Sol. / hint to Problems
Ch. 7

7.1

$$[a]_N = \begin{bmatrix} 1 & 0 \\ 0 & \frac{Z_2 Z_4}{Z_1 Z_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1/F(s) \end{bmatrix}$$

(a) $$[a]_{N_R} = F(s)\begin{bmatrix} 1/F(s) & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & F(s) \end{bmatrix}$$ using (3.19)

Hence N_R is also a GIC

(b) The chain matrix of Fig P. 7.1 is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1/F(s) \end{bmatrix}\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & F(s) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & F(s)\,R \\ 0 & 1 \end{bmatrix}$$

If $F(s) = s$, then the above chain matrix corresponds to a series ~~an inductor~~ an impedance of value sR, i.e., a floating inductor L of value R.

(c) The chain matrix of Antoniou's GIC (p. 211) is

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{Z_2 Z_4}{Z_1 Z_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1/F(s) \end{bmatrix}$$

To make $F(s) = s$, we may choose

$$Z_1 = Z_3 = Z_4 = R_1, \quad Z_2 = \frac{1}{sC_1}, \quad R_1 C_1 = 1.$$

7.1 (contd.)

(d) ~~If~~ $F(s) = 1/s^2$, then the chain matrix of fig P7.1 is

$$\begin{bmatrix} 1 & R/s^2 \\ 0 & 1 \end{bmatrix}$$

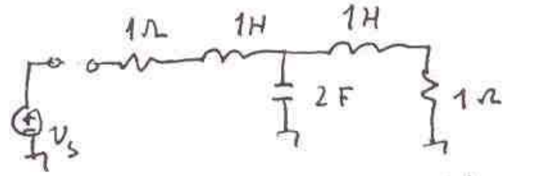
The above corresponds to a floating FDNR whose value $D = 1/R$

(e) choosing $Z_1 = Z_3 = R_1$, $Z_2 = Z_4 = 1/sC_1$, $R_1C_1 = 1$,

we have $F(s) = \dfrac{R_1^2}{\frac{1}{s^2C_1^2}} = s^2 R_1^2 C_1^2 = s^2$.

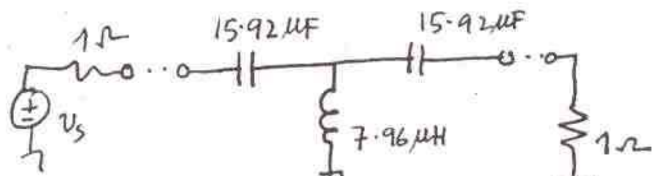
SOLUTION / HINTS TO PRACTICE PROBLEMS IN CH. 7

7.2 3rd order BUT, LC normalized LPF is: (see appendix)

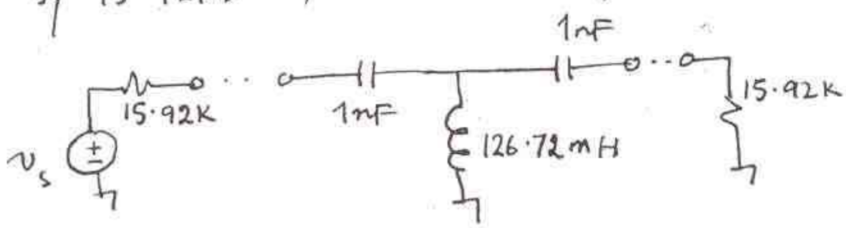


The frequency denormalized HPF is: $L_{HP} \to \frac{1}{\omega_c C_{LP}}$; $C_{HP} \to \frac{1}{\omega_c L_{LP}}$

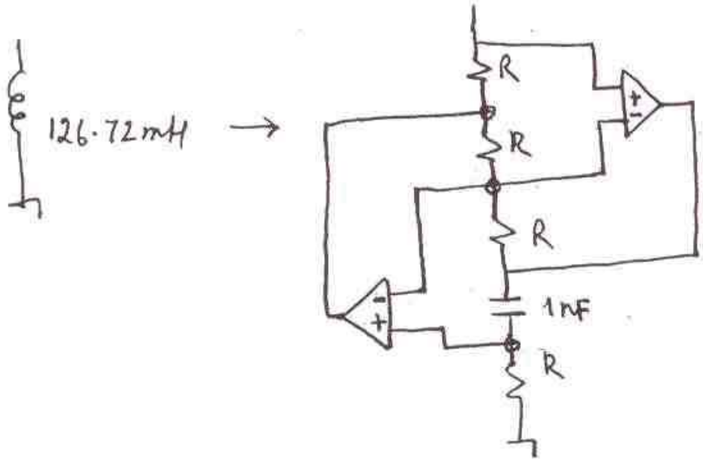
$\omega_c = 2\pi \times 10^4$



If all caps → 1 nF. divide 15.92μF by 15.92×10^3, multiply 7.96μH by 15.92×10^3, multiply R_s, R_L by 15.92×10^3.



One needs to realize the 126.72 mH inductance by GIC. The capacitors in GIC has to be 1nF.



$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

$= sCR^2$ if

$Z_1 = Z_3 = Z_2 = Z_5 = R$

$Z_4 = 1/sC$

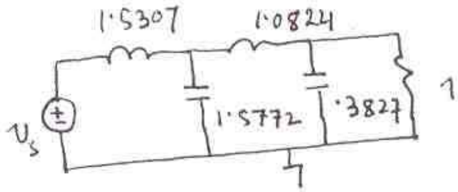
$L = CR^2$, then

For $L = 126.72$ mH

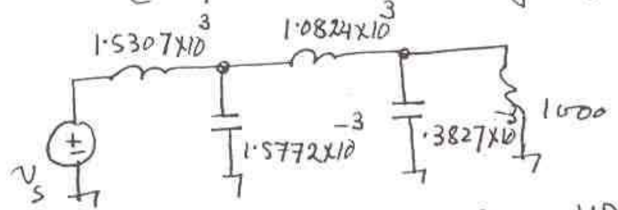
$C = 1$ nF

$R = 11.257$ KΩ

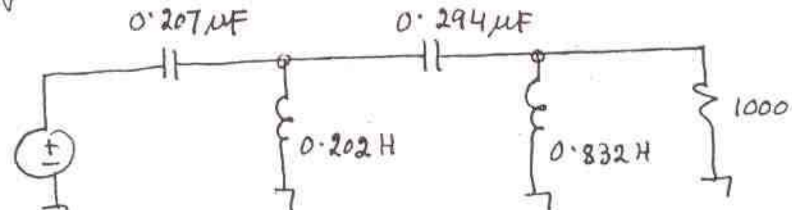
(7.3) 4^{th} order BUT, normalized LPF with 1Ω termination is:



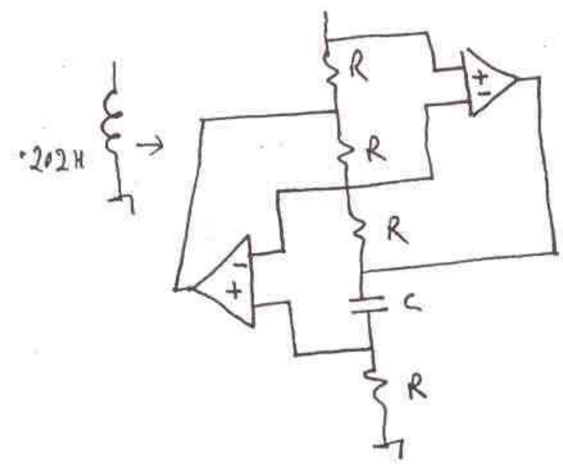
For $R_L = 1000\Omega$, the network will be (impedance scaling..)

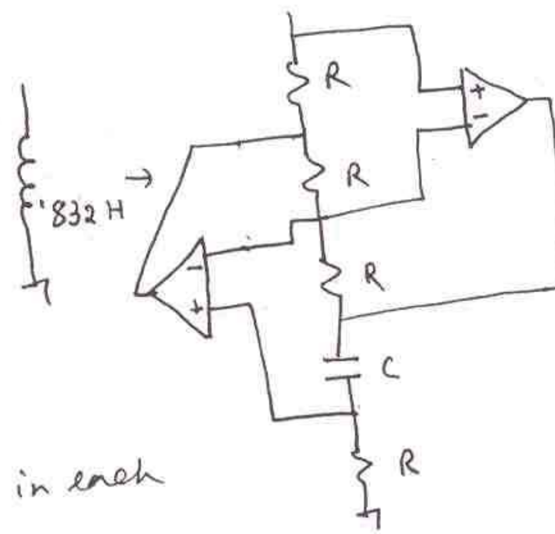


For a 3dB freq. of 500Hz, $w_c = 2\pi \times 500$. The LP $\Leftrightarrow$ HP transformation will give the HPF.



To replace the grounded L by GIC,



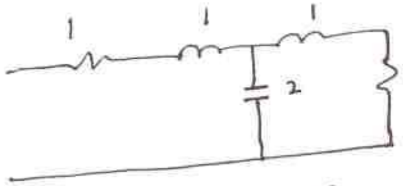


$\because Z_{in} = SCR^2$ with $R = 10K\Omega$ in each case

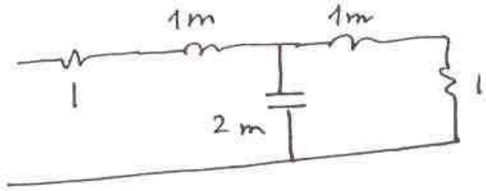
$C\Big|_{.202H} \rightarrow 2.02nF$

$C\Big|_{.832H} \rightarrow 8.32nF.$

(7.4) 3rd order BUT LP network:

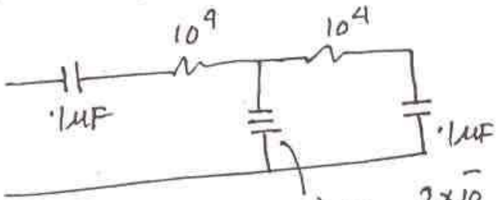


For a bandwidth of 1000 rad/sec, the network is:



$R \to C$, $L \to R$, $C \to D$ ← super capacitor

For an FDNR realization



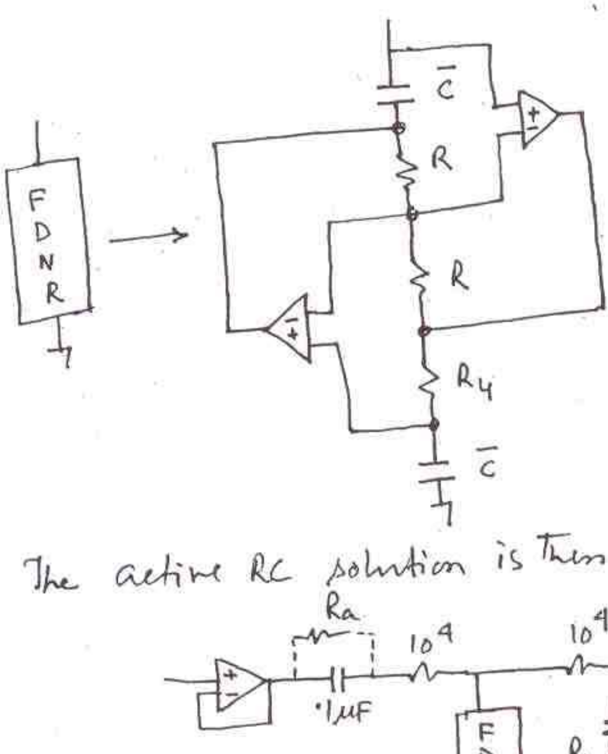
Choose $k = 10^{-7}$

so that all caps. are 0·1μF.

