

**Concordia University**  
**Applied Ordinary Differential Equations**  
**ENGR213**  
**Winter 2011**

**TEAM ASSIGNMENT No. 2**

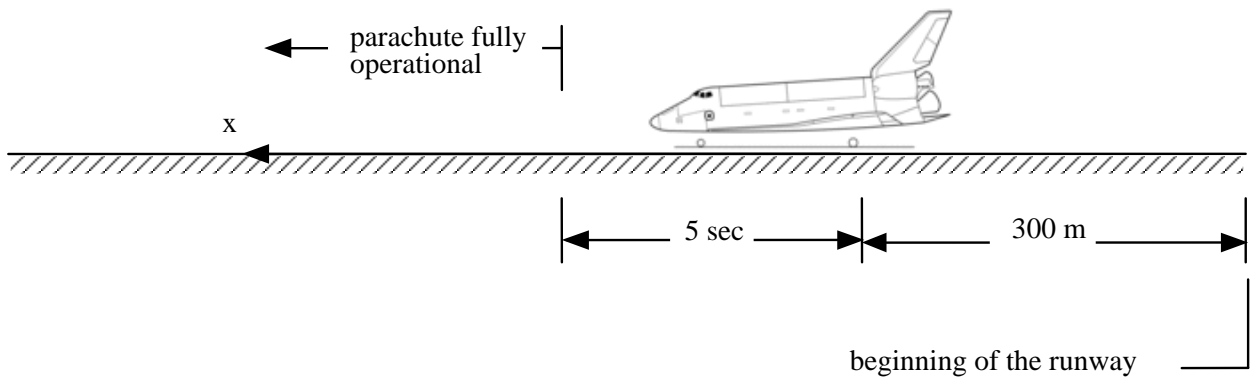
The team assignment **must** be done in groups of **2 to 3** students.

Each group submits one report. A signed "*Expectations of Originality*" form must be attached to the report.

Due date for Sec. G: March 7, 2011.

**PROBLEM No.1.** The Canadian Space Agency has decided to send an experimental probe to Mars. The probe's mass is 40 kg. When the probe is near Mars, it will be attracted by the gravitational field of Mars ( $g_{Mars} = 3.75 \text{ m/s}^2$ ). You are asked to provide the parachute's diameter so that the probe will touch Mars's surface with a velocity of 3 m/s (which is equivalent of dropping the probe from 0.5 m height on Earth). ( $A_{chute} = \pi d^2 / 4$ ,  $C_D = 1.4$ , the density of Mars's atmosphere is 2/3 of Earth's).

**PROBLEM No.2.** Due to an emergency, NASA has decided to land the Space Shuttle in Montreal (at the Pierre Elliott Trudeau International Airport). The longest runway of this airport is 11,000 ft and 200 ft wide (3,353 m, 61 m). The Shuttle's total landing mass is 104,328 kg and its velocity is 350 km/hr (twice the landing speed of a commercial airliner). Upon touchdown, a chute (with  $D = 16.5 \text{ m}$  and  $C_D = 1.4$ ) is deployed. The chute and orbiter's drag, along with the rolling resistance of the tires, slow the orbiter to a speed (say 60 km/hr) where the brakes can then be safely applied. Assess NASA's decision to land the Space Shuttle in Montreal. In your assessment consider only the deceleration of the vehicle due to the chute and neglect all the other effects. The parachute requires 5 seconds to be fully operational from the time of deployment. Assume further that the shuttle touches down 300 m from the beginning of the runway and upon brake application the vehicle needs approximately 200 m to come to a complete stop. (The nearby Mirabel International has a runway that is 12,000 ft long and 300 ft wide.)



Newton's 2<sup>nd</sup> law before parachute deployment

$$m \frac{dV}{dt} = 0$$

Newton's 2<sup>nd</sup> law after parachute deployment

$$m \frac{dV}{dt} = - \frac{1}{2} \rho_{air} C_D A V^2$$

Where:

- $m$  the mass of the Space Shuttle (104,328 kg)
- $V$  the velocity of the Shuttle (m/s)
- $t$  time (s)
- $g$  acceleration of gravity (9.81 m/s<sup>2</sup>)
- $\rho_{air}$  the density of air (1.1774 kg/m<sup>3</sup>)
- $C_D$  the drag coefficient of the parachute  $C_D = 1.4$  (no dimensions, dimensionless number, or clear number,)
- $A$  projected area of the solid on a plane perpendicular to the body's motion (  $\pi d^2/4$ , m<sup>2</sup>)
- $F_D$  drag force (N)
- $d$  diameter of the chute (m)
- $u_o$  landing velocity (350/3.6 m/s)

Dividing the above equations by  $m$  we have:

$$\frac{dV}{dt} = 0 \quad \text{and} \quad \frac{dV}{dt} = - \frac{K}{m} V^2 \quad \text{where} \quad K = \frac{1}{2} \rho_{air} C_D A$$