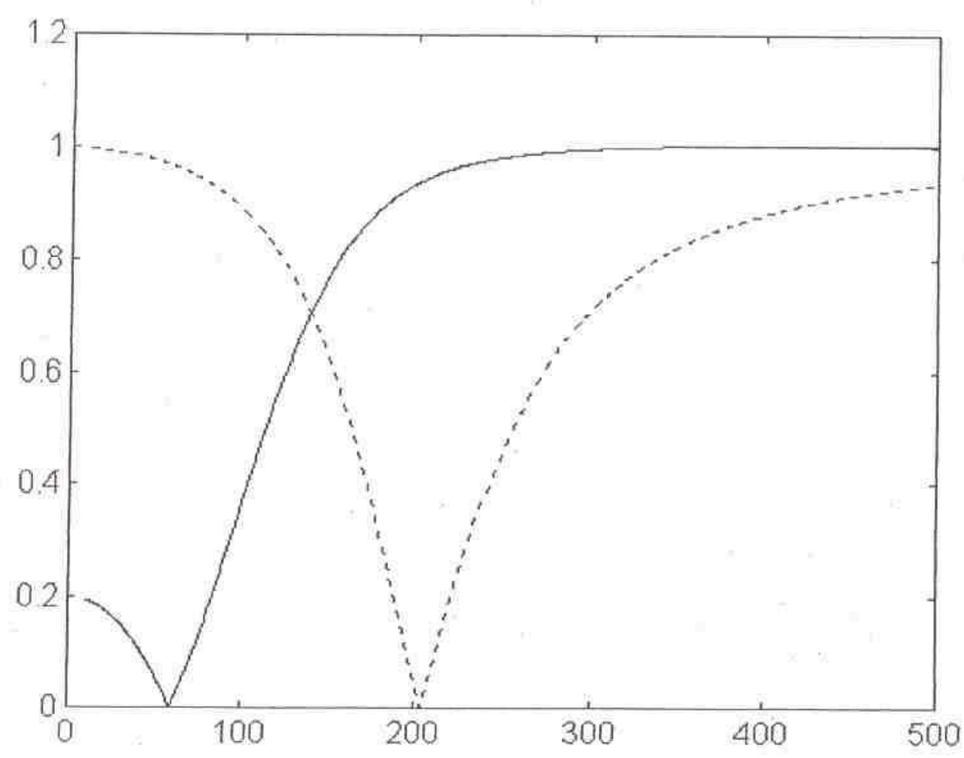
$$H_{2,2}(z) = \frac{I + [G - (I+J)]z^{-1} + (J-H)z^{-2}}{1 + (C+E-2)z^{-1} + (1-E)z^{-2}}$$

So $\quad I = .8909, \quad J - H = 1, \quad G - (I+J) = -1.9922$

$C + E - 2 = -1.9903; \quad 1 - E = 0.9972$

I, J, H, G, C, E are unknowns. $A = B = D = 1$ assumed.

```
%'            1- Ch6-2
%analog filter response
a=[1 1004.2 6.9833e5];
b=[1 0 1.4212e5];
f=linspace(10,500,901);
w=2*pi*f;
h=freqs(b,a,w);
mag1=abs(h);
%scf response with Fs=8000
Fs=8e3;
b=[1 -1.9978 1];
a=[1 -1.8718 0.8821];
f=linspace(10,500,901);
w=2*pi*f;
h=freqz(b,a,f,Fs);
mag2=0.939*abs(h);
%scf response with Fs=128000
Fs=128e3;
%b=[1 -1.999991326 1];
b=[1 -1.9999 1];
%a=[1 -1.992142941 0.992185397];
a=[1 -1.9921 0.9922];
f=linspace(10,500,901);
w=2*pi*f;
h=freqz(b,a,f,Fs);
mag3=0.9960842445*abs(h);
plot(f,mag1,f,mag2,f,mag3,'w:')
end
```



6.3 (contd.)

$$H_{2,2}(z) = I \cdot \frac{1 + \left(\frac{G}{I} - \frac{I+J}{I}\right) z^{-1} + \frac{J-H}{I} z^{-2}}{1 + (C+E-2) z^{-1} + (1-E) z^{-2}}$$

So $I = 0.8909$

$J - H = I$

$G - I - J = -1.9922\, I$

$C + E - 2 = -1.9903$

$1 - E = 0.9972$

$J, H, G, C, E \rightarrow$ Five capacitance ratios (assuming $A, B, D = 1$)

Four equations.

From $1 - E = .9972$, $E = 1 - .9972 = 0.0028$

$C + E - 2 = -1.9903$, then gives $C = .0069$

We have $G - I - J = -1.9922 \times I$

$J - H = I$

So $G - J = -0.9922\, I = -0.88316$

$J - H = .8909 \rightarrow J = H + .8909$

Let $H = 1$, $J = 1.8909$

Then $G = 1.8909 - .88316 = 1.00774$

Thus the solutions are:

$A = 1$, $B = 1$, $D = 1$, $H = 1$, $I = .8909$

$E = .0028$, $C = .0069$, $G = 1.00774$, $J = 1.8909$

Scaling the minimum capacitor to be $= 1$ pF,

let $E \rightarrow 1$ pF, scaling factor $1/.0028 = 357.1428571$

Then $A = B = D = H \cong 357$ pF

$I = .8909 \times 357.1428571 \approx 318$ pF

$E \cong 1$ pF, $C \cong 2.5$ pF, $G \approx 360$ pF, $J = 675$ pF.