$D = 2 \times 10^{-10} \leftarrow RC$

With a FDNR network, $D = \bar{C}^2 R_4$ where $\bar{C}$ is the capacitance used in the FDNR. Thus

$\because \bar{C} = 0{\cdot}1\,\mu F$ as specified



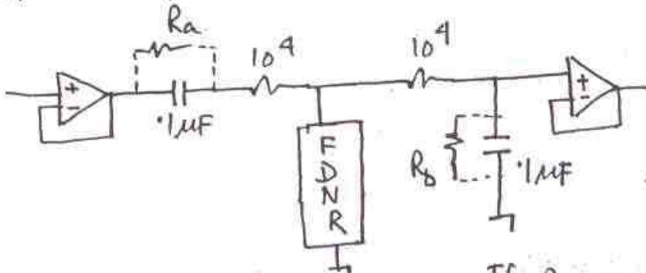
$10^{-14} \cdot R_4 = 2 \times 10^{-10}$

$R_4 = 2 \times 10^{4} = 20\,K$

$R = 1K$ each.

R_a, R_b are DC continuity providing resistances.

The active RC solution is then



Assuming a gain $= \frac{1}{2}$ at DC (for equal R_L, R_s)

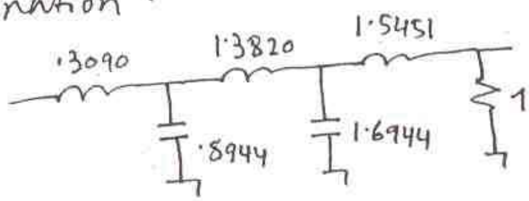
$$\frac{R_b}{R_a + 20K} = \frac{1}{2}$$

If $R_a = 1M\Omega$, $R_b = 510K$

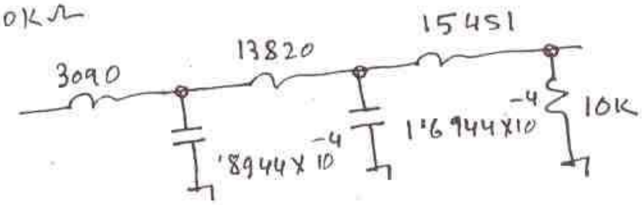
(7.5) Follow methods in (7.3)^{4}. Use impedance scaling for $R_L = R_S = 500\,\Omega$. Use frequency scaling to all the C & L in the prototype LP section.
Use FDNR principle.
For the super capacitor section, $D = \bar{C}^2 R_4$, let $R_4 = 7500 = R$.
Find $\bar{C}$.
Use buffer amplifiers in front and at end. Use DC continuity resistance.

X

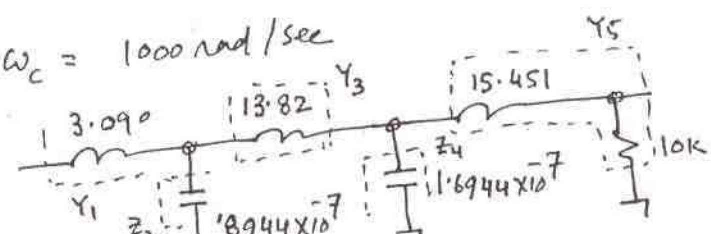
(7.6) Fifth order LP - LC prototype filter. Assume single $R_L = 1\,\Omega$ termination.



For $R_L = 10\,K\Omega$



For a $\omega_c = 1000$ rad/sec



The branch gain functions are, successively, (see Fig. 14, eqn set 8(a))

$T_1 \Rightarrow Y_1 = \frac{1}{sL_1}$ where $L_1 = 3.09$

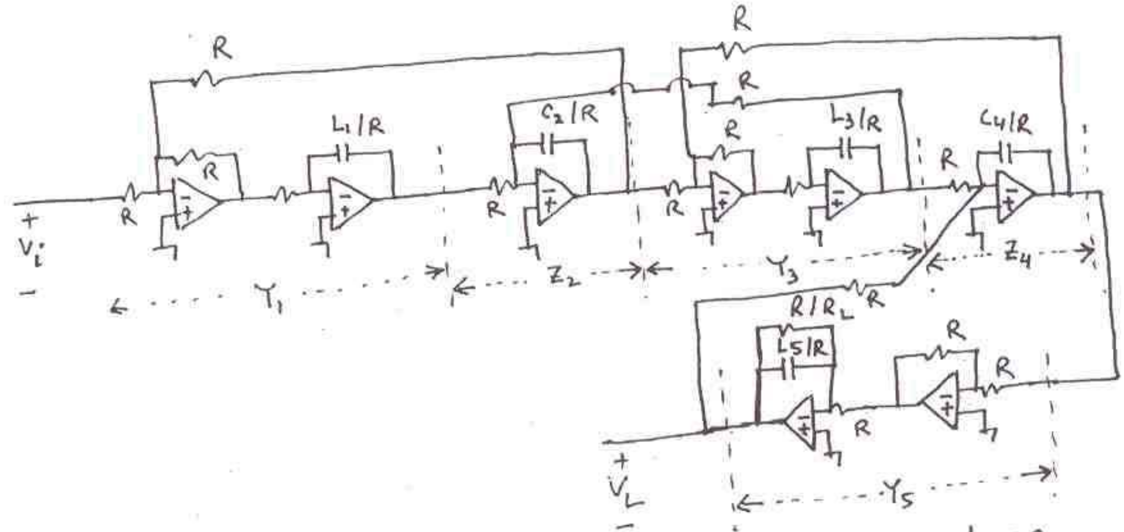
$T_2 \Rightarrow -Z_2 = -\frac{1}{sC_2}$, $C_2 = .8944 \times 10^{-7}$

$T_3 \Rightarrow Y_3 = \frac{1}{sL_3}$, $L_3 = 13.82$

$T_4 \Rightarrow -Z_4 = -\frac{1}{sC_4}$, $C_4 = 1.6944 \times 10^{-7}$

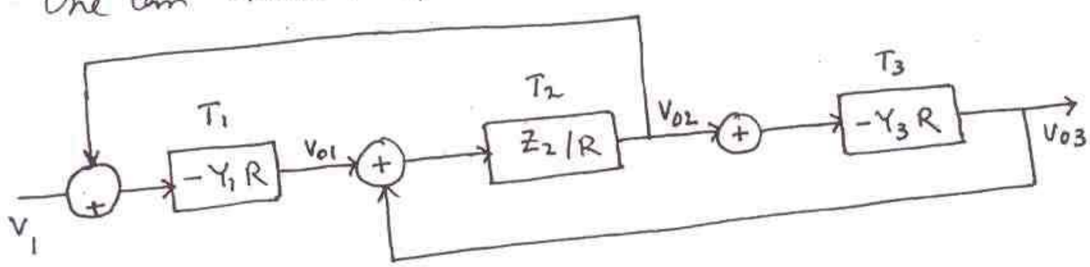
$T_5 \Rightarrow Y_5 = \frac{1/L_5}{s + R_L/L_5}$, $L_5 = 15.451$, $R_L = 10K$

(7.6) Contd. The integrator blocks are realized as:

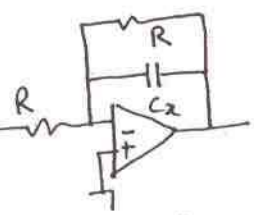


The Leap-Frog interconnections are also shown above. Assume $R = 100\,\Omega$ and obtain the values of the integrating capacitors in the various sections.

(7.7) The first section is an inverting lossy integrator
The second section is a non inverting lossless integrator
The third section is an inverting lossy integrator
One can then adopt the structure shown in Fig. 15, redrawn below.



The section



$C_x = C/2$

corresponds to a voltage T.F. $T_1 = \dfrac{-1/RC_x}{s + 1/R_x C_x}$ with $R_x = R$, $C_x = C/2$

Similarly, $T_2 = \dfrac{1}{sCR}$, $T_3 = -\dfrac{1/RC_x}{s + 1/RC_x}$, $C_x = \dfrac{C}{2}$

$V_{01} = T_1 (V_1 + V_{02})$; $V_{02} = T_2 (V_{01} + V_{03})$

$V_{03} = T_3 \cdot V_{02}$

(7.7) contd. Rearranging

$$V_{01} - T_1 V_{02} = T_1 V_1$$
$$T_2 V_{01} - V_{02} + T_2 V_{03} = 0$$
$$T_3 V_{02} - V_{03} = 0$$

In matrix notation

$$\begin{bmatrix} 1 & -T_1 & 0 \\ T_2 & -1 & T_2 \\ 0 & T_3 & -1 \end{bmatrix} \begin{bmatrix} V_{01} \\ V_{02} \\ V_{03} \end{bmatrix} = \begin{pmatrix} T_1 V_1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta = (1 - T_2 T_3) - T_2 (T_1) = 1 - T_1 T_2 - T_2 T_3$$

$$V_{03} = \frac{1}{\Delta} \begin{vmatrix} 1 & -T_1 & T_1 V_1 \\ T_2 & -1 & 0 \\ 0 & T_3 & 0 \end{vmatrix} = \frac{1}{\Delta} \cdot (-T_2)(-T_1 T_3 V_1) = \frac{T_1 T_2 T_3 V_1}{\Delta}.$$

$$\frac{V_{03}}{V_1} = \text{transfer function} = \frac{T_1 T_2 T_3}{1 - T_1 T_2 - T_2 T_3} = T(s)$$

Subst. for T_1, T_2, T_3 . . .

$$T_1 T_2 T_3 = \frac{1/RC_x}{s + 1/RC_x} \cdot \frac{1}{sCR} \cdot \frac{1/RC_x}{s + 1/RC_x} = \frac{2/CR}{s + 2/CR} \cdot \frac{1}{sCR} \cdot \frac{2/CR}{s + 2/CR}$$

$$T_1 T_2 = -\frac{2/CR}{s + 2/CR} \cdot \frac{1}{sCR} \quad ; \quad T_2 T_3 = -\frac{2/CR}{s + 2/CR} \cdot \frac{1}{sCR}$$

So

$$T(s) = \frac{\left(\frac{2/CR}{s + 2/CR}\right)^2 \cdot 1/sCR}{1 + \frac{2/CR}{s + 2/CR} \cdot \frac{1}{sCR} + \frac{2/CR}{s + 2/CR} \cdot \frac{1}{sCR}}.$$

$$= \frac{\left(\frac{2/CR}{s + 2/CR}\right)^2 \frac{1}{sCR} \cdot sCR \cdot (s + 2/CR)}{sCR\left(s + \frac{2}{CR}\right) + 4/CR}$$

