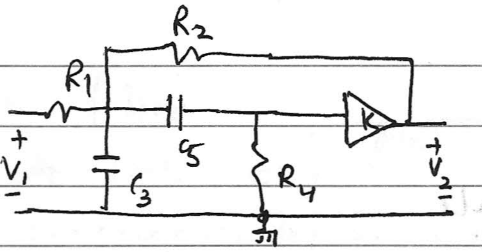
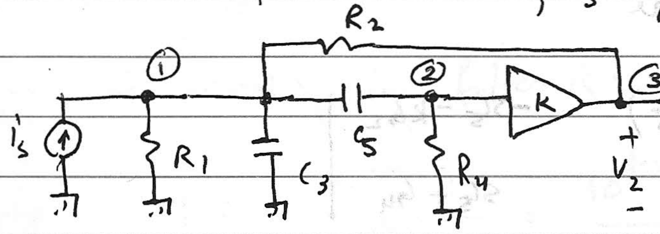
Q1:



After source transformation, $i_s = \frac{V_1}{R_1}$



Three node system, the nodal matrix is: with $G_x = \frac{1}{R_x}$

$$
\begin{array}{c}
\begin{bmatrix} G_1 + G_2 + sC_3 + sC_5 & -sC_5 & -G_2 \\ -sC_5 & sC_5 + G_4 & 0 \\ -G_2 & 0 & G_2 \end{bmatrix}
\begin{bmatrix} V_{①} \\ V_{②} \\ V_{③} \end{bmatrix}
=
\begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix} \\
\quad ① \qquad\qquad\qquad ② \qquad\qquad ③
\end{array}
$$

$V_{①}, V_{②} \ldots$ are the voltages at nodes ①, ② ..

Observation, $V_{③} = K V_{②}$. So we can substitute column ③ elements and absorb them in column ②.
Since ③ is the output node of a voltage amplifier, the KCL @ node ③ is arbitrary. So row ③ of the matrix can be discarded.
Hence, the reduced nodal matrix is:

$$
\begin{array}{c}
\begin{bmatrix} G_1 + G_2 + s(C_3 + C_5) & -sC_5 - KG_2 \\ -sC_5 & sC_5 + G_4 \end{bmatrix}
\begin{bmatrix} V_{①} \\ V_{②} \end{bmatrix}
=
\begin{bmatrix} i_s \\ 0 \end{bmatrix} \\
① \qquad\qquad\qquad ②
\end{array}
$$

Q1. (Contd.)

By using Krammer's rule

$$V_{(2)} = \frac{D_2}{D} \quad \text{where}$$

$$D = \begin{vmatrix} G_1 + G_2 + s(C_3 + C_5) & -sC_5 - kG_2 \\ -sC_5 & sC_5 + G_4 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} G_1 + G_2 + s(C_3 + C_5) & i_s \\ -sC_5 & 0 \end{vmatrix}$$

Hence $$V_{(2)} = \frac{sC_5 \cdot i_s}{[G_1 + G_2 + s(C_3 + C_5)](sC_5 + G_4) - sC_5(sC_5 + kG_2)}$$

Then $V_{(3)} = kV_{(2)}$ & $i_s = \dfrac{V_1}{R_1}$. Substituting

$$\frac{V_{(3)}}{V_1} = \frac{V_2}{V_1} = \frac{1}{R_1} \cdot \frac{kSC_5}{s^2 C_3 C_5 + [G_1C_5 + (1-k)G_2C_5 + G_4(C_3 + C_5)]s + \cancel{G_1(G_2+G_4)}}$$

$G_4(G_1 + G_2)$

$$= \frac{k\, s\, G_1/C_3}{s^2 + \left[\frac{G_1}{C_3} + (1-k)\frac{G_2}{C_3} + G_4\left(\frac{1}{C_3} + \frac{1}{C_5}\right)\right]s + \frac{\cancel{G_1(G_2+G_4)}}{C_3C_5}}$$

$G_4(G_1 + G_2)$

Q2: MFM filter.

For $A_p = 1\,dB$, $\epsilon = \sqrt{10^{0.1\times1} - 1} = \sqrt{10^{.1} - 1} = 0.50885$

The attenuation to pass-band frequency ratio is $\omega_s = \frac{40}{10} = 4$.