$C_{Total} = 2784.5$ pF $\rightarrow 2.7845$ nF.

6.3 Contd.

$1-E = .9972$ gives $E = 0.0028$

$C+E-2 = -1.9903$ gives $C = 2-1.9903-E = 0.0069$

Then $J-H=1$; $G-I-J=-1.9922$ gives

$G-J = -1.9922 + .8909 = -1.1013$

$J-H = 1$. Three unknowns G, J, H, two equations.

$$\left.\begin{aligned} G-J &= -1.1013 \\ J-H &= 1 \end{aligned}\right\} \quad G-H = -.1013$$

If we let (assume) $H=1$, $J=2$, $G = 1-.1013 = 0.8987$.

Thus, the preliminary solutions are:

$A = B = D = 1$ (assumed)
$I = .8909$
$G = .8987$
$J = 2$
$H = 1$ (assumed)
$E = .0028$
$C = .0069$

We can scale up the capacitors making the lowest capacitor = 1 PF. Thus let $E = 1$ pF, a scale factor $1/.0028 = 357.14286$ is to be applied to all others.

So
$E = 1$ pF
$C = 2.46$ PF
$H = 357.14286$ pF $= 357.143$ pF.
$J = 714.286$ pF.
$G = 320.964$ pF.
$I = 318.178$ pF.
$A = B = D = 357.143$ pF

A possible set of solutions

6.4

Using the results from P 4.5 (Ch 4), the HPF is

$$T(s) = \frac{s^5}{s^5 + 396482\, s^4 + 0.6504039898 \times 10^{11} s^3 + 0.9069665315 \times 10^{16} s^2 + 0.5173875291 \times 10^{21} s + 0.415936312 \times 10^{26}}$$

We ignored pre-warping since $f_s = 128$ kHz is $\gg$ filter cut off frequencies.

Now applying BLT, $s \to \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} = 2 \times 128 \times 10^{3} \frac{1 - z^{-1}}{1 + z^{-1}}$

$$H(z) = 0.1099511628 \times 10^{11} \frac{(1 - z^{-1})^5}{D(z)}$$

$$D(z) = 0.157007829 \times 10^{9} z^{-5} + 0.1885127413 \times 10^{11} z^{-4} - 0.6444713465 \times 10^{11} z^{-3} + 0.1171053664 \times 10^{12} z^{-2} - 0.1049767527 \times 10^{12} z^{-1} + 0.4662020098 \times 10^{11}$$

6.5

Around OA1

$$V_1^{(1)} = V_i^{(2)} \frac{z^{-\frac{1}{2}}}{1-z^{-1}} \cdot \frac{H}{D} - V_i^{(1)} \frac{1}{1-z^{-1}} \cdot \frac{G}{D} - V_2^{(1)} \frac{1}{1-z^{-1}} \frac{C}{D} - \frac{E}{D} V_2^{(1)}$$

$$= \frac{(H/D)\, z^{-1}}{1-z^{-1}} V_i^{(1)} - \frac{G/D}{1-z^{-1}} V_i^{(1)} - \frac{C/D}{1-z^{-1}} V_2^{(1)} - (E/D) V_2^{(1)}$$

Around OA2 (ignoring $(\phi_1), (\phi_2)$ phase pair)

$$V_2^{(1)} = V_1^{(2)} \frac{z^{-\frac{1}{2}}}{1-z^{-1}} \frac{A}{B} - \frac{K}{B} V_i^{(1)} - \frac{F}{B} \cdot \frac{1}{1-z^{-1}} V_2^{(1)}$$

$$= \frac{(A/B)\, z^{-1}}{1-z^{-1}} V_1^{(1)} - (K/B) V_i^{(1)} - \frac{F/B}{1-z^{-1}} V_2^{(1)}$$

$$V_2^{(1)} = \frac{(A/B)\, z^{-1}}{1-z^{-1}} \left[\frac{(H/D)\, z^{-1}}{1-z^{-1}} V_i^{(1)} - \frac{G/D}{1-z^{-1}} V_i^{(1)} - \frac{C/D}{1-z^{-1}} V_2^{(1)} - (E/D) V_2^{(1)} \right]$$

$$- (K/B) V_i^{(1)} - \frac{F/B}{1-z^{-1}} V_2^{(1)}$$

$$V_2^{(1)} \left[1 + \frac{F/B}{1-z^{-1}} + \frac{(A/B)(C/D)\, z^{-1}}{(1-z^{-1})^2} + \frac{(A/B)(E/D)\, z^{-1}}{1-z^{-1}} \right]$$

$$= \frac{(A/B)(H/D)\, z^{-2}}{(1-z^{-1})^2} V_i^{(1)} - \frac{(A/B)(G/D)\, z^{-1}}{(1-z^{-1})^2} V_i^{(1)} - \frac{K}{B} V_i^{(1)}$$

Let $D(z) = 1 + \frac{F/B}{1-z^{-1}} + \frac{(AC/BD)\, z^{-1}}{(1-z^{-1})^2} + \frac{(AE/BD)\, z^{-1}}{1-z^{-1}}$

$$N(z) = \frac{(AH/BD)\, z^{-2}}{(1-z^{-1})^2} - \frac{(AG/BD)\, z^{-1}}{(1-z^{-1})^2} - (K/B)$$