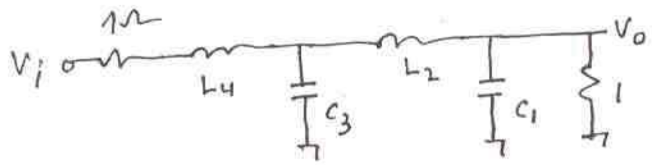
$$= \frac{\frac{(2/CR)^2}{(s + 2/CR)}}{s^2 CR + 2s + 4/CR} = \left(\frac{2}{CR}\right)^2 \cdot \frac{1}{s + 2/CR} \cdot \frac{1}{s^2 CR + 2s + \frac{4}{CR}}$$

$$= \left(\frac{2}{\cancel{C}R}\right)^2 \cdot \frac{\cancel{CR}}{2 + sCR} \cdot \frac{\cancel{CR}}{s^2 C^2 R^2 + 2sCR + 4} = \frac{4}{(sCR + 2)(s^2 C^2 R^2 + 2sCR + 4)}$$

$$4 / C^3 R^3$$

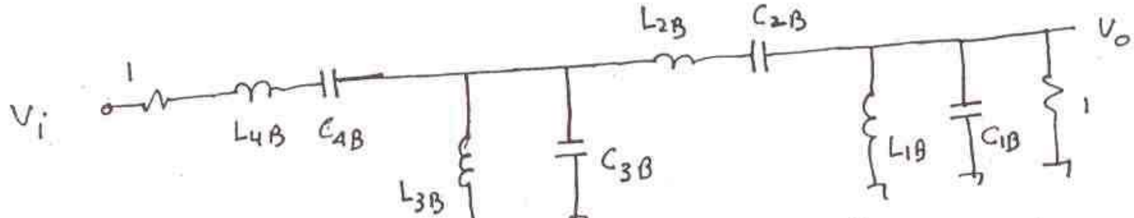
Subst. $R = 10^4$, $C = 10^{-8}$ now.

(7.8) 4^{th} order MFM filter (assume BUT) with 1Ω double termination is:



$L_4 = 0.7654$
$C_3 = 1.8478$
$L_2 = 1.8478$
$C_1 = .7654$

For the BPF with $B = 1$ rad/sec, $\Omega_0 = 1$ rad/sec, component transformation is used:



$L_{4B} = \frac{L_4}{B} = L_4$

$C_{4B} = \frac{B}{L_4 \Omega_0^2} = \frac{1}{L_4}$

$L_{3B} = \frac{B}{\Omega_0^2 C_3} = \frac{1}{C_3}$

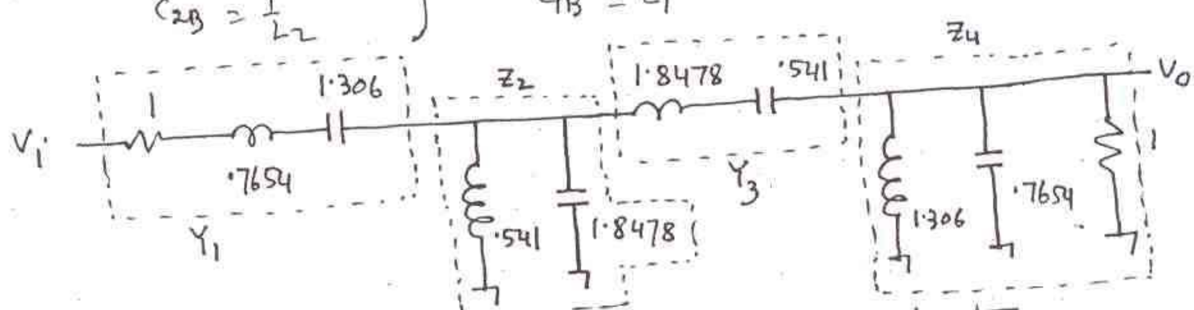
$C_{3B} = C_3$

$L_{2B} = L_2$

$C_{2B} = \frac{1}{L_2}$

$L_{1B} = \frac{1}{C_1}$

$C_{1B} = C_1$



is to be implemented in Leap-Frog architecture. Now use the technique suggested in Figures 22(a)-(b).

x

(7.9) For a fourth order BPF, the associated prototype LPF will be of order $4/2 = 2$. For BUT magnitude response (MFM is assumed here as BUT since Ap data is lacking).

$H_{..}| = \frac{-H}{\quad}$??

(7.9) Contd. On applying LP → BP transformation, the above TF. will assume the form:

$$\frac{H_{01}\left(\frac{\omega_{p1}}{Q_p}\right)s}{s^2+\left(\frac{\omega_{p1}}{Q_p}\right)s+\omega_{p1}^2}\cdot\frac{H_{02}\left(\frac{\omega_{p2}}{Q_p}\right)s}{s^2+\left(\frac{\omega_{p2}}{Q_p}\right)s+\omega_{p2}^2}$$, where $H_{01}H_{02}=H$

If the resonant gain is $=1$, it means $H_{01}=H_{02}=1$. So $H=1$

Hence $\quad H_{LP}|_N = \dfrac{-1}{s^2+\sqrt{2}s+1}$, Here $b_2=1,\ b_1=\sqrt{2},\ b_0=1$

Then $\quad a_0 = H b_0 = 1$

$a_1 = b_{n-1} - nC = b_1 - 2C = \sqrt{2}-2C$. If $a_1=0$, $C=\frac{1}{\sqrt{2}}$

$a_2 = b_0 - \binom{2}{2}C^2 = 1 - 1\cdot\frac{1}{2} = \frac{1}{2}$

Then $H_0 = \frac{1}{C} = \sqrt{2}$, $Q_p = \frac{Q}{C} = 10\sqrt{2} = 14.14$ $\because Q = \frac{1\,\text{rad}}{0.1\,\text{rad}} = 10$

The PRB network then has the TF.

$$T_s = \frac{(H_0/Q_p)\,s}{s^2+s/Q_p+1}$$, $\because \omega_0 = 1$ is given

$$= \frac{0.1\,s}{s^2+\frac{s}{14.14}+1}$$

We need to use two such second order BPF with feedback to realize the overall 4th order BPF.

Realize each T_s using IGSAB architecture (see Ch 4).

Possible solution is: $\quad C_2 = C_3 = 1\text{F}$

$$R_1 = \frac{Q_p}{|H_0|} = \frac{14.14}{1.414} = 10\ \Omega$$

$$R_5 = \frac{Q_p}{2Q_p^2-|H_0|} = \frac{14.14}{2\times 14.14^2 - 1.414} = 0.035\ \Omega$$

$$R_6 = 2Q_p = 28.28\ \Omega$$

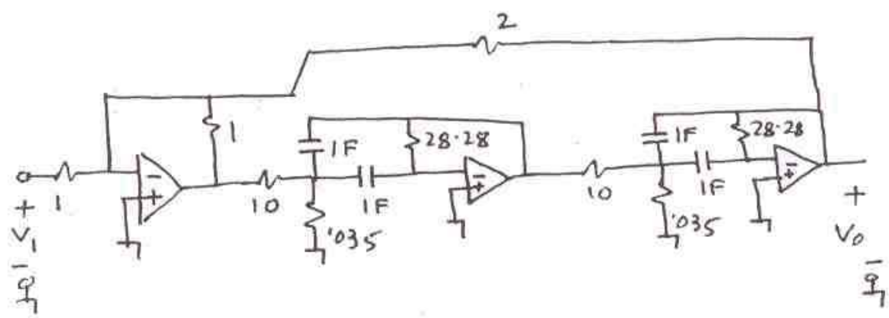
$\because H=1$, $\quad R_0 = \frac{R_f}{1} = R_f$. We can set $R_0 = R_f = 1\,\Omega$

(7.9) Contd. For the feedback connections

$a_1 \to 0$ implies $R_1 \to \infty$

$a_2 \to \frac{1}{2}$ implies $R_2 = 2\ \Omega$

Final configuration is:



(7.10) 3rd order BUT prototype LP transfer function is:

$$H_{LP}|_N = \frac{-H}{s^3 + 2s^2 + 2s + 1}$$

After applying LP $\leftrightarrow$ BP transformation $s \to Q \cdot \frac{p^2+1}{p}$ where $\Omega_0 = 1$ rad/s has been used)

$$H_{BP} = \frac{-H.}{\left(Q\frac{s^2+1}{s}\right)^3 + 2\cdot\left(Q\frac{s^2+1}{s}\right)^2 + 2\cdot\left(Q\frac{s^2+1}{s}\right) + 1} = \frac{-H.\, s^4/Q^4}{D_1(s)}$$

$$D_1(s) = \sum_{i=0}^{4} \frac{b_{4-i}\, s^i\, (s^2+1)^{4-i}}{Q^i}$$, Given $Q = 1/.1 = 10$.

The gain at the resonant frequency $\Omega_0 = 1$ i.e. $s = j$ is H and this is given as 2.

Then $a_0 = H \cdot b_0 = 2$

$a_1 = b_3 - 4C = 2 - 4C$. For $a_1 = 0$, $C = \frac{1}{2}$

Then $a_2 = b_2 - \binom{4}{2} C^2 - \sum_{i=1}^{1} \binom{4-i}{2-i} a_i C^{2-i} = 2 - \frac{4\cdot 3}{2}\cdot\left(\frac{1}{2}\right)^2 = 0{\cdot}5$

$$Q_p = \frac{n}{b_{n-1}} \cdot Q = \frac{Q}{C} = \frac{10}{\frac{1}{2}} = 20$$

(7.10) contd.

$$a_3 = b_1 - \binom{4}{3} c^3 - \sum_{i=1}^{2} \binom{4-i}{3-i} a_i c^{3-i}$$

$$= b_1 - 4 \cdot c^3 - \binom{3}{2} a_1 c^2 - \binom{2}{1} a_2 c^1$$

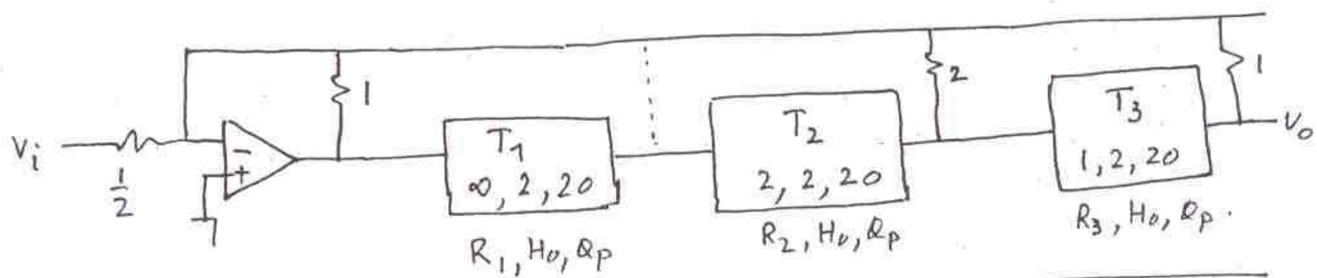
$$= 2 - 4 \cdot \left(\tfrac{1}{2}\right)^3 - 2 \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} = 2 - \tfrac{1}{2} - \tfrac{1}{2} = 1.$$

