Frequency Hopping CDMA: A Review *

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Abstract

To obtain insight of frequency-hopping (FH) spread spectrum system, a review of the current studies on the performance and system capacity of FH code division multiple-access (CDMA) is carried out in this paper. Throughout the reviewing, several meaningful topics for the future studies are proposed.

1 Introduction

Code division multiple access (CDMA) technique has attracted considerable attention in personal and mobile communications due to its good anti-jamming capability and its potential for high capacity. Consider two types of modulation: PSK and FSK. If the pseudo-random (PN) sequence at the modulator is used to shift the phase of PSK signal. The resulting modulated signal is called a *direct sequence* (DS) spread spectrum signal. On the other hand, if the PN sequence is used to select the frequency of the transmitted signal pseudo-randomly, the resulting signal is called a *frequency hopping* (FH) spread spectrum signal [**?**].

Most of studies on spread spectrum systems focused on a DS CDMA because of its potential for higher capacity over an FH one and the capability of offering diversity reception in a RAKE receiver [**?**]. However, in the DS-CDMA, a stringent synchronization is required inherently, and a very good power-control algorithm is needed in order to minimize the multiple-access interference (MAI). As an alternative to DS-CDMA, FH-CDMA has its own advantages such as non-stringent timing requirement and good immunity to the "near-far" problem. So far, various efforts have been devoted to studying the performance and system capacity of FH-CDMA in the presence of partial-band interference, co-channel interference, and fading detriments. In particular, [**?**, **?**] examined the system capacity of an FH system with MFSK for possible application to digital mobile application. [**?**, **?**, **?**, **?**, **?**] studied the performance of FH-FSK in fading channels. The performance of coded systems was also studied as shown in [**?**, **?**]. In addition, in order to combine the advantages of DS and FH and to overcome their weaknesses, the hybrid DS/FH CDMA was therefore proposed and its related studies

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can be found in [?] and the references therein. Furthermore, the effect of the FH multiple access network on the throughput was also examined [?]. Recently, for the goal of providing effortless service for mobile and business users by means of a small, short-range radio-based network, a Bluetooth system has been defined [?]. In this system, fast FH is proposed to avoid interference.

To get a better understanding of the problems and objectives in FH-CDMA area, this paper will review the current studies related to FH-CDMA. Then, several meaningful topics are proposed for future research.

2 System Description

In FH system, FSK modulation with non-coherent detection is usually employed due to the fact that it is difficult to maintain phase coherence for a PSK modulation as the signal is hopped from one frequency to another over a wider bandwidth [?]. The FSK modulation could be either binary or M-ary. A simplified block diagram of an FH system is described in Fig. 1 [?].