Then $D(z) = \left[(1-z^{-1})^2 + (F/B)(1-z^{-1}) + (AC/BD) z^{-1} + (AE/BD) z^{-1}(1-z^{-1}) \right] \Big/ (1-z^{-1})^2$

$$= \frac{1 - 2z^{-1} + z^{-2} + F/B - (F/B) z^{-1} + (AC/BD) z^{-1} + (AE/BD) z^{-1} - (AE/BD) z^{-2}}{(1-z^{-1})^2}$$

$$= \frac{1 + F/B + (AC/BD) z^{-1} + (AE/BD - F/B - 2) z^{-1} + (1 - AE/BD) z^{-2}}{(1-z^{-1})^2}$$

6.5 (contd.)

$$N(z) = \frac{(AH/BD)z^{-2} - (AG/BD)z^{-1} - (K/B)(1-z^{-1})^2}{(1-z^{-1})^2}$$

$$= \frac{(AH/BD)z^{-2} - (AG/BD)z^{-1} - (K/B)(1-2z^{-1}+z^{-2})}{(1-z^{-1})^2}$$

$$= \frac{-K/B + \left(\frac{2K}{B} - AG/BD\right)z^{-1} + (AH/BD - K/B)z^{-2}}{(1-z^{-1})^2}$$

Hence, $\frac{V_2(1)}{V_1(1)} = \frac{N(z)}{D(z)}$,

$$= \frac{-\frac{K}{B} + \left(\frac{2K}{B} - \frac{AG}{BD}\right)z^{-1} + \left(\frac{AH}{BD} - \frac{K}{B}\right)z^{-2}}{1 + \frac{F}{B} + \frac{AC}{BD}z^{-1} + \left(\frac{AE}{BD} - \frac{F}{B} - 2\right)z^{-1} + (1 - AE/BD)z^{-2}}$$

$$= -\frac{KD + (AG - 2KD)z^{-1} + (KD - AH)z^{-2}}{BD + FD \cdots + (AE + AC - FD - 2BD)z^{-1} + (BD - AE)z^{-2}}$$

$$= -\frac{KD + (AG - 2KD)z^{-1} + (KD - AH)z^{-2}}{(B+F)D + (AC + AE - FD - 2BD)z^{-1} + (BD - AE)z^{-2}}$$

6.6

From Fig 6.20, in absence of L, K, F

around OA1:

$$V_1^{(1)} = \frac{H}{D} \cdot \frac{z^{-\frac{1}{2}}}{1-z^{-1}} V_i^{(2)} - \frac{G}{D} \cdot \frac{1}{1-z^{-1}} V_i^{(1)} - \frac{C}{D} \frac{1}{1-z^{-1}} V_2^{(1)} - \frac{E}{D} V_2^{(1)}$$

$$V_1^{(1)} = \frac{H}{D} \frac{z^{-1}}{1-z^{-1}} \cdot V_i^{(1)} - \frac{G}{D} \cdot \frac{1}{1-z^{-1}} \cdot V_i^{(1)} - \frac{C}{D} \cdot \frac{1}{1-z^{-1}} V_2^{(1)} - \frac{E}{D} V_2^{(1)}$$

$$= \frac{z^{-1}(H/D) - (G/D)}{1-z^{-1}} V_i^{(1)} - \frac{C/D}{1-z^{-1}} V_2^{(1)} - \frac{E}{D} \cdot V_2^{(1)}$$

$$= \frac{z^{-1} \frac{H}{D} - \frac{G}{D}}{1-z^{-1}} V_i^{(1)} - \left(\frac{E}{D} + \frac{C/D}{1-z^{-1}} \right) V_2^{(1)}$$

$$= \frac{z^{-1}(H/D) - G/D}{1-z^{-1}} V_i^{(1)} - \frac{(E/D)(1-z^{-1}) + C/D}{1-z^{-1}} V_2^{(1)}$$

around OA2, ignoring the $(\phi_1), (\phi_2)$ phases (F → absent)

$$V_2^{(1)} = \frac{A}{B} \frac{z^{-\frac{1}{2}}}{1-z^{-1}} V_1^{(2)} - \frac{I}{B} \frac{1}{1-z^{-1}} V_i^{(1)} + \frac{J}{B} \frac{z^{-\frac{1}{2}}}{1-z^{-1}} V_i^{(2)}$$

$$V_2^{(1)} = \frac{(A/B) z^{-1}}{1-z^{-1}} V_1^{(1)} - \frac{I/B}{1-z^{-1}} V_i^{(1)} + \frac{(J/B) z^{-1}}{1-z^{-1}} V_i^{(1)}$$ (Using $V_i^{(2)} = z^{-\frac{1}{2}} V_i^{(1)}$, $V_i^{(2)} = z^{-\frac{1}{2}} V_i^{(1)}$)

$$V_2^{(1)} = \frac{(A/B) z^{-1}}{1-z^{-1}} \left[\frac{z^{-1}(H/D) - G/D}{1-z^{-1}} V_i^{(1)} - \frac{(E/D)(1-z^{-1}) + C/D}{1-z^{-1}} V_2^{(1)} \right]$$

