ELEC 441/6081, MT # 1 Soln

Winter 09-10 term

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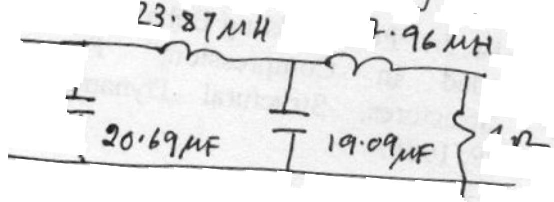
Q1: (a) Using frequency scaling by $a = 2\pi \times 10^9$ rad/s



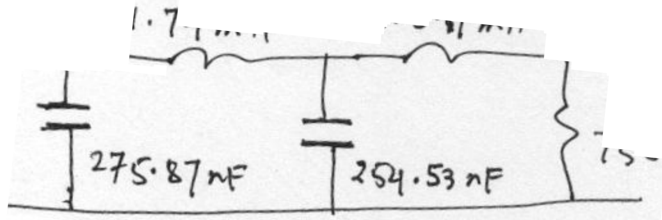
$1.3\,F \rightarrow \frac{1.3}{2\pi \times 10^9}\,F$

$1.5\,H \rightarrow \frac{1.5}{2\pi \times 10^9}\,H.$

$R \rightarrow$ does not change

Using impedance scaling by 75

$C \rightarrow C/75,\quad L \rightarrow L \times 75;\quad R \rightarrow R \times 75$



is the Final Circuit

(b) For hpf we apply component transformation first

Thus $L_i \rightarrow C_i$, $\quad C_i = \frac{1}{L_i \omega_{cn}}$; $\omega_{cn} = 2\pi \times 400$ rad/s

$C_j \rightarrow L_j$, $\quad L_j = \frac{1}{C_j \omega_{cn}}$

Thus:

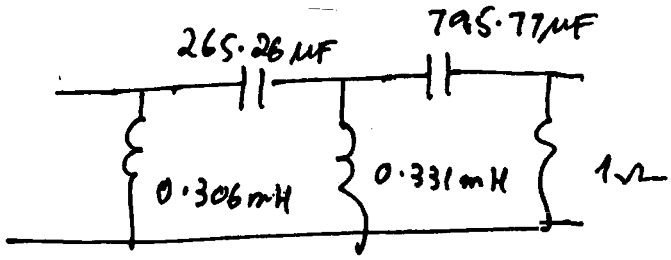
$1.3\,F \rightarrow$ inductance $L_j = \frac{1}{1.3 \times 2\pi \times 400} = 0.306\,mH$

$1.2\,F \rightarrow 0.331\,mH$

$1.5\,H \rightarrow C_i = \frac{1}{1.5 \times 2\pi \times 400} = 265.26\,\mu F$

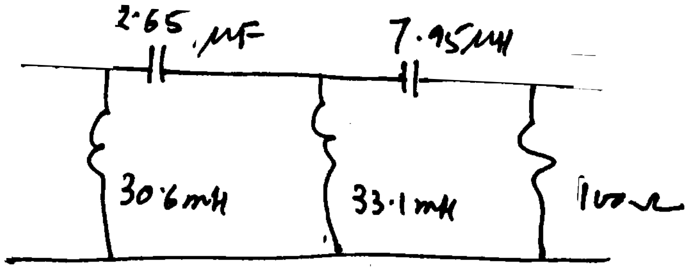
$0.5\,H \rightarrow \quad = 795.77\,\mu F$

Q1 (b) First cut



For 100Ω termination, use impedance scaling

L → 100 × L , C → C/100 , R → R × 100 .



Final answer

Q2:

At the passband edge, $\omega_n = 1$, $A_p = 1$ dB

So $\epsilon = \sqrt{10^{0.1A_p} - 1} = \sqrt{0.2589} = 0.5088$

a) $$|H_N(j\omega_n)|^2 = \frac{1}{1 + .5088^2 \omega_n^{2m}} = \frac{1}{1 + .2589\, \omega_n^{2m}}$$

At $f = 10$ kHz, $\omega_n = \frac{10}{1} = 10$

$$|H_N(j\omega_n)|^2 = \frac{1}{1 + .2589 \times 10^{10}}$$ $m = \text{order} = 5$

$$\approx \frac{10^{-10}}{.2589}$$

In dB → $10\log_{10}|\ \ |^2 = -100\text{ dB} - 10\log_{10}(0.2589)$ -94.13

$= -100 + 5.87 =$

Attenuation $=$ 94.13 dB Ans

(b) Now $$10\log_{10}|H_N(j\omega_n)|^2 = -40 \quad \text{at} \quad \omega_n = 40 = \frac{40k}{1k}$$

$$10\log_{10}\left[\frac{1}{1 + .2589 \times (40)^{2m}}\right] = -40$$ $\log_{10} 1 = 0$

$$-10\log_{10}[1 + .2589 \times (40)^{2m}] = -40$$

$$10^4 = 1 + .2589 \times (40)^{2m}; \quad 0.2589 \times (40)^{2m} \approx 10^4$$

$$(40)^{2m} = \frac{10^4}{.2589} = 38.6249 \times 10^3$$

$$2m \log_{10}(40) = \log_{10}(38.6249 \times 10^3) = 4.5869$$

$$m = \frac{4.5869}{2 \times 1.6020} = 1.43 \rightarrow 2 \text{ order}$$ → Ans

Q.3 Passband ripple implies a CHEB approximation

$A_p = 0.5\,dB$ leads to $\epsilon = 0.3493$

$$D = \frac{10^{.1A_a}-1}{10^{.1A_p}-1} = \frac{10^{3.5}-1}{\epsilon^2} = 25908.2$$