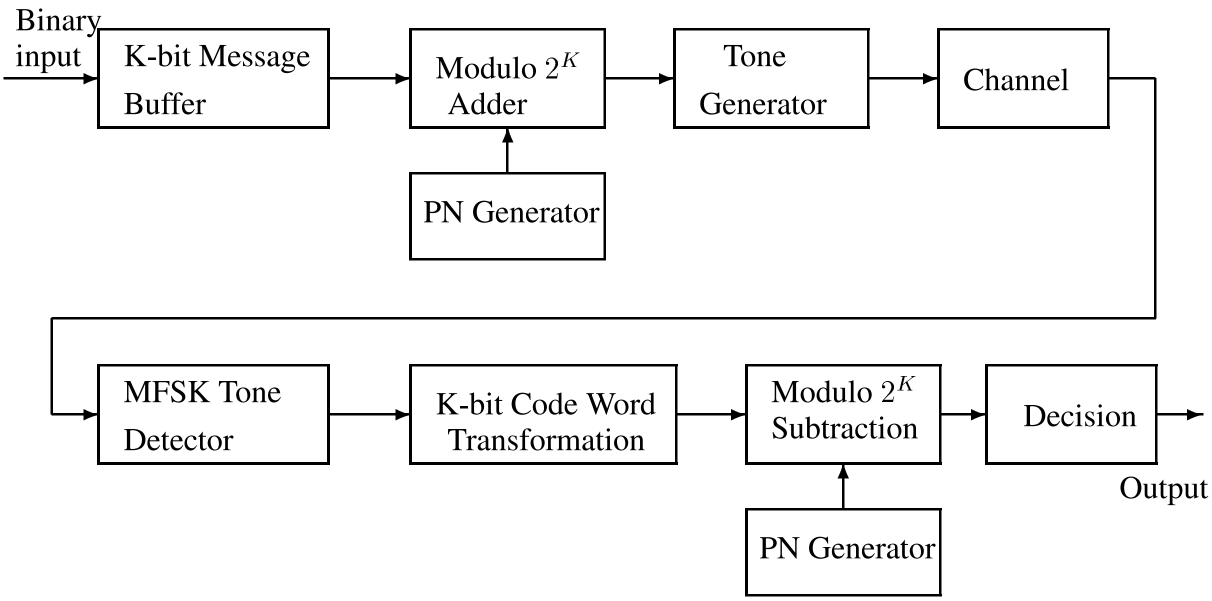

Fig. 1: A simplified block diagram of an FH system

As shown in the figure, the transmitted binary input is mapped into a K-bit code word and then the code word is mixed with the PN sequence. The resulting signal is translated into frequency and transmitted over the channel. During any signal interval, the transmitted signal could occupy one or more of the available frequency slots. In other words, the frequency used to transmit the information bearing signal is hopped from one to another. For this reason, the system is called frequency-hopping (FH) system and the PN sequence is thus named the hopping pattern.

When the transmitted signal is passing through the channel, the signal may suffer from background Gaussian noise, interference, and fading. In FH system, the most serious type of interference is partial-band interference which is modeled as a zero-mean Gaussian random process with a flat power spectral density ρ of the total bandwidth W and zero elsewhere. In mobile communications, the channel fading, such as Raleigh, Rician, Log-normal, or Nakagami-m, can seriously deteriorate the system performance. The fading effect will be discussed later.

At the receiving end, the received signal is demodulated by means of an FSK demodulator. The resultant signal is then transformed to the K-bit code word and mixed with the PN sequence which is identical to that in the transmitter to remove the pseudo-random frequency translation introduced at the transmitter. Finally, the original information bearing signal is extracted.

Depending on whether the FH rate is equal to or faster than the symbol rate, the system is called the fast or slow FH system. The fast FH has the advantage in antijam applications. For instance, it can prevent a so-called following jammer from having sufficient time to intercept the frequency and to create interfering signal components. However, the penalty paid for this is the energy which is combined non-coherently from several frequency-hopped elements [**?**].

The performance of FH system will be discussed next.

3 Performance Analysis

The performance analysis on FH system can be divided into two categories: performance analysis of a single-user system and performance analysis of a multiuser system. In a single-user system, the analysis is focusing on the effects of partial-band interference, fading, and diversity combining. In a multi-user system, the model of co-channel interference (CCI), the effect of the CCI, and the system capacity are of interest. The studies on these issues are reviewed in the following.

3.1 Single User System

In a single-user FH system, after dehopping, the received signal is similar to that for the conventional (no frequency hopping) FSK signaling and hence the performance analysis of the conventional FSK is applicable to the case of FH-FSK. Of the current studies for noncoherent FSK signals, the analysis in correlated fading has attracted a lot of attention [**?**, **?**]. Specifically, [**?**] studied the bit error probability in Rayleigh and Rician fading with correlated diversity and [**?**] investigated the maximal ratio combining (MRC) performance in correlated Nakagami fading. However, the FH-FSK signal, unlike the conventional FSK counterpart, may suffer from an interference called the partial band interference in addition to fading. The performance in the presence of partial band interference has been analyzed by many researchers.