$$- \frac{I/B}{1-z^{-1}} V_i^{(1)} + \frac{(J/B) z^{-1}}{1-z^{-1}} V_i^{(1)}$$

$$= \frac{(A/B) z^{-1}}{1-z^{-1}} \cdot \frac{z^{-1}(H/D) - G/D}{1-z^{-1}} V_i^{(1)} - \frac{I/B}{1-z^{-1}} V_i^{(1)} + \frac{(J/B) z^{-1}}{1-z^{-1}} V_i^{(1)}$$

$$- \frac{(A/B) z^{-1}}{1-z^{-1}} \cdot \frac{(E/D)(1-z^{-1}) + C/D}{1-z^{-1}} V_2^{(1)}$$

$$V_2^{(1)} \left[1 + \frac{(A/B) z^{-1}}{1-z^{-1}} \cdot \frac{(E/D)(1-z^{-1}) + C/D}{1-z^{-1}} \right]$$

$$= \left[\frac{(A/B) z^{-1}}{1-z^{-1}} \cdot \frac{(H/D) z^{-1} - G/D}{1-z^{-1}} - \frac{I/B}{1-z^{-1}} + \frac{(J/B) z^{-1}}{1-z^{-1}} \right] V_i^{(1)}$$

6.6 (cont.)

$$V_2^{(1)}\, D(z) = N(z)\, V_1^{(1)}$$

$$D(z) = \frac{(1-z^{-1})^2 + \frac{A}{B} z^{-1}\left\{ \frac{E}{D}(1-z^{-1}) + C/D \right\}}{(1-z^{-1})^2}$$

$$= \frac{(1-z^{-1})^2 + \frac{A}{B}\frac{E}{D} z^{-1} - \frac{A}{B}\frac{E}{D} z^{-2} + \frac{A}{B}\frac{C}{D} z^{-1}}{(1-z^{-1})^2}$$

$$= \frac{1 - 2z^{-1} + z^{-2} + \left(\frac{AE}{BD} + \frac{AC}{BD}\right) z^{-1} - \frac{AE}{BD} z^{-2}}{(1-z^{-1})^2}$$

$$= \frac{1 - \left[2 - \left(\frac{AE}{BD} + \frac{AC}{BD}\right)\cancel{B}\right] z^{-1} + \left(1 - \frac{AE}{BD}\right) z^{-2}}{(1-z^{-1})^2}$$

$$N(z) = \frac{(A/B)(H/D) z^{-2} - (A/B)(G/D) z^{-1}}{(1-z^{-1})^2} - \frac{I/B}{1-z^{-1}} + \frac{(J/B) z^{-1}}{1-z^{-1}}$$

$$= \frac{(AH/BD) z^{-2} - (AG/BD) z^{-1} - (I/B)(1-z^{-1}) + (J/B) z^{-1}(1-z^{-1})}{(1-z^{-1})^2}$$

$$= \frac{(AH/BD) z^{-2} - (AG/BD) z^{-1} - (I/B) + (I/B) z^{-1} + (J/B) z^{-1} - (J/B) z^{-2}}{(1-z^{-1})^2}$$

$$= \frac{-(I/B) + (I/B - AG/BD + J/B) z^{-1} - (J/B) z^{-2} + (AH/BD) z^{-2}}{(1-z^{-1})^2}$$

So
$$\frac{V_2^{(1)}}{V_1^{(1)}} = \frac{N(z)}{D(z)} = \frac{-\frac{I}{B} + \left(\frac{I}{B} - \frac{AG}{BD} + \frac{J}{B}\right) z^{-1} + \left(\frac{AH}{BD} - J/B\right) z^{-2}}{1 - \left[2 - \left(\frac{AE}{BD} + \frac{AC}{BD}\right)\right] z^{-1} + \left(1 - \frac{AE}{BD}\right) z^{-2}}$$

$$= -\frac{DI + [AG - D(I+J)] z^{-1} + (DJ - AH) z^{-2}}{BD + (AE + AC - 2BD) z^{-1} + (BD - AE) z^{-2}}\,.$$

6.7

Eq (6.40) is :

$$H_{22}(z) = \frac{V_2^{(1)}}{V_1^{(1)}} = -\frac{DI + [AG - D(I+J)]z^{-1} + (DJ - AH)z^{-2}}{DB + [A(C+E) - 2DB]z^{-1} + (DB - AE)z^{-2}}$$

$$= -\frac{Y}{X} \text{ (say)}$$

i.e.
$$\frac{Y}{X} = \frac{DI + [AG - D(I+J)]z^{-1} + (DJ - AH)z^{-2}}{DB + [A(C+E) - 2DB]z^{-1} + (DB - AE)z^{-2}}$$

$$Y\left[DB + [A(C+E) - 2DB]z^{-1} + (DB - AE)z^{-2}\right] = X\left[DI + [AG - D(I+J)]z^{-1} + (DJ - AH)z^{-2}\right]$$

Let $Y_1 = DB$, $Y_2 = A(C+E) - 2DB$; $Y_3 = (DB - AE)$

$X_1 = DI$, $X_2 = AG - D(I+J)$, $X_3 = DJ - AH$

Then $Y \cdot Y_1 = X(X_1 + X_2 z^{-1} + X_3 z^{-2}) - Y \cdot (Y_2 z^{-1} + Y_3 z^{-2})$

