

## Some new lower bounds of van der Waerden numbers

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### 1. Some new lower bounds of $w(2; 5, t_1)$

#### 1.1. $w(2; 5, 7) \geq 260$

A good partition of the set  $\{1, 2, \dots, 259\}$  corresponding to  $w(2; 5, 7)$  is as follows:

```
11111101 11101111 10000111 10000100 01110111 10100111 11001011
11011100 01000010 11001001 10100001 00011101 11101001 11110010
11110111 00010000 10110010 01101000 01000111 01111010 01111100
10111101 11000100 00101100 10011010 A0010001 11011111 10011011
00101111 0111B011 00011011 01011110 111. (AB arbitrary)
```

#### 1.2. $w(2; 5, 8) \geq 331$

A good partition of the set  $\{1, 2, \dots, 330\}$  corresponding to  $w(2; 5, 8)$  is as follows:

```
A1000011 01100011 11110011 11011110 11010101 01111010 100001B1
01111010 01101101 11101111 01110111 00011110 01001011 10101011
11011110 11110001 11110100 10001100 00111010 00101111 11101101
10010111 00001000 01001011 10100011 11011111 11010001 01110000
10010110 00111111 00111101 11101101 10110111 10111000 10010111
100C0110 11D11100 11110111 01110001 1110001E 11000F10 1G.
(AB..FG arbitrary)
```

#### 1.3. $w(2; 5, 9) \geq 473$

A good partition of the set  $\{1, 2, \dots, 472\}$  corresponding to  $w(2; 5, 9)$  is as follows:

```
11101A10 0B110111 01011C10 1100111D 10EF1101 010G0110 11100110
01011101 01111010 H1110010 10110111 11111001 00100011 10101110
10100111 11011011 10111111 11001110 11110111 01101111 10010101
11010011 00011010 11101110 10110001 10010111 01010011 11101101
11011111 11100111 11111011 10110111 11001010 11101001 10001101
01110111 01011000 11001011 10101001 11110110 11101111 01110011
11111101 11011011 11100101 01110101 11000100 10011111 11101101
```

0100111I 01011110 10111010 01J00111 0110K010 10111L01 M1110011  
 01N11010 11101100 01P10111. (AB..OP arbitrary)

#### 1.4. $w(2; 5, 10) > 557$

A good partition of the set  $\{1, 2, \dots, 557\}$  corresponding to  $w(2; 5, 10)$  is as follows:

```
11111001 10011011 01110110 10111000 11011111 00111010 11101101
11111100 11000011 11111101 01110100 11110111 10110111 10101111
01100001 01111101 01011100 00110011 10110111 10100111 11111011
10001111 11011011 10101110 11100110 11011111 01000101 10001010
01101011 10110110 01011111 10010111 11110111 11010001 01111110
11110110 00100110 01101111 00011110 11101111 00101101 11101011
11101101 01111111 01011011 10101110 10000111 11011101 01011011
01101111 10000101 10011110 11111011 11110011 11111011 11111001
11011100 01100001 11010100 11101111 00111000 11101010 11110111
11111010 11010001 11111011 01001111 11010011 11011011 11101.
```

#### 1.5. $w(2; 5, 11) > 736$

A good partition of the set  $\{1, 2, \dots, 736\}$  corresponding to  $w(2; 5, 11)$  is as follows:

```
11001111 11011111 10111011 01101111 11101110 11101101 11001011
10011000 01111101 11010101 10111110 01001010 11111110 11110011
11111011 10111101 01101010 11110111 11010010 01010011 10111110
11110101 11111111 01011000 11111101 11011011 10101000 11000011
10111011 01111111 01111110 01110111 11011011 11110101 11110111
00110100 10110001 00101100 11010110 11111101 11100010 01110111
01111111 11101001 11110101 01011100 11011111 10111010 10010110
11011101 10001111 10011111 11101111 11110110 10100110 10101111
00101101 10111010 11110101 01111001 11010011 10011111 11001010
10111111 10110110 10000111 11101011 00111111 01111011 11111111
01111000 11110011 01101010 01001111 11101101 01101101 01111100
01001101 00001111 11011110 01110010 11111111 01010111 00110111
11110101 11111100 11101010 10111111 11011001 11111101 00010110
11110010.
```

## 2. Some new lower bounds of $w(3; t_0, t_1, t_2)$

Here, we present seven new lower bounds of  $w(3; t_0, t_1, t_2)$  and for each we provide an example of a good partition.

### 2.1. $w(3; 2, 4, 8) > 155$

22212212 12222221 12221122 22212221 22112222 22212222 01121112  
 22122222 12121121 22211222 22211121 22212212 22222212 11211122  
 22212222 22122122 22111212 21222122 22221221 222.

## 2.2. $w(3; 3, 3, 7) > 149$

20222022 22221201 22222212 20011200 20212220 12222221 22222202  
 22222102 22020210 22122220 10222202 20211220 12222221 22122221  
 20202122 00221201 22022212 22012220 22222.

## 2.3. $w(3; 3, 3, 8) > 185$

02122221 22122222 21222221 02201222 20220212 22212222 20020022  
 21002022 22221222 21202222 22210220 02122211 22212102 20122220  
 22121222 21222222 02001221 00201222 22122221 21220222 21022002  
 22221222 22202222 2.

## 2.4. $w(3; 3, 3, 9) > 221$

22222212 12202222 02121122 22222022 22222010 22222221 21220220  
 20212222 22112222 12222222 02101220 20222222 00222022 22212202  
 22222202 12122122 21020212 02221220 21222222 11222122 00212222  
 20012202 22222202 22211222 20220222 00222222 22102001 12222.

## 2.5. $w(3; 3, 3, 10) > 265$

22220222 12222022 22022221 10220022 22222220 22222211 22222222  
 02022222 21122222 22110202 21022022 21122121 22222222 00222022  
 22200112 22222222 12122222 21222222 02222021 10202212 22220020  
 02222222 22021221 22222222 12221100 22222222 00222222 21121222  
 21222221 22200220 22220212 22222220 20212221 2.

## 2.6. $w(3; 3, 4, 5) > 163$

02221211 12222122 22022221 11212221 21121101 20021222 12100202  
 21222212 22211021 22212012 10112112 02220201 12222022 22022021  
 11212221 21121110 20121222 12100222 21222212 22200120 222.

## 2.7. $w(3; 3, 5, 5) > 243$

22121122 22121001 11212212 11020021 21220122 21221210 02120012  
 11211010 01112221 01112122 12220120 20111210 22121121 11121121  
 22212111 22221222 21221211 12122211 11211112 10112221 21102202  
 12202100 22011212 22010120 01121110 22212111 12211200 22221211  
 10012112 10112122 112.