3.1.1 Non-Fading

We first consider the non-fading case. The partial-band interference can be modeled as a zero mean Gaussian random process with a flat power spectral density ($= N_I/\rho$) over a fraction ρ of the total bandwidth W and zero elsewhere. Thus, the received signal will be jammed with the probability ρ and not be jammed with the probability $1-\rho$. By noting that bit error probability in pure additive white Gaussian noise (AWGN) is $P_b = \frac{1}{2}e^{-E_b/N_o}$ [?], where E_b is the bit energy and N_o the noise power spectral density, the average probability of error in the presence of partial band interference is

$$P_2(\rho) = \frac{1}{2}\rho e^{-\rho E_b/2N_I} \tag{1}$$

If we consider a worst case in which the jammer can select a value of ρ to maximize the error probability, the corresponding bit error probability (P_b) is given by [?]

$$P_b = \begin{cases} \frac{1}{2}e^{(-E_b/2N_I)}, & E_b/N_I \le 2, \quad \rho = 1 \\ \frac{e^{-1}}{E_b/N_I}, & E_b/N_I \ge 2, \quad \rho = 2/(E_b/N_I) \end{cases} \tag{2}$$

3.1.2 Fading Effect

It is well-known that different fading channels (Rayleigh, Rician, Log-Normal, Nakagami-m, etc.) will yield different results. Here, for the sake of simplicity, we discuss only the performance in the generalized (Nakagami-m) fading. When the FH-FSK signal is passing through a fading channel, the signal amplitude will be affected by a random variable so that the received signal energy per bit would be R^2E_b, where R is an attenuation factor representing the channel fading. In Nakagami-m fading, the probability density function (pdf) of R is given by

$$f(R) = \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m}e^{-mR^2/\Omega}, \quad R \ge 0,\, m \ge 1/2 \tag{3}$$

where the constant m is the fading parameter and $\Gamma(m)$ is the Gamma function, and $\Omega := E\{R^2\}$.

The average performance in partial-band interference and Nakagami-m fading can be obtained by replacing E_b of (2) with R^2E_b and then averaging (2) over R. The resultant bit error probability is given by [?]

$$P_b = \begin{cases} \frac{1}{2}\left[\frac{m}{m+E_b/2N_I}\right]; & E_b/N_I \le 2m/(m-1), \quad \rho = 1 \\ \frac{\left(\frac{m-1}{m}\right)^{m-1}}{E_b/N_I}; & E_b/N_I \ge 2m/(m-1), \quad \rho = 2m/[(m-1)(E_b/N_I)] \end{cases} \tag{4}$$

In the above, setting $m = 1$, we can obtain the performance in Rayleigh fading channel while as $m \to \infty$, we have the performance in non-fading channel.

There are also other studies on the performance in the presence of large Doppler shift [?], band multitone jammer [?], logical and diversity combining for packet radio networks [?], effect of delay spread [?], etc.. Here, we will not review them one by one in order to focus on a more practical multiuser system.

3.2 Multiuser System

In an FH-CDMA system, each user is assigned a PN hopping pattern, based on which the user selects the frequency to transmit the signal. Owing to the fact that many users are simultaneously occupying a common channel, the bit error rate (BER) performance of FH-CDMA system is mainly determined by the probability of hits. Here, a hit means that two or more signals are hopped into the same frequency slot and thus incurs the interference to each other. To achieve an acceptable BER, the probability of hit must be kept quite low, implying that the number of simultaneous users to be much lower than that of available frequency slots [**?**]. In the following, the studies on the system capacity and co-channel interference models are reviewed.

3.2.1 System Capacity

Here, we focus on the system model described by Goodman in 1980 [**?**]. The block diagram of this system is the same to that in Fig. 1 (see also Figs.1,2 in [**?**]). We consider that each user is assigned a unique address, which is a sequence of L K-bit code words. As a result, each code word occupies a time slot of duration $\tau = T/L$ and takes on a value between zero and $2^K - 1$. At the receiver, each of the 2^K frequency slots is determined in a threshold detection manner. Thus, all detected tones are formed a $2^K \times L$ decision matrix. Due to the fact that the detected tones may come from both the desired user and other users in the system, the majority logic decision rule is used, which regards the row having the largest number of entries as the correct one.

In practice, the background noise or fading may cause a transmitted tone to be omitted (this is called deletion) or a false tone to be detected (called false-alarm). Letting r denote the envelope of the received signal and b the threshold used in the system, the probabilities of deletion and false-alarm are respectively given by [**?**, **?**]

$$P_D = Pr\{r < b\} \tag{5}$$

$$P_F = e^{-b_0^2/2} \tag{6}$$

where $b_0 = b/\sqrt{N_o}$ is the threshold level normalized by the noise power N_o.