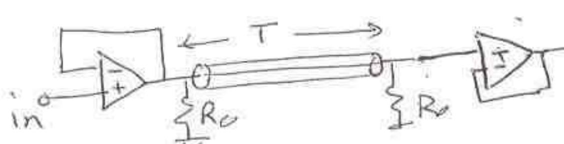
$$Y = X\left(\frac{X_1}{Y_1} + \frac{X_2}{Y_1}z^{-1} + \frac{X_3}{Y_1}z^{-2}\right) - Y\left[\frac{Y_2}{Y_1}z^{-1} + \frac{Y_3}{Y_1}z^{-2}\right]$$

$$= X\frac{X_1}{Y_1} + \left(X\frac{X_2}{Y_1} - Y\frac{Y_2}{Y_1}\right)z^{-1} + \left(X\frac{X_3}{Y_1} - Y\frac{Y_3}{Y_1}\right)z^{-2}$$

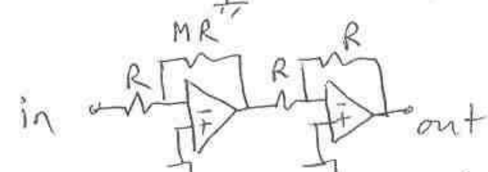
Let $\frac{X_1}{Y_1} = B_0$; $\frac{X_2}{Y_1} = B_1$; $\frac{Y_2}{Y_1} = B_2$; $\frac{X_3}{Y_1} = B_3$; $\frac{Y_3}{Y_1} = B_4$

$$Y = B_0 X + (B_1 X - B_2 Y)z^{-1} + (B_3 X - B_4 Y)z^{-2}$$

Let



represent $\Rightarrow$ $\boxed{z^{-1}}$ $\rightarrow$ Block of delay by 1 unit of T



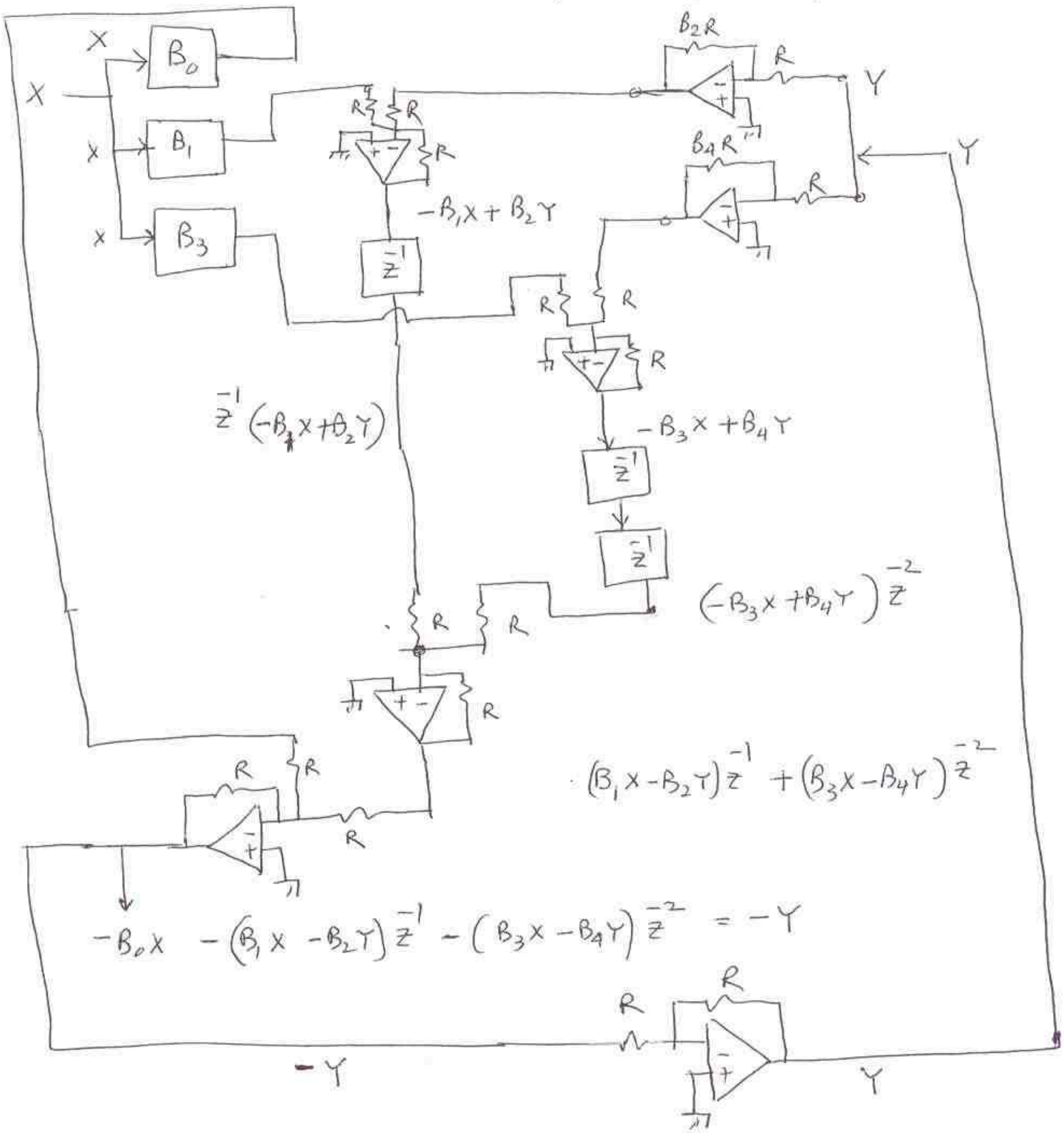
represent $\boxed{M}$ — gain of M block

Each block will be a sub circuit in the simulation program.