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$$\frac{\omega_a}{\omega_c} = \frac{15}{7.8} = \omega_s = 1.923$$

$$\sqrt{D} = 160.96$$

$$\cosh^{-1}(160.96) = 5.77 \quad ; \quad \cosh^{-1}(1.923) = 1.271$$

$$\text{Order of filter} \geq \frac{\cosh^{-1}\sqrt{D}}{\cosh^{-1}(\omega_s)} = 4.539 \rightarrow 5$$

The normalized LPF transfer function is

$$H_N(s) = \frac{1}{2^{5-1}\epsilon} \cdot \frac{1}{s^5 + 1.17251s^4 + 1.9374s^3 + 1.3096s^2 + 0.7525s + 0.1789}$$

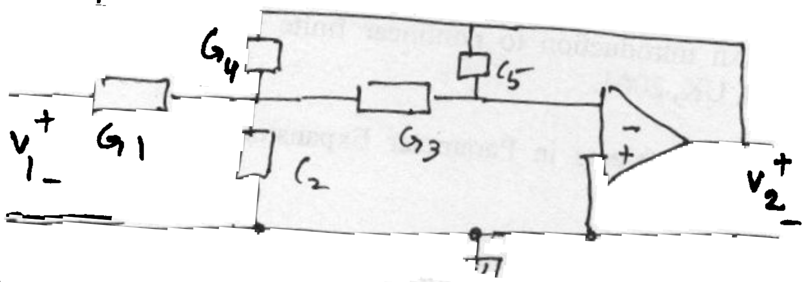
$$= \frac{0.1789}{s^5 + 1.1725s^4 + 1.9374s^3 + 1.3096s^2 + 0.7525s + 0.1789}$$

For the HPF with $\omega_{CH} = 15 \times 10^3 \times 2\pi$, the frequency de-normalized transfer function will be: $s \rightarrow \omega_{CH}/s$

$$H(s) = \frac{0.1789}{\left[\left(\frac{2\pi \times 15 \times 10^3}{s}\right)^5 + 1.1725\left(\frac{2\pi \times 15 \times 10^3}{s}\right)^4 + 1.9374\left(\frac{2\pi \times 15 \times 10^3}{s}\right)^3 + 1.3096\left(\frac{2\pi \times 15 \times 10^3}{s}\right)^2 + 0.7525\left(\frac{2\pi \times 15 \times 10^3}{s}\right) + 0.1789\right]}$$

Q 4

The given circuit is a LPF using infinite gain voltage amplifier It can be compared with



Thus, $R_1 \to G_1$, $C_1 \to C_2$, $R_2 \to G_4$, $R_3 \to G_3$, $C_2 \to C_5$ $\to$ Design guidelines for these are given in Table 4.3

$$\frac{E_2}{E_1} = \frac{G\omega_0^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$ can be compared with

$$\frac{V_2}{V_1} = \frac{H}{s^2 + \left(\frac{\omega_p}{Q_p}\right)s + \omega_p^2}$$

So $H = G\omega_0^2$. $\omega_0 \equiv \omega_p$, $Q \equiv Q_p$

From given data $H = 5\omega_0^2 = 5$, $\omega_0^2 = 1$ given $= b_0$

$b_1 = \frac{\omega_0}{Q} = \frac{\omega_p}{Q_p} = 1.2 = \frac{1}{Q_p}$

So $Q_p = \frac{1}{1.2} = 0.833$

Let's use the design guidelines

Q4: contd.

$$G_1 = \frac{H}{\omega_p} = \frac{5}{1} = 5 \qquad \text{So } R_1 = \frac{1}{5} = 0.2\,\Omega$$

$$C_2 = \frac{\alpha_p(2\omega_p^2 + H)}{\omega_p^2} = \frac{0.833\,(2+5)}{1} = 5.831\text{ F}$$

Thus $C_1 = 5.831\text{ F}$ in the given circuit

$G_3 = \omega_p = 1$ So $R_3 = G_3 = 1\,\Omega$ in the given circuit

$G_4 = G_3 = 1,$ $R_2 = 1\,\Omega$ in the given circuit

$$C_5 = \frac{\omega_p^2}{\alpha_p(2\omega_p^2 + H)} = \frac{1}{5.831} = 0.1715\text{ F}$$

$C_5 \to C_2 = 0.1277\text{ F}$ in the given circuit

Hence:

$R_1 = 0.2\,\Omega$
$R_2 = 1\,\Omega$
$R_3 = 1\,\Omega$
$C_1 = 5.831\text{ F}$
$C_2 = 0.1715$

needed design values

Proof:

$$\frac{1}{R_2 R_3 C_1 C_2} = \frac{1}{1.1.\,5.831 \times .1715} \approx 1 = b_0 = \omega_0^2$$

$$\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{1}{5.831}(5+1+1) \cong 1.2 = b_1$$

$$\frac{1}{R_1 R_3 C_1 C_2} = \frac{1}{.2}\cdot\frac{1}{1}\cdot\frac{1}{1} = 5 = G b_0$$