Based on the decision matrix, it has been shown by Goodman that the corresponding BER can be computed in the following [**?**].

1. Probability of an interfering tone being inserted in a position of the decision matrix is

   $$P_I = p + P_F - pP_F$$

   where

   $$p = [1 - (1 - 2^{-K})^{M-1}](1 - P_D)$$

2. Probability of m entries in an incorrect row is

   $$P_s(m) = \binom{L}{m} P_I^m (1 - P_I)^{L-m}$$

3. Probability of exactly k rows having as many as n entries, where n is the maximum number of entries, is

   $$P(n,k) = \binom{2^K - 1}{k} [P_s(n)]^k \left[\sum_{l=0}^{n-1} P_s(l)\right]^{2^K - 1 - k}$$

4. Probability of i entries in the correct row is

$$P_c(i) = \binom{L}{i}(1-P_D)^i P_D^{L-i}$$

5. Upper bound on BER

$$P_b \leq \frac{2^{K-1}}{2^K-1}\left(1 - \sum_{i=0} LP_c(i)\left[P(i,0) + \frac{1}{2}P(i,1)\right]\right)$$

By using the above formulas, it has been found that for bit rate= $32kb/s$, bandwidth= $20MHz$, and $P_b \leq 10^{-3}$, the optimum values of K and L are 8 and 19, respectively [**?**]. Also, for the perfect transmission (considering mutual interference alone) and the required $P_b \leq 10^{-3}$, the system can accommodate up to 209 simultaneous users, while only 170 users can be supported by the system in the presence of the transmission impairments including white Gaussian noise and frequency-selective Rayleigh fading with an average rf SNR of 25dB.

The effect of other fading channels on the system capacity has also been studied recently. From the above formulas, it can be seen that the false-alarm probability P_F is determined by the noise power alone while the deletion probability P_D is due to both fading and noise. Hence, the other fading channels will affect P_D, through which they influence the system capacity. In [**?**], the study on system capacity is extended over Rician, shadowed Rician, and general Nakagami fading channels. It is shown that in a Rician fading channel with the Rician factor (ratio of power in specular component to that in multipath component) of 2dB will reduce the system capacity by 13% as compared to the capacity for the non-fading case. In Nakagami fading, the system capacity is found to decrease by 50% for the fading parameter $m = 0.5$.

Remarks

- In the above analysis, the statistical independence between the rows of the decision matrix is assumed [**?**, **?**]. This assumption has been examined and shown valid in [**?**].

- The system is using hard-limited combining, i.e., the logic decision matrix. In [**?**], Yue compared the performance of the linear combining with the one of the hard-limited combing and inferred that for large SNR the system with hard-limited combining as shown in [**?**, **?**] can accommodate more users than that with the linear combining because in realistic FH-CDMA system the interference which limits the performance has non-Gaussian nature while the linear combining is optimal for Gaussian noise.

- The analysis has one weakness: the probability of deletion P_D is assumed to be equal in all the cases. In my opinion, however, when there is a hit, the probability of deletion would be different from that in non-hit case.

To obtain insight on the independence assumption and the non-Gaussian nature of the interference, the related work is reviewed in the following.

3.2.2 Independence assumption

For the sake of simplicity, we consider a slow FH system in a single hop per symbol basis [**?**]. Consider MFSK modulation and let n_F denote the total number of available frequency slots. The received signal is given by

$$r(t) = S_m(t) + I(t) + n(t) \tag{7}$$

where $n(t)$ is the background Gaussian noise with spectral density N_o, $S_m(t)$ is the information bearing waveform, i.e.,

$$S_m(t) = \sqrt{2S} cos[(\omega_o^{(0)} + \omega_m)t + \theta_m^{(0)}], \quad 0 \le t \le T_c \tag{8}$$

where S is the received signal power from the desired transmitter, T_c is the chip duration, $\omega_o^{(0)}$ is the FH carrier for the 0-th transmitter (we assume that the symbol is transmitted in the 0-th frequency slot), ω_m is the frequency of the m-th tone, and $\theta_m^{(0)}$ is the phase angle.