6.7
(contd.)

Consider the eqn

$$Y = B_0 X + (B_1 X - B_2 Y)\, z^{-1} + (B_3 X - B_4 Y)\, z^{-2}$$



The above represents the simulation system

6.8 For (P6.3) Given $H(s) = \dfrac{0.891975\, s^2 + 1.140926 \times 10^8}{s^2 + 356.0475 s + 1.140926 \times 10^8}$

$= 0.891975 \dfrac{s^2 + 1.27910087 \times 10^8}{s^2 + 356.0475 s + 1.140926 \times 10^8}$

Using $f_s = 64$ KHz, $T = \dfrac{1}{64{,}000}$, and pre-warping, we get (see Table 6.4)

$$H(z) = 0.8902645313 \; \frac{1 - 2.001691586 z^{-1} + z^{-2}}{1 - 1.966764535 z^{-1} + 1.004208024 z^{-2}}$$

* * * * * * * *

For (P6.4) Given (using pre-warping, since $f_s = 64$ KHz is not $>> 15$ KHz, the HPF cut off frequency) we get n = order = 5. Hence

$$H(s) = s^5 \Big/ \left[s^5 + 396482 s^4 + 0.6504039898 \times 10^{11} s^3 + 0.9069665315 \times 10^{16} s^2 + 0.5173875291 \times 10^{21} s + 0.4159336312 \times 10^{26} \right]$$

Using $f_s = 64$ KHz and BLT, we will get

$$H(z) = 0.1099511628 \times 10^{11} \; \frac{(1 - z^{-1})^5}{D(z)}$$

$$D(z) = 0.15700782 9 \times 10^{9} z^{-5} + 0.1885127413 \times 10^{11} z^{-4} - 0.6444713465 \times 10^{11} z^{-3} + 0.1171053664 \times 10^{12} z^{-2} - 0.1049767527 \times 10^{12} z^{-1} + 0.4662020098 \times 10^{11}$$

To implement this 5th order function by using the structure in Fig. 6.20, we need to decompose $H(z)$ as

$H(z) = H_1(z) H_2(z) H_3(z)$ where H_1, H_2 are second order functions and H_3 is a first order function in z^{-1}.

One can use MATLAB for the decomposition.

6.8 contd. from CAP7 Let $D = m, \quad A = m$

Then $h_D = DI ; \quad I = \frac{h_D}{m}$

$$AG - DI - DJ = mG - mI - mJ = m(G - I - J)$$

$$= m\left(G - \frac{h_D}{m} - J\right) = m(G - J) - h_D = h_D a_{1N}$$

$$DJ - AH = m(J - H) = h_D a_{2N}$$

$DB = 1 \quad ; \quad D = m ; \quad B = \frac{1}{m}$

$$AC + AE - 2DB = mC + mE - 2 = -a_{1D}$$

$$m(C + E) = 2 - a_{1D}.$$

$$DB - AE = 1 - mE = a_{2D} \quad ; \quad E = \frac{1 - a_{2D}}{m}$$

$A = m \quad B = \frac{1}{m}$

$D = m$

$I = \frac{h_D}{m}$

$E = \frac{1 - a_{2D}}{m}$.

$C = \frac{1 - a_{1D} + a_{2D}}{m}$

$$\left.\begin{aligned} m(G - J) &= h_D(1 + a_{1N}) \\ m(J - H) &= h_D a_{2N} \end{aligned}\right\}$$

$$m(C + E) = 2 - a_{1D}$$

$$mC = 2 - a_{1D} - mE = 2 - a_{1D} - 1 + a_{2D}$$

$$mC = 1 - a_{1D} + a_{2D}$$

So only G, J, H remain unknown with two eqns.

$$\left.\begin{aligned} G - J &= \frac{h_D(1 + a_{1N})}{m} \\ J - H &= \frac{h_D a_{2N}}{m} \end{aligned}\right\}$$

Let $J = m$?

$$G - m = \frac{h_D(1 + a_{1N})}{m}$$

$$G = m + \frac{h_D(1 + a_{1N})}{m}$$

$$m - H = \frac{h_D a_{2N}}{m}$$

$$H = m - \frac{h_D a_{2N}}{m}$$

'm' should be such that all Caps ≥ 0

$$m - \frac{h_D a_{2N}}{m} > 0 \quad \& \quad m + \frac{h_D(1 + a_{1N})}{m} > 0$$

6-8 (cont.)