Thus, $a_1 = 0$; $a_2 = \frac{1}{2}$; $a_3 = 1$

If $R_f = 1$, $R_o = \dfrac{R_f}{Hb_o} = \dfrac{R_f}{2} = \dfrac{1}{2}\ \Omega$

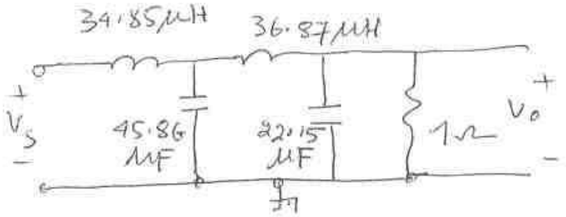
$a_2 = \dfrac{R_f}{R_2}$; $R_2 = \dfrac{R_f}{a_2} = 2R_f = 2\ \Omega$

$a_3 = 1 = \dfrac{R_f}{R_3}$; $R_3 = R_f = 1\Omega$



7.11 / 4.4

The filter circuit is



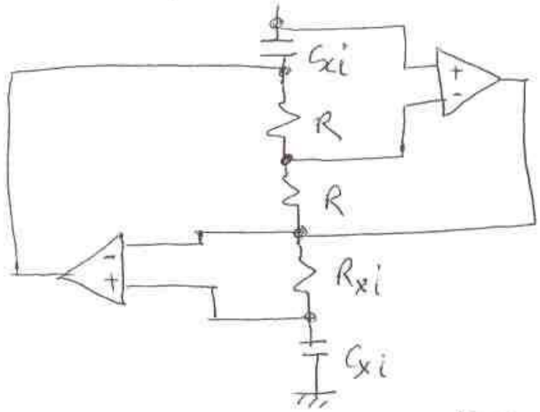
We could use FDNR technique with a scale factor $k = 10^{-6}$ say.

Then $34.85\,\mu H \to 34.85\,\Omega$; $36.87\,\mu H \to 36.87\,\Omega$

$$1\,\Omega \to \frac{10^{-6}}{1} = 1\,\mu F$$

$45.86\,\mu F \to 45.86 \times 10^{-12} = D_1$; $22.15\,\mu F \to 22.15 \times 10^{-12} = D_2$

D_1, D_2 are implemented as



$D_1 = C_{x1}^2 R_{x1}$

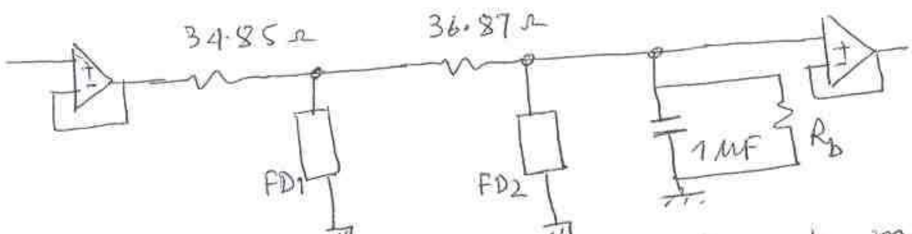
$D_2 = C_{x2}^2 R_{x2}$

with $R_{x1} = R_{x2} = 1\,\Omega$

$C_{x1} = \sqrt{45.86}\,\mu F = 6.77\,\mu F$

$C_{x2} = \sqrt{22.15}\,\mu F = 4.7\,\mu F$.

The active RC filter is then

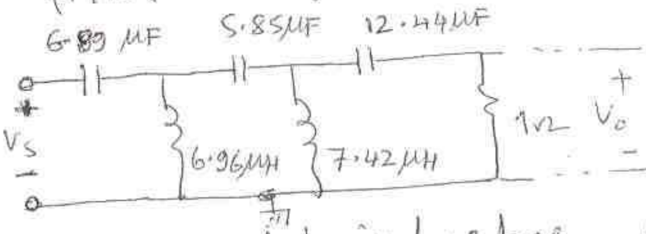


FD1 ← C_{x1}, R_{x1} circuit

FD2 ← C_{x2}, R_{x2} circuit

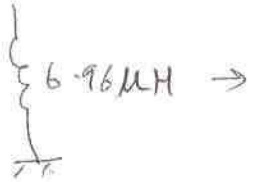
We can choose $R_b >> 71.72\,\Omega$ to maintain a DC gain ≈ 1. ← $\frac{R_b}{R_b + 36.87 + 34.85}$, $R_b >> 36.87 + 34.85$

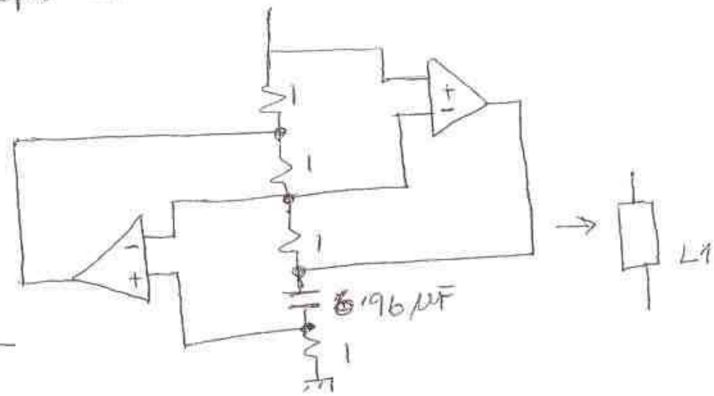
$\frac{7.11}{4.5}$ The filter circuit is

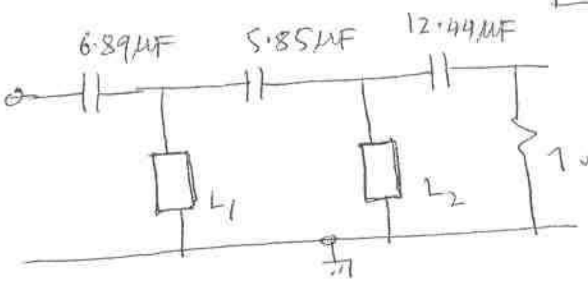


Because of grounded inductors, GIC-s with only one capacitor can be used to replace the inductors.

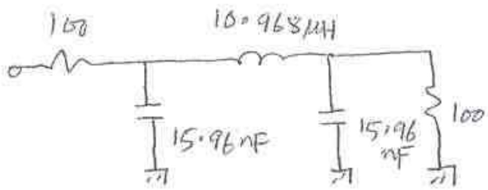
Thus







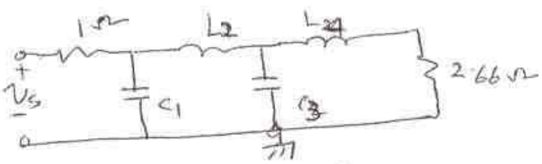
$\frac{7.11}{4.6}$



→ use FDNR technique as in 7.11/4.4

Choose R_a, R_b as in Fig. 7.12(b)

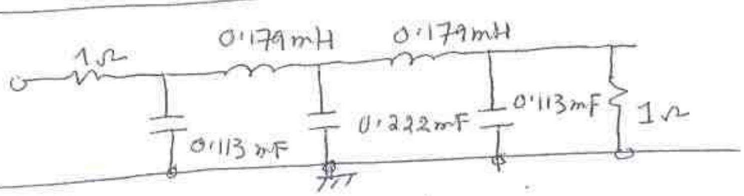
$\frac{7.11}{4.7}$



$C_1 = 1.0495$
$C_3 = 1.7067$
$L_2 = 1.3095$
$L_4 = 2.791$

Use FDNR technique

$\frac{7.11}{4.8}$



We could use Leap frog structure as in Fig. 7.18 with $R_s = 1$

$L_1 = 0$, $C_2 = .113$ mF,

$L_3 \Rightarrow 0.179$ mF / R

$C_4 = 0.222$ mF, $L_5 \Rightarrow 0.179$ mF / R, $C_6 = 0.113$ mF

(7.11) Hints: • Use FDNR or Leap-frog technique for the Low-pass filters
• Use GIC structure for the High-pass filters

x

(7.12) For $A_p = 0.1$ dB, $\epsilon = 0.1526$
Use the formulae set in Ch 6, 4.2 (p. 131), assuming double termination with $R_L = 1450\,\Omega$

$$R_L = \left[1 + 2\epsilon^2 \pm 2\epsilon\sqrt{1+\epsilon^2}\,\right] R_s$$

For $n = 8$ $\quad \dfrac{L_n}{R_L} = \dfrac{2}{q}\sin\dfrac{\pi}{2n}$ $\rightarrow$ terminal element

$$q = \frac{1}{2}\left[\left(\frac{1}{\epsilon} + \sqrt{\frac{1}{\epsilon^2}+1}\right)^{1/n} - \left(\frac{1}{\epsilon} + \sqrt{\frac{1}{\epsilon^2}+1}\right)^{-1/n}\right]$$

$$C_1 = \frac{2}{qR_s}\sin\frac{\pi}{2n}, \quad C_{2k-1}L_{2k} = \frac{4\sin\left(\frac{4k-1}{2n}\pi\right)\sin\left(\frac{4k-3}{2n}\pi\right)}{q^2 + \sin^2\left(\frac{2k-1}{n}\pi\right)}$$

$$C_{2k+1}L_{2k} = \frac{4\sin\left(\frac{4k-1}{2n}\pi\right)\sin\left(\frac{4k+1}{2n}\pi\right)}{q^2 + \sin^2\left(\frac{2k}{n}\pi\right)}$$

$k = 1, 2, \cdots k_m$ where k_m = integer $< \frac{n}{2}$.

In this case $k_m = 3$

Considering the + sign in the expression of R_L,

$$R_s = \frac{R_L}{1 + 2\epsilon^2 + 2\epsilon\sqrt{1+\epsilon^2}} = 1088.6 \approx 1089\,\Omega$$

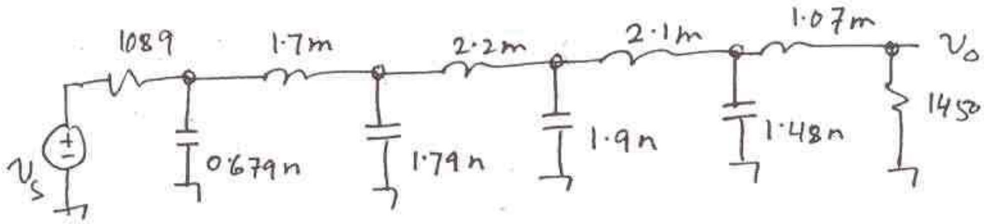
$q = 0.6559$
$C_1 = 5.4623 \times 10^{-4}$
$L_2 = 1.3763 \times 10^{3}$
$C_3 = .0014$
$L_4 = 1.7605 \times 10^{3}$
$C_5 = .0015$

$L_6 = 1.6628 \times 10^{3}$
$C_7 = .0012$
$L_8 = 862.57$

The above values correspond to $\omega_c = 1$ rad/sec.