In (7), $I(t)$ is the total interference from the interferers. We assume there are J transmitted signals in the system. The total interference during the chip interval $(0, T_c)$ can be written as

$$I(t) = \sum_{j=1}^{J} \gamma_j \sqrt{2I_j} cos[(\omega_o^{(j)} + \omega_k^{(j)})t + \theta_k^{(j)}] \tag{9}$$

where γ_j is an indicating parameter and it is set to 1 if j-th interferer is on and 0 otherwise. $\{\gamma_j\}$ are assumed to be independent random variables. I_j is the received signal power from the j-th transmitter. $\omega_o^{(j)}$ is the FH carrier for the j-th transmitter, $\omega_k^{(j)}$ is the k-th tone transmitted from the j-th transmitter, and $\theta_k^{(j)}$ is the phase angle uniformly distributed between 0 and 2π.

It has been shown in [**?**] that if the most likely location of the transmitters is the neighborhood of the ring of radius r_o, the I_j/S ratios can be modeled as independent, identically distributed (i.i.d) exponential random variables with a common density function

$$f_{I/S}(x) = \alpha e^{-\alpha x} \tag{10}$$

where $\alpha = \frac{5}{4}\left(r_o/r\right)^4$, in which r is the distance between the receiver and the desired transmitter.

After dehopping and suitable normalizing, the l-th energy detector has the output R_l given by [**?**]

$$R_l = X_l^2 + Y_l^2, \quad 1 \le l \le M \tag{11}$$

where

$$X_l + iY_l = \delta_{ml} e^{\phi_m} + \sum_{j=1}^{J} \beta_l^{(j)} \sqrt{I_j/S} e^{\theta_l^{(j)}} + z_l \tag{12}$$

where δ_{ml} is the Kronecker delta, z_l is a complex-valued Gaussian random variable, $\beta_l^{(j)}$ is an indicating parameter defined as

$$\beta_l^{(j)} = \begin{cases} 1, & \text{if } j\text{-th interferer transmits } l\text{-th tone} \\ 0, & \text{otherwise} \end{cases}$$

Now, we further define the sets A_l, $1 \leq l \leq n_F$, as

$$A_l = \{j : \beta_l^{(j)} = 1, 1 \leq j \leq J\}$$

and let $n_l = |A_l|$ denote the number of elements in A_l. If $\{I_j/S\}$ are i.i.d. random variables, the decision variable R_l will depend only on n_l. Based on different assumption of n_l, the independence and correlated models are established.

A. Independence model

In this model, all n_l are assumed independent. Thus, each frequency slot has a probability of $1/n_F$ being hit by any one of the J transmitters. For FH/BFSK, the probability of symbol error is given by [**?**]

$$P_e(m) = \frac{1}{2} \sum_{j=0}^{J} \sum_{k=0}^{J} \hat{B}_j \hat{B}_k e^{-\frac{1}{2[\sigma_j^2 + \sigma_k^2]}} \tag{13}$$

where

$$\hat{B}_k = \binom{J}{k} \left(\frac{1}{L}\right)^k \left(1 - \frac{1}{L}\right)^{J-k} \tag{14}$$

The independent model permits relatively easy evaluation of system performance. However, in reality, each interferer can transmit in only one of n_F frequency slots. Thus, $\{n_l\}$ are not independent but satisfy $\sum_{l=1}^{n_F} \leq J$. Next, we consider correlated model.

B. Correlated model

Since $\cap_{l=1}^{n_F} A_l = \phi$, $\{R_l\}$ are conditionally independent for any given set of $\{n_l\}$. Therefore, the evaluation of the symbol error probability turns to be an enumeration procedure for each possible outcome of $\{n_l\}$ satisfying the constraint $\sum_{l=1}^{n_F} \leq J$. The resulting expression can be found in Appendix B of [**?**] and not shown here.

Remarks

- Numerical results show that for large $n_F(\geq 10^3)$ both models produce nearly identical results. Also, even for small $n_F(= 100)$, the results from both models are different but still comparable. The numerical results have been further substantiated in [**?**]. Therefore, we can use the independence assumption in performance analysis.