Then, the order of the filter is: $n = \frac{\log \eta}{2 \log \omega_s}$

where $\eta = \frac{10^{.1A_s} - 1}{10^{.1A_p} - 1} = \frac{10^{2.5} - 1}{10^{.1} - 1} = 1217.45$

$$n = \frac{\log(1217.45)}{2\log 4} = 2.56 \rightarrow 3$$

We can start with a 3rd order BUT filter function:

$$H_N(s)\Big|_{BUT} = \frac{1}{s^3 + 2s^2 + 2s + 1} \quad \text{for which } \epsilon = 1$$

Since $\epsilon = 0.5088$ here, we can get the normalized LPF function by scaling $s|_{BUT}$ to $\epsilon^{\frac{1}{n}}.s$

where $\epsilon^{\frac{1}{n}} = (0.50885)^{1/3} = 0.7984$

So $$H_N(s) = \frac{1}{(.7984s)^3 + 2(.7984s)^2 + 2(.7984s) + 1}$$

$$H_N(s) = \frac{1}{0.5089 s^3 + 1.2749 s^2 + 1.5968 s + 1}$$

The de-normalized transfer function is obtained by frequency scaling $s \rightarrow s/2\pi f_c = \frac{s}{2\pi \times 10^4}$

Then $H(s)$, the frequency denormalized transfer

Q2/ function becomes

$$H(s) = \frac{1}{0.20508 \times 10^{-14} s^3 + 0.32285 \times 10^{-9} s^2 + 0.00002541 s + 1}$$

Q3:

$$\Omega_{p_1} = 10^5 \; ; \; \Omega_{p_2} = 4\times10^5$$

$$\Omega_0 = \sqrt{4\times10^{10}} = 2\times10^5$$, geometric mean frequency.

$$\Omega_{a_1} = 15.263\times10^5$$

$$\Omega_{a_2} = \frac{\Omega_0^2}{\Omega_{a1}} = \frac{4\times10^{10}}{15.263\times10^5} = 2.6207\times10^4$$

Thus for the normalized LPF associated with the given BPF, the stop-band to pass-band ratio

$$\omega_s = \frac{15.263\times10^5 - 2.6207\times10^4}{4\times10^5 - 10^5} \approx 5$$

; the band width $B = 3\times10^5$ rad/sec.

Remember $B = 3\times10^5$; $\Omega_0 = 2\times10^5$

Now $A_p = 1$ dB, $A_a = 60$ dB, $\omega_s = 5$

The filter has equiripple pass-band. So it is a CHEB filter.

We calculate:

$$\eta = \frac{10^{.1A_a}-1}{10^{.1A_p}-1} = \frac{10^6-1}{10^{.1}-1} = 3862112.23$$

$$\epsilon = \sqrt{10^{.1}-1} = 0.5088$$

order n of the filter $\geq \dfrac{\cosh^{-1}\sqrt{\eta}}{\cosh^{-1}\omega_s} = \dfrac{\cosh^{-1}(1965.226)}{\cosh^{-1}(5)}$

$n \geq 3.61$. Take $n=4$.

Hence, using Tables of CHEB filter function

$$H_N(s) = \frac{1}{2^{4-1}\epsilon}\cdot\frac{1}{s^4+0.953s^3+1.454s^2+0.743s+0.276}$$

Q3 (Contd.)

Ans

(a) $H_N(s) = \dfrac{0.246}{s^4 + 0.953 s^3 + 1.454 s^2 + 0.743 s + 0.276}$

(b) LC ladder realization

The filter is an all-pole system. Let us write $D(s) = s^4 + 1.454 s^2 + 0.276 + 0.953 s^3 + 0.743 s$

$= M(s) + N(s)$

$M(s) \to$ even degree component of $D(s)$,

$N(s) \to$ odd degree component of $D(s)$

With an ideal voltage source i.e. $R_s = 0$ and a 1Ω termination, i.e., $R_L = 1$, the DC gain $= 1$.

So the realized structure will not meet the DC gain feature of $H_N(s)$. But response relative to frequency is important in filters.