(7.12) Contd. That will make $L \to L_N / (2\pi \times 128 \times 10^3)$

and $C \to C_N / (2\pi \times 128 \times 10^3)$, where L_N, C_N are the values already calculated. So the final LC filter is:



Use Leap-Frog architecture now. Note

$Y_1 \to R_s$

$Z_2 \to .679\,nF$

$Y_3 \to 1.7mH$ .. & so on

$Y_9 \to$ 1.07mH in series with 1450Ω

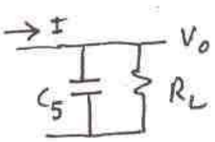
$Z_2, Y_3, \ldots, Z_8$ will consist of ideal integrators.

Y_1 will be a simple inverter.

Y_9 will be a lossy integrator.

x

(7.13) Follow same technique as in (7.12). The load end is now



with $V_o = I \cdot Z_6$

Where $Z_6 = \dfrac{R_L \frac{1}{sC}}{R_L + \frac{1}{sC}} = \dfrac{R_L}{sCR_L + 1}$

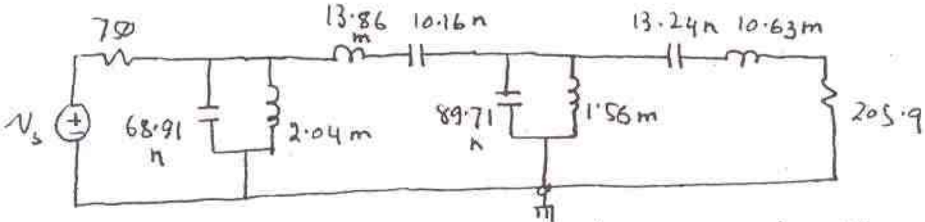
ie $Z_6 = \dfrac{1/C}{s + 1/CR_L}$

$$V_o = I \cdot \frac{1/C}{s + 1/CR_L} = I.R. \frac{1/CR}{s + 1/CR_L} = V_R' \frac{1/CR}{s + 1/CR_L}$$

$V_o = V_R' T_R'$ where T_R' is the transfer function of a lossy integrator (non inverting), ie.

(7.14) The structure is not an all-pole filter. It has transmission zeros. So adopt the method to realize general ladder (see 7.5, p.145).

Given:



One can associate the normalized admittance and impedance functions, as:

$$t_{y1} = R_p Y_1 = \frac{1}{R_1/R_p} \quad , R_1 = 750 \quad \text{[see Fig. 23(a)]}$$

$$-t_{z2} = -Z_2/R_p = \frac{1}{sR_pC_{p2} + \frac{1}{sL_2/R_p}} \quad ; C_{p2} = 68.91, \; L_2 = 2.04$$

$$t_{y3} = R_p Y_3 = \frac{1}{sL_3/R_p + \frac{1}{sC_{p3}R_p}} \quad ; L_3 = 13.86, \; C_{p3} = 10.16$$

$$-t_{z4} = -Z_4/R_p = \frac{1}{sR_pC_{p4} + \frac{1}{sL_4/R_p}} \quad ; C_{p4} = 89.71, \; L_4 = 1.56$$

$$t_{y5} = R_p Y_5 = \frac{1}{R_5/R_p + sL_5/R_p + \frac{1}{sC_{p5}R_p}} \quad ; R_5 = 205.9, \; L_5 = 10.63, \; C_{p5} = 13.24$$

$R_p \rightarrow$ convenient normalizing resistance.

Consider figure 25 and the normalized voltage gain relation (eq. 16) for each of the t_y above

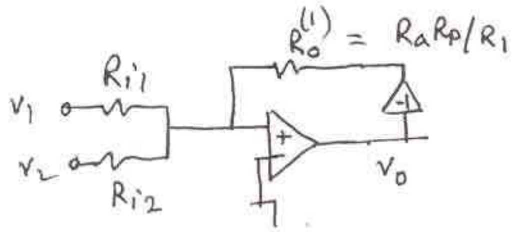
$$V_o = + \frac{1}{R_aY_0T_0 + R_aY_1T_1 + R_aY_2T_2 + \frac{1}{\frac{Y_3T_3}{R_aG_3G_{c3}} + \frac{Y_4T_4}{R_aG_3G_{c3}}}} \left(\frac{R_a}{R_{i1}} V_1 + \frac{R_a}{R_{i2}} V_2 \right)$$

7.14 contd.

For t_{y1}, we can set $-T_0 = -1$, $Y_0 = G_0 = \frac{1}{R_0}$, so that

$$\frac{R_1}{R_p} = R_a Y_0 = \frac{R_a}{R_0} \quad \text{i.e. } R_0^{(1)} = \frac{R_a R_p}{R_1}.$$

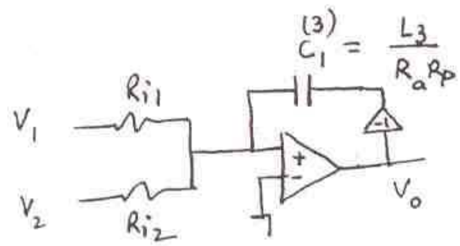
The sub-network is:



For t_{y3}, $R_a Y_1 T_1 \rightarrow sL_3/R_p$. Let $Y_1 = sC_1$, $-T_1 = -1$, then $R_a s C_1 = \frac{sL_3}{R_p}$

$$\text{i.e. } C_1^{(3)} = \frac{L_3}{R_a R_p}$$

The sub-network is:



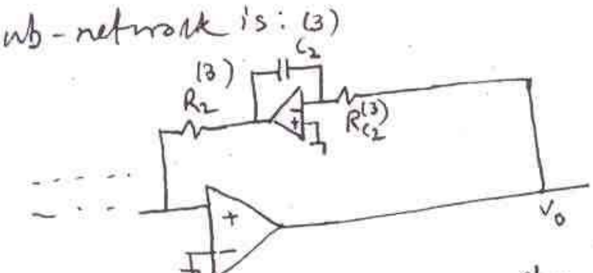
Further $R_a Y_2 T_2 \rightarrow \frac{1}{sC_{p3} R_p}$

Let $Y_2 = G_2$, $-T_2 = -\frac{1}{sC_2 R_{c2}}$

$$R_a \, G_2 \cdot \frac{1}{sC_2 R_{c2}} = \frac{1}{sC_{p3} R_p}$$

$$\text{i.e. } C_2^{(3)} = C_{p3} \cdot \frac{R_p R_a}{R_2^{(3)} R_{c2}^{(3)}}$$

The sub-network is:



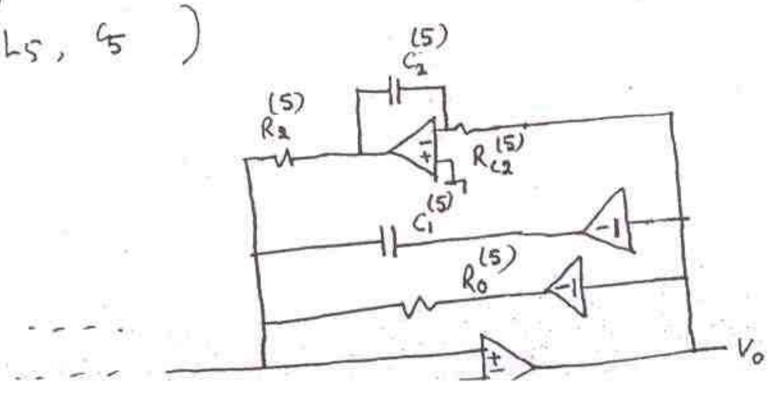
The superscript (1), (3) .. etc. relate to t_{y1}, t_{y3} .. and so on.

For t_{y5}, similarly, the subnetwork will be (considering R_5, L_5, C_5)

$$R_0^{(5)} = \frac{R_a R_p}{R_5}$$

$$C_1^{(5)} = \frac{L_5}{R_a R_p}$$

$$C_2^{(5)} = C_{p5} \cdot \frac{R_p R_a}{R_2^{(5)} R_{c2}^{(5)}}$$



(7.14) contd.

If we let $C_1^{(\ )} = C_2^{(\ )} = C$ where ( ) includes 3, 5

Then $\frac{L_x}{C} = R_a R_p = r^2$, $x = 3, 5$

$R_0^{(y)} = \frac{r^2}{R_y}$, $y = 1, 5$

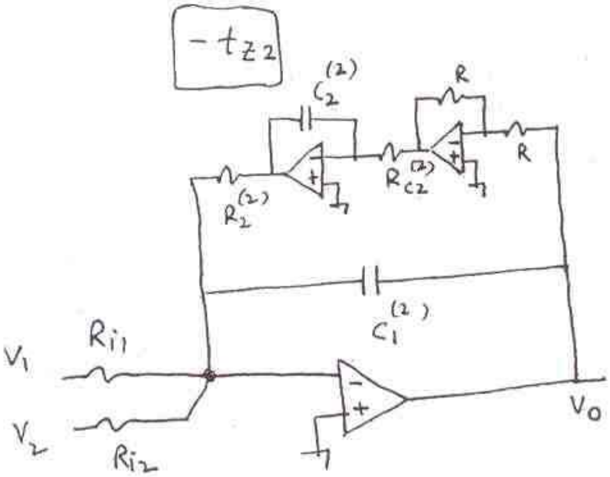
$R_2^{(z)} R_{C_2}^{(z)} = \frac{r^2}{C_{pz} C_2^{(z)}}$, $z = 3, 5$

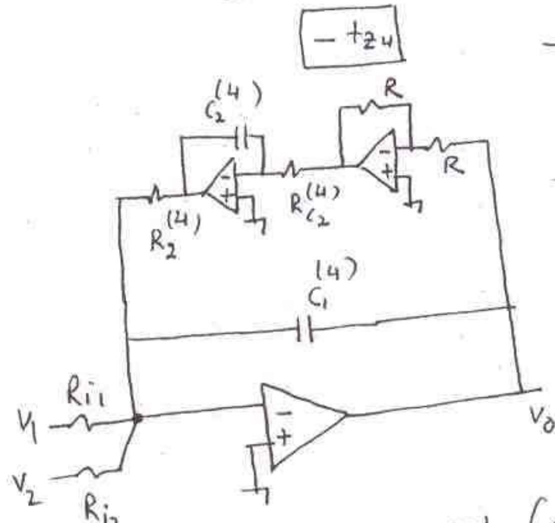
Note, R_p has a unique value for the entire L, C ladder. R_a can be different for different sections of the ladder i.e. for t_{y1}, t_{y3}, t_{y5} .. and so on.