3.2.3 Non-Gaussian model of interference

Consider a detector which projects the received signal onto a set of M real and orthogonal basis functions. After the correlation, the received signal can be expressed into an M-dimensional vector

$$\mathbf{Z} = a(r)\mathbf{S} + \mathbf{Y} + \mathbf{N} \tag{15}$$

where $\mathbf{S}$ is the desired signal, $\mathbf{N}$ is the Gaussian RV, and $\mathbf{Y}$ is the interference random vector which is given by

$$\mathbf{Y} = \sum_{i=1}^{\infty} a(r_i)\mathbf{X}_i \tag{16}$$

where $a(r) = K/r^m$ is the signal amplitude loss function over distance r and $\mathbf{X}_i$ is the random vector for the i-th interferer.

Considering the l-th branch of $\mathbf{Y}$ and assuming that there is a large (infinite) number of transmitters ($J \to \infty$), we have [**?**]

$$Y_l = \lim_{J\to\infty} \sum_{i=1}^{J} a(r_i)X_l \tag{17}$$

where $X_l := [cos\phi_l, sin\phi_l]$, in which ϕ_l is uniformly distributed over $[0, 2\pi)$.

It is shown in [**?**] that if the random variables $\{X_l\}$ are i.i.d. and spherically symmetric (SS), and the interferers using the same frequency form a Poisson field in the plane with density λ_t, the characteristic function of Y_l is SS α-stable, i.e.,

$$\phi_{Y_l}(\omega) = e^{-\gamma||\omega||^{\alpha}} \tag{18}$$

where $\alpha = 2/m$, $0 < \alpha < 2$, and the parameter γ is the dispersion given by

$$\gamma = -\lambda_t \pi K^{\alpha} \int_0^{\infty} \frac{\Phi_0'}{x^{\alpha}} dx$$

Here, $\Phi_0(x)$ is the CF of the SS random variable X_l and $\prime$ denotes the differentiation.

As a result, $\mathbf{Y}+\mathbf{N}$ in (15) form a Gaussian and α-stable mixture model. Again, we assume that the symbol is transmitted in the 0-th frequency slot. Based on the SS α-stable model, the BER is upper bounded by [**?**]

$$P_e \leq \frac{M}{2(M-1)}(M-1)Pr\{W_1 > W_0|0\} \tag{19}$$

where W_0 and W_1 denote the outputs of the envelop detectors of two branches of the FSK system. The W_0 has a Rician distribution and other $W_j, j = 1, 2, ..., M-1$ have Rayleigh distribution. The probability density functions for $\{W_j\}$ can be found in [**?**] and are not reproduced here.

The shadowing and fading effects on the interference are also studied by Ilow [**?**]. Besides, in order to improve the performance, the Cauchy receiver and soft-limiting receiver are discussed.

Remarks

- The above mixture model shows that the interference has a non-Gaussian nature and it somehow explains that the hard-limited combining could provide better performance than the linear combining [**?**].
- However, the mixture model has several weaknesses.

1. Infinite variance: It is well-known the the α-stable has infinite variance, which is incurred by the assumption of infinite powers for the terminals which are infinitely close to the receiver [?]. In practice, we know that there is no way the terminal has an infinite power no matter how close it is to the receiver.

2. Same frequency for all interferers: In the derivation, it is implied that all the interferers could be present at all frequency slots. In reality, due to the frequency hopping, it is impossible.

It is noteworthy to mention that there are a lot of related work on the performance analysis in multiuser system by considering fading effect [?], new two-level FSK modulation process [?], and even hybrid DS/FH scheme [?]. Here, we will not review them for the sake of simplicity.

4 Conclusions

In this report, the studies in literature for FH-CDMA system have been reviewed to get a better understanding of the problems and objectives in FH-CDMA. Although many performance issues have been addressed, there are still relevant and meaningful problems remaining un-touched. In the context of this review report, several interesting topics are:

1. As mentioned in the remarks of Section 3.2.1, the analyses of [?, ?] ignored the fact that P_D in the presence of hit is different from that in non-hit case. Therefore, if we modify the analysis accordingly, is there any notable difference?

2. In [?], the performance of linear combining was compared with that of hard-limited combining. What is the performance if we use the soft-limited combining?

3. In [?], the performance of FSK in correlated fading was analyzed but the BER evaluation required two-dimensional integration. Actually, there is a simple way to reduce that computational complexity to no integration.

References