Filter coefficients

$$h_D = .8909\,, \quad a_{1D} = 1.9903\,; \quad a_{1N} = -1.9922$$

$$a_{2D} = 0.9972\,, \quad a_{2N} = 1$$

Condition $\quad m - \dfrac{h_D a_{2N}}{m} > 0$ requires

$$m > \sqrt{h_D a_{2N}} \rightarrow \sqrt{.8909 \times 1} \rightarrow 0.94387$$

So let $\quad m = 0.94387 + \alpha$

Condition $\quad m + \dfrac{h_D (1 + a_{1N})}{m} > 0$

$$m + \frac{.8909\,(1 - 1.9903)}{m} > 0$$

$$m - \frac{.8909 \times .9903}{m} > 0 \quad ; \; m > \sqrt{.8909 \times .9923}$$

$$m > \quad 0.94018\ldots$$

So $m = .94387 + \alpha$ will suffice both conditions

Hence $\quad \Sigma C = 5\,(.94387 + \alpha) + \dfrac{h_D\,(2 + a_{1N} - a_{2N}) + 2 - a_{1D}}{(.94387 + \alpha)}$

$\beta = 5 \times .94387$

$$= \beta + 5\alpha + \frac{.8909\,(2 - 1.9922 - 1) + 2 - 1.9903}{0.94387 + \alpha}$$

6.8 (cont.)

$$\sum C = 5(.94387+\alpha) + \frac{h_D(2+a_{1N}-a_{2N})+2-a_{1D}}{.94387+\alpha}$$

$\frac{\partial}{\partial\alpha}\left[\sum C\right]=0$ will give a 'real' value of α provided

$$h_D\,(2+a_{1N}-a_{2N})+2-a_{1D} > 0.$$

In the present case, the above is not true. An alternative topology could provide the desired uni-variable optimization solution.

6.9 The transfer function in problem 5.3 is:

$$\frac{600\,s}{s^2+600s+3\times10^8}.$$

Thus, $\omega_p^2 = 3\times10^8$, $\omega_p/Q_p = 600$.

Using pre-warping and BLT, the SC-transfer function for $f_s = 120$ kHz is:

$$H(z) = 2.4809\times10^{-3}\,\frac{1-z^{-2}}{1-1.9744\,z^{-1}+0.99504\,z^{-2}}$$

Follow Example 6.3 or Example 6.4 or 6.5.

6.10 The s-domain transfer function is

$$H(s) = \frac{(\omega_p/Q_p)s}{s^2+(\omega_p/Q_p)s+\omega_p^2} = \frac{(2\pi\times10^4/10)s}{s^2+\left(\frac{2\pi\times10^4}{10}\right)s+(2\pi\times10^4)^2}$$

The associated SC-TF is, for $f_s = 100$ KHz

$$H(z) = 0.0278018249\,\frac{(1-z^{-2})}{1-1.594983015\,z^{-1}+0.9443963502\,z^{-2}}$$

Follow up as in P 6.9.

6.11

$1\ dB = \text{antilog}_{10}(1/20) = 10^{.05} = 1.122$

The hpf is then $H(s) = \dfrac{1.122\ s^2}{s^2 + \dfrac{2\pi \times 1.2 \times 10^3}{\sqrt{2}} + (2\pi \times 1.2 \times 10^3)^2}$

Using $f_s = 256$ kHz, the SC-TF becomes

$$H(z) = 1.110195461 \frac{1 - 2z^{-1} + z^{-2}}{1 - 1.978528770\, z^{-1} + 0.9793873116\, z^{-2}}$$

- Follow up as in P6.9.
- Find the highest valued capacitor, say C_{max}. → to be 20 pF.

Scale all capacitors as $[20/C_{max}]$ pF values.

x

6.12 The lpf TF is:

$$H(s) = \frac{(2\pi \times 3.4 \times 10^3)^2}{s^2 + \dfrac{2\pi \times 3.4 \times 10^3}{1.3} s + (2\pi \times 3.4 \times 10^3)^2}$$, given l.f. gain of 0 dB

The SC-TF is: (with $f_s = 256$ kHz)

$$H(z) = 0.0016844 \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.93116\, z^{-1} + 0.937902\, z^{-2}}$$

Follow the procedure as in P6.11

x

6.13 Notch filter in P 6.3 is $\dfrac{.891975 s^2 + 1.140926 \times 10^8}{s^2 + 356.047 s + 1.140926 \times 10^8}$

Using $f_s = 69$ kHz,

$$H(z) = 0.890263 \frac{1 - 1.969014\, z^{-1} + z^{-2}}{1 - 1.966905\, z^{-1} + 0.9944904\, z^{-2}}$$

Use Tables 6.5, 6.6 for further work

6.14

$$T(s) = \frac{12\times10^6}{s^2 + 6\times10^3 s + 12\times10^6}$$

So $\omega_p^2 = 12\times10^6$; $\omega_p = \sqrt{12}\times10^3 = 3.4641\times10^3$

Let us choose $f_s \sim 5$ times ω_p = 20 kHz.

We can derive $H(z) = h_D \frac{1 + a_{1N} z^{-1} + a_{2N} z^{-2}}{1 - a_{1D} z^{-1} + a_{2D} z^{-2}}$

as $H(z) = 0.0064795 \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.714903 z^{-1} + 0.7408207 z^{-2}}$

Use the method ~~8~~ as in Example 6.3 to design the capacitances. To simulate the 'designed' response, use these capacitance values to re-calculate the coefficients, for ~~eee~~ example, A, B, C, D, ... (eq 6.41) and then find H(z).
Compare using MATLAB plots the response $|H(z)|$ and $|T(s)|$.