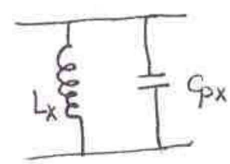
Since no specifications for the dynamic range adjustment has been given, we can assume, for simplicity

$R_{i1} = R_{i2} = R_a$ and $R_{2K} = R_{CK}$, $K = 3, 5$.

Similarly, (see Fig. 27), the subnetworks for t_{z2}, t_{z4} will be: (each has only C and L in parallel)







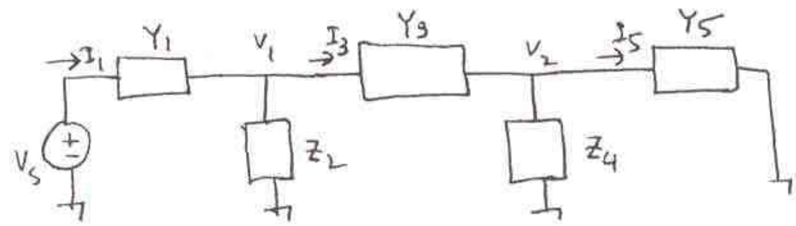
Choosing $C_1^{(\ )} = C_2^{(\ )} = C$ for (2), (4), we get (see eqn. 22)

$C_{px} / C = m = \frac{R_a}{R_p}$; , $R_2^{(x)} R_{C2}^{(x)} = m \frac{L_x}{C}$, $x = 2, 4$

We can further assume $R_{i1} = R_{i2} = R_a$; $R_K^{(x)} = R_{CK}^{(z)}$, $K = 2$

To find out the interconnections, we need to set up the I-V equations in different sections of the ladder. In a block diagram form we can write:

(7.14) contd/



$I_1 = (V_s - V_1)Y_1$; $V_1 = (I_1 - I_3)Z_2$

$I_3 = (V_1 - V_2)Y_3$; $V_2 = (I_3 - I_5)Z_4$

$V_o = I_5 \cdot R_L$; $I_5 = V_2 Y_5$

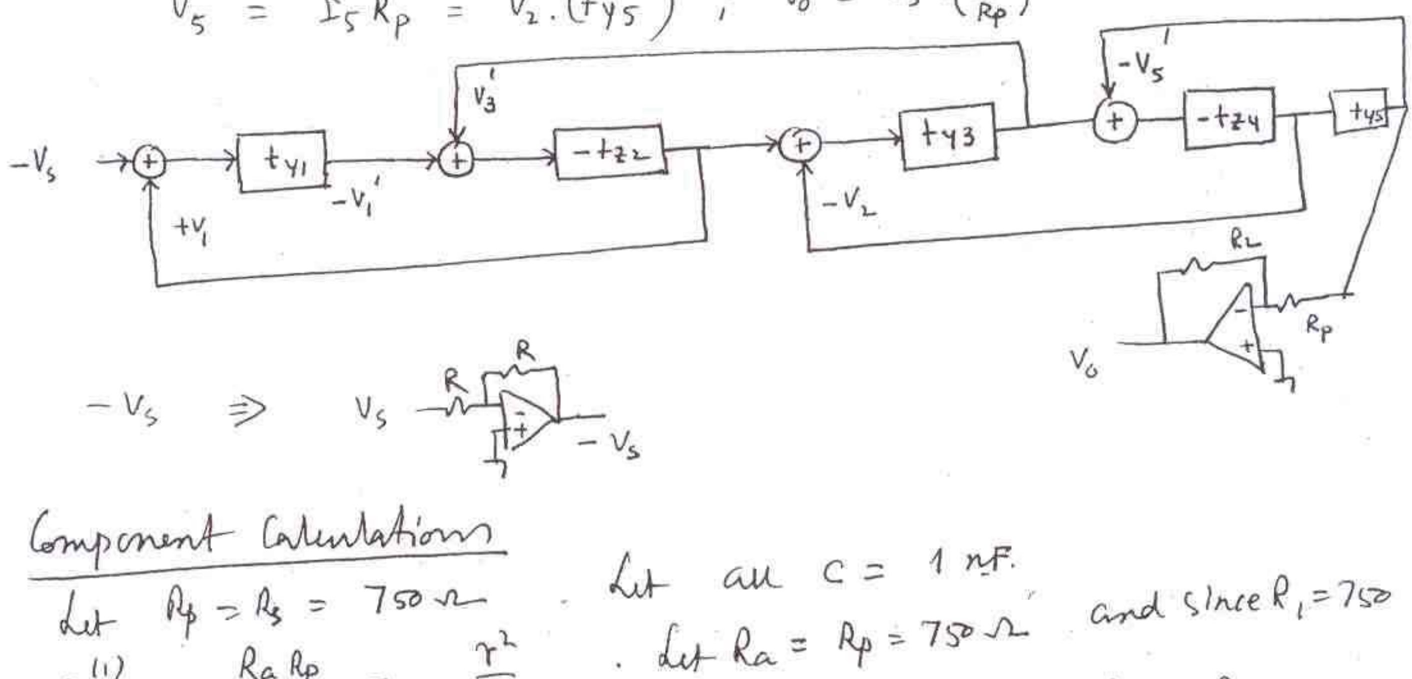
So using R_p

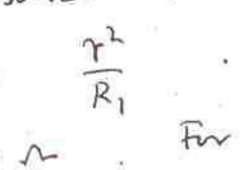
$V_1' = (V_s - V_1)\, Y_1 R_p = (V_s - V_1)\, t_{y1}$

$V_1 = (V_1' - V_3')\left(\frac{Z_2}{R_p}\right) = -(V_1' - V_3')(-t_{z2})$

$V_3' = (V_1 - V_2)\, t_{y3}$; $V_2 = -(V_3' - V_5')(-t_{z4})$

$V_5' = I_5 R_p = V_2 \cdot (t_{y5})$; $V_o = V_5' \cdot \left(\frac{R_L}{R_p}\right)$



$-V_s \Rightarrow$ 

Component Calculations

Let $R_p = R_s = 750\,\Omega$. Let all $C = 1$ nF.

$R_o^{(1)} = \frac{R_a R_p}{R_1} = \frac{r^2}{R_1}$. Let $R_a = R_p = 750\,\Omega$ and since $R_1 = 750$

$R_o^{(1)} = 750\,\Omega$. For t_{y1}, we'll keep $R_{i1} = R_{i2} = R_a = 750\,\Omega$

~~For t_{y3}:~~ So for t_{y1} : $R_o = 750\,\Omega$ $R_{i1} = R_{i2} = 750\,\Omega$, let $R = 750\,\Omega$

(7.14) contd. For t_{y3}:

$$C_1^{(3)} = \frac{L_3}{R_a R_p} = 1nF$$

$$R_a = \frac{L_3}{R_p C_1^{(3)}} = \frac{13.86\ mH}{750 \times 1nF} = 18.480 K\Omega$$

$$C_2^{(3)} = C_{p3} \cdot \frac{R_p R_a}{R_2^{(3)} R_{c2}^{(3)}} = 1nF \therefore\ R_2^{(3)} R_{c2}^{(3)} = \frac{10.16n}{1n} \cdot R_p R_a$$

$\therefore R_p = 750,\ R_a = 18.48\ K$; $R_2^{(3)} = R_{c2}^{(3)}$ assumed,

$$R_2^{(3)} = R_{c2}^{3} = \sqrt{R_p R_a \times 10.16} = 11.866\ K\Omega$$

$R_{i1} = R_{i2} = R_a = 18.48K$; $R = 1K$ (say)

For t_{y5}: $C_1^{(5)} = C_2^{(5)} = 1nF$ assumed

$$R_a = \frac{L_5}{R_p C_1^{(5)}} = \frac{10.63m}{750 \times 1n} = 14.173\ K\Omega$$

$$R_2^{(5)} R_{c2}^{(5)} = \frac{C_{p5}}{C_2^{(5)}} \cdot R_p R_a = \frac{13.24}{1} \cdot 750 \times 14.173 \times 10^3$$

$R_2^{(5)} = R_{c2}^{(5)}$ (assumed) $= 11.863\ K\Omega$

Let $R_{i1} = R_{i2} = R_a = 14.173K$; all $R = 1K$.

Proceed similarly for t_{z2}, t_{z4}. Connect up as in the system diagram.

For t_{z2}: $m = 68.91 = \frac{R_a}{R_p}$; $R_a = 51.682\ K = R_{i1} = R_{i2}$

all $C = 1nF$.

$R_2 R_{c2} = 68.91 \times \frac{2.04m}{1n}$; $R_2 = R_{c2} = 11.86K$

Let all $R = 1K$

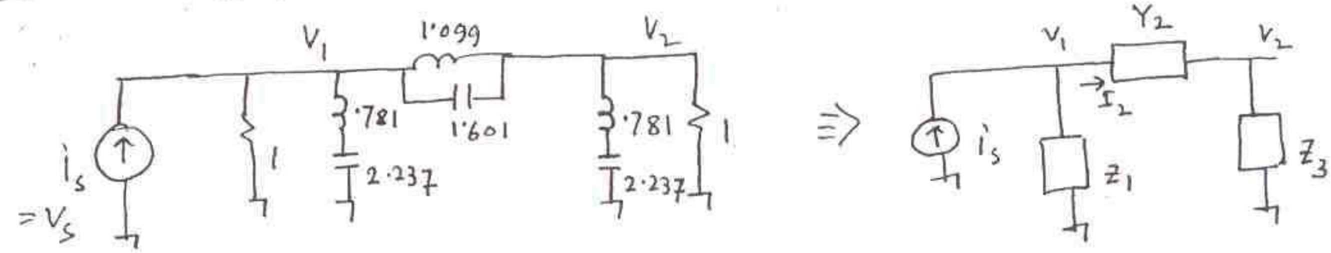
For t_{z4}: $m = 89.71$; $R_a = 67.282\ K = R_{i1} = R_{i2}$

$R_2 R_{c2} = 89.71 \times 1.56 \times 10^6$; $R_2 = R_{c2} = 11.83K$

Let all $R = 1K$.

ALT: One could source transform the R_s thereby reducing the number of active circuits. Try it. Follow the ... (Example 4).