Here $M(s) = s^4 + 1.454 s^2 + 0.276$ So $\deg(M(s))$

$N(s) = 0.953 s^3 + 0.743 s$ $> \deg(N(s))$

We perform CF expansion of $M(s)/N(s)$ around $s = \infty$ to realize Y_{22}

$0.953 s^3 + 0.743 s \,)\, s^4 + 1.454 s^2 + 0.276 \,(\, 1.049 s \to C = 1.04$

$s^4 + 0.779 s^2$

$0.674 s^2 + 0.276 \,)\, 0.953 s^3 + 0.743 s \,($

$0.674 s^2 + 0.276 \,)\, 0.933 s^3 + 0.743 s \,(\, 1.414 s \to L = 1.414$

$0.953 s^3 + 0.390 s$

$0.353 s \,)\, 0.674 s^2 + 0.276 \,(\, 1.909 s \to C = 1.909$

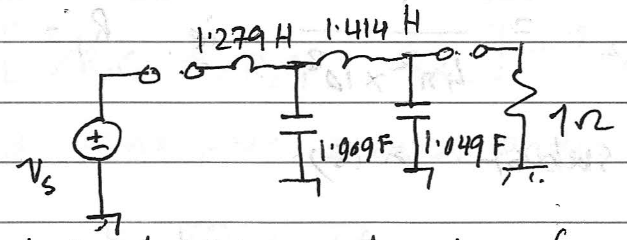
$0.674 s^2$

$0.276 \,)\, 0.353 s \,(\, 1.279 s \to L = 1.279$

$.353 s$

Q3(b)

Beginning from load end



is the L, C ladder realization for $H_N(s)$

Q. 4:

With $k = 1$, the transfer function is:

$$\frac{V_2}{V_1} = \frac{-s \cdot 2/R_1C_2}{s^2 + s\left[1/R_2C_2 + 1/R_2C_1 - 1/R_1C_2\right] + 1/R_1R_2C_1C_2} \quad \ldots (1)$$

The standard band-pass second order function is

$$T(s) = \frac{H_0 \cdot (\omega_P/Q_P)\, s}{s^2 + (\omega_P/Q_P)\, s + \omega_p^2} \quad \ldots (2)$$

Where ω_p = pole-frequency = $2\pi f_p$

Q_p = pole-Q.

By comparing (1) and (2)

$$H_0 \cdot \frac{\omega_P}{Q_P} = -\, 2/R_1C_2$$ (The negative sign is unimportant for magnitude response)

$$\frac{\omega_P}{Q_P} = \frac{1}{R_2C_2} + \frac{1}{R_2C_1} - \frac{1}{R_1C_2}$$

$$\omega_p^2 = \frac{1}{R_1R_2C_1C_2}$$

Given $C_1 = C_2 = 0{\cdot}1\ \mu F$ each

Let $C_1 = C_2 = 1\ F$ for now. We can do impedance scaling later.

Then $$\omega_p^2 = 4\pi^2 \times (10^4)^2 = \frac{1}{R_1R_2} \quad \ldots (3)$$

$$\frac{2\pi \times 10^4}{5} = \frac{2}{R_2} - \frac{1}{R_1} = \frac{2R_1 - R_2}{R_1R_2} \quad \ldots (4)$$

From (3) and (4) by division i.e. (3) ÷ (4)

$$\frac{4\cancel{\pi}^2 \times 10^8 \times 5}{2\pi \times 10^4} = \frac{1}{2R_1 - R_2} = 2\pi \times 10^4 \times 5$$

Q 4: (contd.)

$$2R_1 - R_2 = \frac{1}{\pi \times 10^5} \quad \cdots (5)$$

from (3) $R_1 R_2 = \frac{1}{4\pi^2 \times 10^8}$ ie. $R_1 = \frac{1}{R_2 \times 4\pi^2 \times 10^8} \quad \cdot (6)$

From (6), subst. in (5)

$$\frac{2}{R_2 \times 4\pi^2 \times 10^8} - R_2 = \frac{1}{\pi \times 10^5}$$; Let $\pi^2 \simeq 10$.

$$\frac{2}{4 \times 10 \times 10^8} - R_2^2 = \frac{R_2}{\pi \times 10^5}$$

or $$R_2^2 + \frac{R_2}{\pi \times 10^5} - \frac{2}{4 \times 10^9} = 0.$$

Solving for R_2 :

$$R_2 = 0.000021$$

Then $$R_1 = \frac{1}{R_2 \times 4\pi^2 \times 10^8} = 0.000012$$

First cut design values are

$C_1 = C_2 = 1F$

$R_1 = 0.000012$

$R_2 = 0.000021$

For $C_1 = C_2 = 0.1\mu F = 10^{-7}$ F,

we divide C_1, C_2 by 10^7 } impedance

multiply R_1, R_2 by 10^7 } scaling.

Second cut design values: $C_1 = C_2 = 0.1\mu F$

Let $R_A = R_B = 1K\Omega$ each. $R_1 = 120\,\Omega$

$R_2 = 210\,\Omega$

Q-4 (Contd.)

The values $R_1 = 120\,\Omega$, $R_2 = 210\,\Omega$,

$C_1 = C_2 = 0.1\ \mu F$ will satisfy

$f_p = 10^4$ Hz and $Q_p = 5$

$R_A = R_B = 1\,K\Omega$ is a free choice.