6.15 In P6.3 $H(z) = 0.8909255285 \frac{1 - 1.992198066 z^{-1} + z^{-2}}{1 - 1.990275499 z^{-1} + 0.9972254644}$

Using notations of DA1, DA2, D networks (see eq. 6.55)

$$h_D = \frac{1}{(1 + C_{0N}/C_{IN})(1 + C_{0D}/C_{ID})}$$

$$a_{1N} = \frac{C_{1N}}{C_{IN}}, \quad a_{2N} = \frac{C_{2N}}{C_{IN}}$$

$$a_{1D} = \frac{C_{1D}}{(C_{0D} + C_{ID})}; \quad a_{2D} = \frac{C_{2D}}{C_{0D} + C_{ID}}$$

Let $h_D' = \sqrt{0.8909255285} = 0.943888514868$

$$1/h_D' = 1.05944715318 \rightarrow (1.02929449293)^2$$

6.15 (cont.)

We can let $\frac{C_{ON}}{C_{IN}} = .0293 = \frac{C_{OD}}{C_{ID}}$

So that $h_D = \frac{1}{(1.0293)^2} \cong 0.94387$ is realized

If $C_{ON} = C_{OD} = 1$ pF, $C_{IN} = C_{ID} = 31.1296 \approx 31.13$ pF.

Then $a_{1N} \cong 1.992 = \frac{C_{1N}}{C_{IN}}$; $C_{1N} = 1.992 \times 31.13 = 62.01$ pF.

$a_{2N} = 1$; $C_{2N} = C_{IN} = 31.13$ pF.

$a_{1D} = \frac{C_{1D}}{C_{OD} + C_{ID}} \cong 1.99 = \frac{C_{1D}}{C_{OD} + 31.13}$; ~~Let $C_{1D} = 1$ pF.~~

~~Then $C_{OD} + 31.13 = \frac{1}{1.99} = 0.5025$; C~~

$C_{1D} = (C_{OD} + C_{ID}) \times 1.99 = 32.13 \times 1.99$, assuming $C_{OD} = 1$ pF

$C_{1D} = 63.94$ pF.

$a_{2D} = \frac{C_{2D}}{C_{OD} + C_{ID}} = 0.9972$; $C_{2D} = 32.13 \times 0.9972 = 32.04$ pF.

So the design capacitances are

$C_{ON} = C_{OD} = 1$ pF ; $C_{1N} = 62.01$ pF ; $C_{2N} = 31.13$ pF.

$C_{IN} = C_{ID} = 31.13$ pF. ; $C_{1D} = 63.94$ pF ; $C_{2D} = 32.04$ pF.

For 6.4, we have

$$H(z) \cong 0.10995 \times 10^{11} \frac{(1 - z^{-1})^5}{D(z)}$$

$D(z) \cong 0.1370078 \times 10^9 z^5$ ... etc.

Factor $D(z)$ as $D_1(z) D_2(z) D_3(z)$ where D_1, D_2 are of second order while D_3 is of order 1.

Then decompose $H(z)$ as $= h_1 \frac{(1-z^{-1})^2}{D_1(z)} \cdot h_2 \frac{(1-z^{-1})^2}{D_2(z)} \cdot h_3 \frac{1-z^{-1}}{D_3(z)}$

Realize the sub-functions $h_1 \frac{(1-z^{-1})^2}{D_1(z)}$, $h_2 \frac{(1-z^{-1})^2}{D_2(z)}$ ...

as above (using DA1, DA2, D SCF-networks)

6.16 In 6.12 we had

$$H(z) = 0.0016844 \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.93116\, z^{-1} + 0.937902\, z^{-2}}$$

In terms of the CDA network

$$H(z)\Big|_{CDA} = \frac{A_1 + A_2 z^{-1} + A_3 B_3 z^{-2}}{1 - A_3 B_1 z^{-1} + A_3 B_2 z^{-2}}$$

Thus $A_1 = .0016844$, $A_2 = 2A_1$; $A_3 = A_1$

$A_3 B_1 = 1.93116$; $A_3 B_2 = .937902$

Design capacitances are given by

$$A_1 = \frac{C_1}{C_1 + C_{S1}} \; ; \; A_2 = \frac{C_2 C_{S1}}{C_{01}(C_1 + C_{S1})} \; ; \; A_3 = \frac{C_3 C_{S1}}{C_{01}(C_1 + C_{S1})}$$

$$B_1 = \frac{C_4}{C_4 + C_{S2}} \; ; \; B_2 = \frac{C_5 C_{S2}}{C_{02}(C_4 + C_{S2})} \; ; \; B_3 = \frac{C_6 C_{S2}}{C_{02}(C_4 + C_{S2})}$$

Six equations, ten capacitors ; so 4 free choices possible.

Let $C_1 = 1$ PF., then $C_{S1} = \frac{1}{A_1} - C_1 = 592.68$ PF.

$$A_2 = 2 \times .0016844 = \frac{C_2}{C_{01}} \cdot \frac{592.68}{593.68}$$

If we let $C_2 = 1$ PF., $C_{01} = \frac{592.68}{593.68} \cdot \frac{1}{2 \times .0016844} = 296.34$ PF.

$$A_3 = .0016844 = \frac{C_3}{C_{01}} \cdot \frac{C_{S1}}{C_1 + C_{S1}} = \frac{C_3}{296.34} \frac{592.68}{593.68} \; ; \; C_3 = 0.5 \text{ PF}.$$

continue this way. & use capacitance scaling to bring $C_{min} = 1$ PF.

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* Design impossible! Since $B_1 < 1$ while by the numbers $A_3 B_1 = 1.93116$ and $A_3 = .0016844$, $B_1 \gg 1$.