(7.15) Apply source transformation



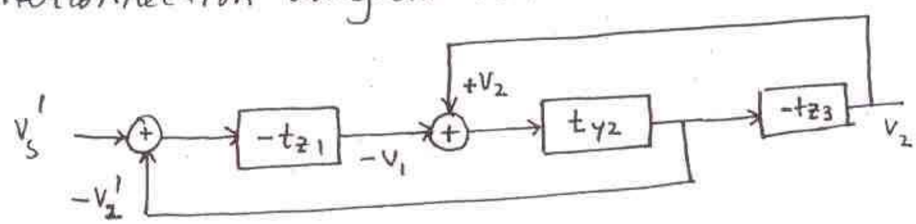
R_p ⇒ scaling resistance for the entire ladder

$V_1 = (i_s - I_2) Z_1$: $V_1 = (V_s' - V_2')\left(\frac{Z_1}{R_p}\right) = -(V_s' - V_2')(-t_{z1})$

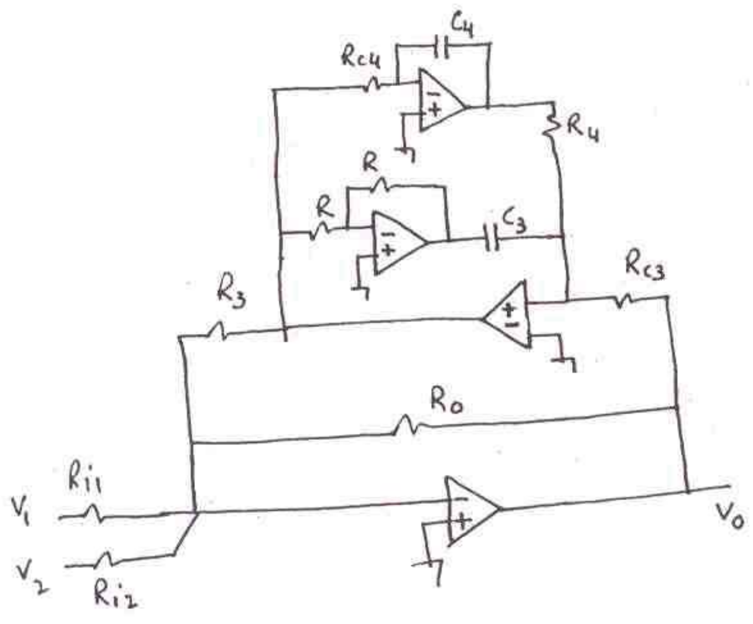
$I_2 = (V_1 - V_2) Y_2$; $V_2' = (V_1 - V_2)(Y_2 R_p) = (V_1 - V_2)(t_{y2})$

$V_2 = I_2 Z_3 = V_2' \cdot \left(\frac{Z_3}{R_p}\right) = -V_2'(-t_{z3})$

The interconnection diagram is:

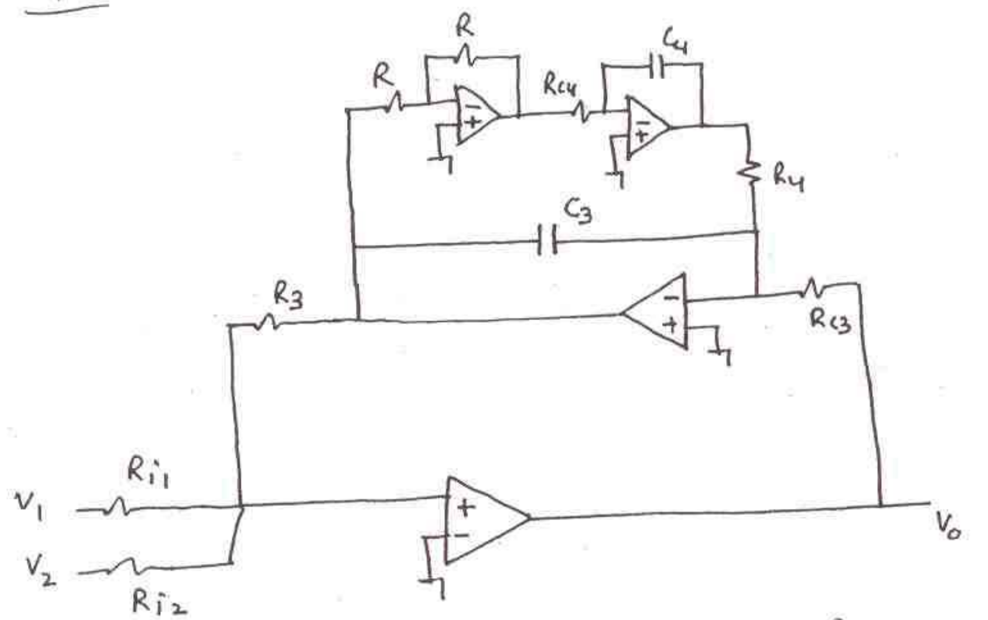


$-t_{z1}$ can be implemented as in fig. 28. So also $-t_{z3}$



Similarly, t_{y2} can be implemented using the sub circuit shown on next page (see fig-26)

(7.15) Contd.



Continue as in Example 4 Case and in Problem 7.14 :

7.16 A 4^{th} order CHEB filter with $A_p = 1$ dB has a normalized TF.

$$H(s) = \frac{1}{2^3 \times 0.5089} \cdot \frac{1}{(s^2+0.279s+0.987)} \frac{1}{(s^2+0.674s+0.279)}$$

The DC gain is $\frac{1}{8 \times .5089} \frac{1}{0.987} \frac{1}{0.279} = 0.8919 \approx 0.892$

So in the active RC implementation an additional gain stage may be required.

Consider $\frac{V_2}{V_1} = \frac{K}{s^4 + 0.953s^3 + 1.454s^2 + 0.743s + 0.276}$

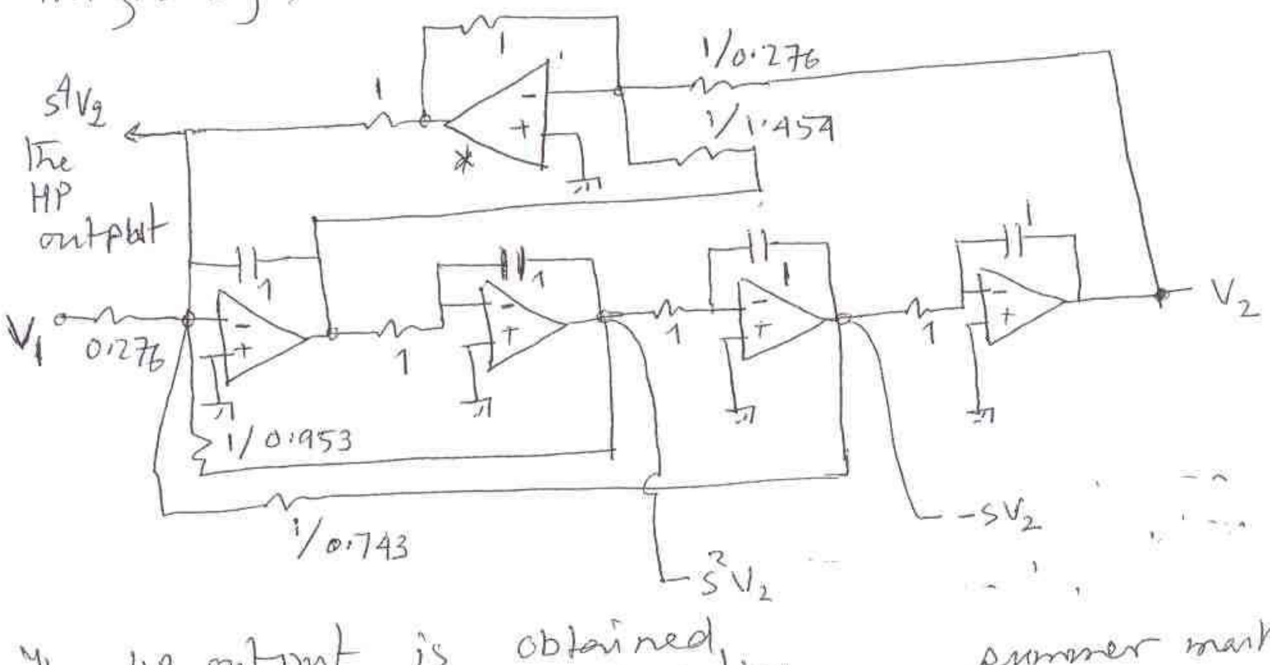
where we will choose $K = 0.276$ for a DC gain $= 1$.

Comparing the above with eq. 7.26

$$\frac{K}{s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

we see $b_3 = 0.953$, $b_2 = 1.454$, $b_1 = 0.743$, $b_0 = 0.276$

The above function can be realized by the structure in Fig 7.29 containing 3 integrators (inverting), 1 inverting integrating summer and 1 inverting summer.



The HP output is obtained at the output of the inverting summer marked with an *.

7.17 Using $T_i = -\frac{1}{s+\alpha}$, we get using FLF structure

$$\frac{V_2}{V_1} = \frac{K}{(-1)^n (s+\alpha)^n + (-1)^{n-1} f_{n-1}(s+\alpha)^{n-1} + (-1)^{n-2} f_{n-2}(s+\alpha)^{n-2} + \cdots + (-1)^1 f_1(s+\alpha) + f_0}$$

$$D(s) = (-1)^n (s+\alpha)^n + (-1)^{n-1} f_{n-1}(s+\alpha)^{n-1} + (-1)^{n-2} f_{n-2}(s+\alpha)^{n-2} + \cdots + (-1)^2 f_2 (s+\alpha)^2 + (-1)^1 f_1 (s+\alpha) + f_0$$

This is to be matched with

$$s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots + b_1 s + b_0$$

Let
$$D'(s) = (s+\alpha)^n + (-1)^{-1} f_{n-1}(s+\alpha)^{n-1} + (-1)^{-2} f_{n-2}(s+\alpha)^{n-2} + (-1)^3 f_{n-3}(s+\alpha) + \cdots + (-1)^{1-n} f_1 (s+\alpha) + f_0 = \frac{D(s)}{(-1)^n}$$

$$= (s+\alpha)^n - f_{n-1}(s+\alpha)^{n-1} + f_{n-2}(s+\alpha)^{n-2} \mp f_{n-3}(s+\alpha)^{n-3} + \cdots + (-1)^{1-n} \cdot f_1 \cdot (s+\alpha) + f_0 (-1)^{-n}$$

Then $b_{n-1} = n\alpha - f_{n-1}$; $f_{n-1} = -b_{n-1} + n\alpha$ [equating coeffts. of s^{n-1}]

$$\left.\begin{aligned} b_{n-2} &= \frac{n(n-1)}{2} \cdot \alpha^2 - (n-1)\alpha f_{n-1} + f_{n-2} \\ \text{So } f_{n-2} &= b_{n-2} + (n-1)\alpha f_{n-1} - \frac{n(n-1)}{2}\alpha^2 \end{aligned}\right\} \text{equating coefft. of } s^{n-2}$$

Equating coeffts. of s^{n-3}

$$b_{n-3} = \frac{n(n-1)(n-2)}{2 \cdot 3}\alpha^3 - \frac{(n-1)(n-2)}{2}\alpha^2 f_{n-1} + (n-2)\alpha f_{n-2} - f_{n-3}$$

$$\text{So } f_{n-3} = -b_{n-3} + \frac{n(n-1)(n-2)}{6}\alpha^3 - \frac{(n-1)(n-2)}{2}\alpha^2 f_{n-1} + (n-2)\alpha f_{n-2}$$

$$\text{or } f_{n-3} = -b_{n-3} + \frac{n(n-1)(n-2)}{6}\alpha^3 + (n-2)\alpha f_{n-2} - \frac{(n-1)(n-2)}{2}\alpha^2 f_{n-1}$$

$\vdots$

Equating coeffts. of s^0 i.e. constant terms

$$b_0 = \alpha^n - \alpha^{n-1} f_{n-1} + \alpha^{n-2} f_{n-2} - \alpha^{n-3} f_{n-3} + \cdots + (-1)^{1-n} \cdot f_1 \alpha + (-1)^{-n} \cdot f_0$$

$$(-1)^{-n} f_0 = b_0 - (-1)^{1-n} f_1 \alpha - (-1)^{2-n} \cdot f_2 \alpha^2 - \cdots + \alpha^{n-3} f_{n-3} - \alpha^{n-2} f_{n-2} + \alpha^{n-1} f_{n-1} - \alpha^n$$

Summary:

7.17 cont.

$$f_{n-1} = -b_{n-1} + n\alpha$$
$$f_{n-2} = b_{n-2} + (n-1)\alpha f_{n-1} - \frac{n(n-1)}{2}\alpha^2$$
$$f_{n-3} = -b_{n-3} + (n-2)\alpha f_{n-2} - \frac{(n-1)(n-2)}{2}\alpha^2 f_{n-1} + \frac{n(n-1)(n-2)}{6}\alpha^3$$
$$\vdots$$
$$(-1)^{-n} f_0 = b_0 - (-1)^{1-n} f_1\alpha - (-1)^{2-n} f_2\alpha^2 - \cdots + \alpha^{n-3} f_{n-3} - \alpha^{n-2} f_{n-2} + \alpha^{n-1} f_{n-1} - \alpha^n$$

For the given function

$$\frac{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

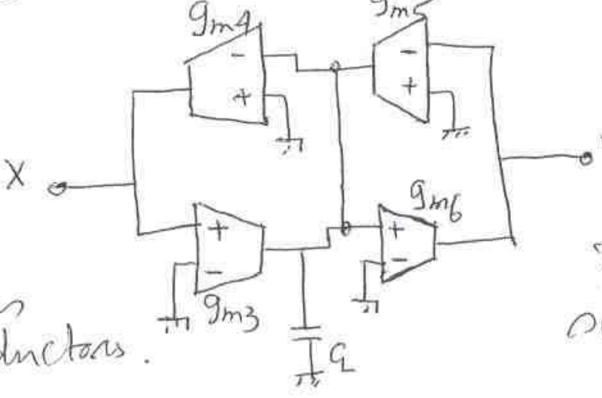
$$f_{4-1} = f_3 = -b_3 + 4\alpha$$
$$f_{4-2} = f_2 = b_2 + 3\alpha f_3 - 6\alpha^2$$
$$f_{4-3} = f_1 = -b_1 + 2\alpha f_2 - 3\alpha^2 f_3 + 4\alpha^3$$
$$(-1)^{-4} f_0 = b_0 + f_1\alpha + f_2\alpha^2 + f_1\alpha^3 = f_0$$

The filter can be organized as in Fig. 7.26 for the feedback paths. For the forward paths, one can add the feedforward coefficients $a_4, a_3 \cdots a_0$ as in Fig. 7.31

×

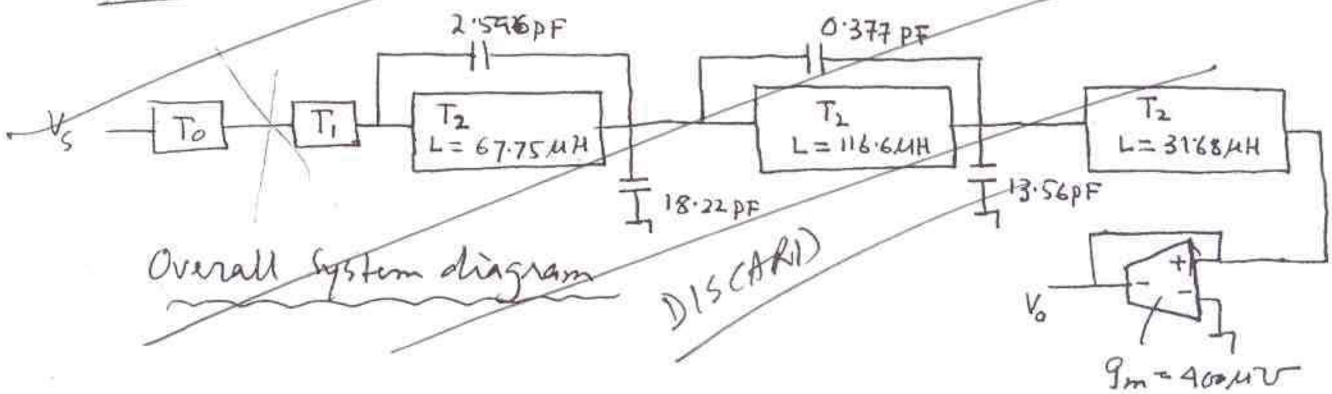
7.18 Follow the example as in Fig. 7.36. Each of the floating inductor can be realized using the OTA-based structure

$L_1 = 67.75$
$L_2 = 116.6$
$L_3 = 31.68$
are to be simulated as floating inductors.



The floating inductor L_{XY} between XY is generated by C_L with $C_L = g^2 L_{XY}$. 'g' is a convenient scaling transconductance.

(7.18) Contd.

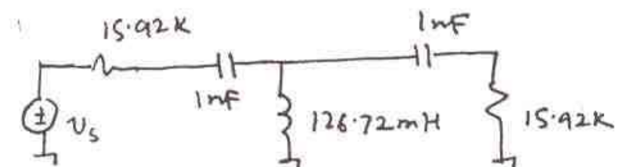


Overall system diagram

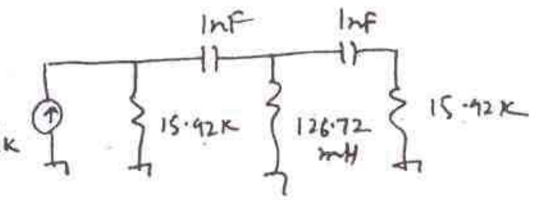
DISCARD

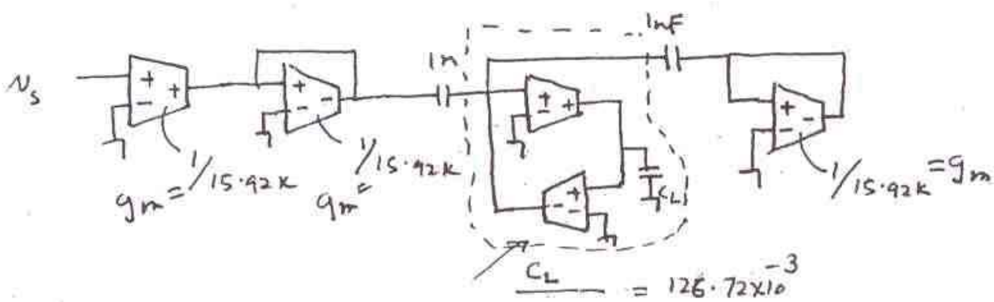
(7.19/7.2)

OTA-C solutions for



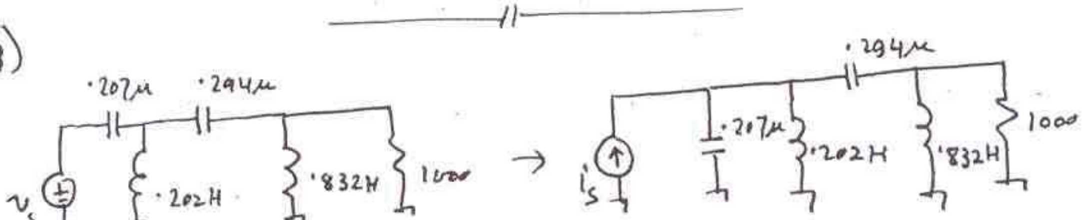
$i_s = \frac{v_s}{15.92K}$





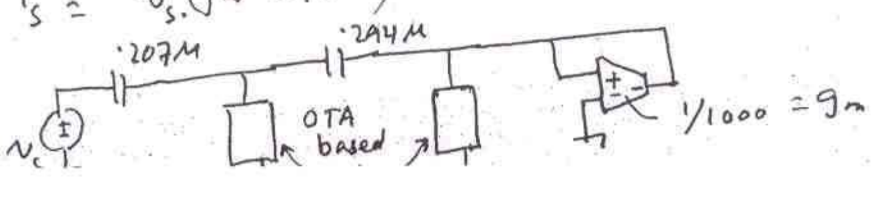
$\frac{C_L}{g_{m1}g_{m2}} = 126.72 \times 10^{-3}$

(7.19)/(7.3)



$i_s = v_s(j\omega \cdot 207 \times 10^{-6}) \rightarrow v_s \cdot sC$, not possible.

So try



7.19/7.6

Use floating L simulated by OTA.

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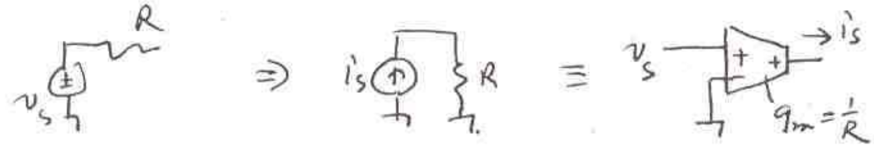
7.19/7.8

For series resistance at input, use Thevenin - Norton transformation to make



7.19 For 'L' floating or grounded use OTA based gyrators.

—— // ——

7.9 - 7.15 to . Follow methods as above. For the PRB and cascaded biquad sections, use corresponding OTA base biquad networks. Remember that the ratio R_1/R_2 in active RC becomes g_2/g_1 in OTA based networks where $g_1 = 1/R_1$, $g_2 = 1/R_2$. The assumptions that are critical in these cases are that the output of the OTA must behave as VCVS i.e. low output impedance. In practical implementation one has to ensure this situation with additional buffer networks interposed between adjacent OTA devices.

—— x ——

7.20 Fourth order BUT filter has a normalized

$$TF = \frac{1}{s^2+0.765s+1} \cdot \frac{1}{s^2+1.848s+1}$$

For 1 KHz cut-off freq

$$H(s) = \frac{0.394889\times10^8}{s^2+4807.26\,s+0.394889\times10^8} \; \frac{0.394889\times10^8}{s^2+11612.83\,s+0.394889\times10^8}$$

Since $f_s = 100\,\text{kHz}$ while $f_P = 1\,\text{KHz}$, we may ignore the pre-wraping.

Applying $s \to z$ BLT. $s \to 2f_s \frac{1-z^{-1}}{1+z^{-1}}$, the SCF - transfer function becomes

$$H(z) = 0.00095\,\frac{1+2z^{-1}+z^{-2}}{1-1.949\,z^{-1}+0.95312\,z^{-2}} \times 0.00095\,\frac{1+2z^{-1}+z^{-2}}{1-1.8866\,z^{-1}+0.8903\,z^{-2}}$$

Now follow the methods in Ch 6 to design the two biquads as appear in cascade above.

— X —