# Multilevel Quantized Soft-Limiting Detector for an FH-SSMA System

Jian F. Weng[1], Tho Le-Ngoc[2], Guo Q. Xue[3], Sofiène Tahar[4]

[1] Sapphire R&D Inc., Canada

[2] Dept. of Electrical & Computer Engineering, McGill University, Canada

[3] Nortel Networks, Canada

[4] Dept. of Electrical & Computer Engineering, Concordia University, Canada

**Abstract:** A new multi-level quantized soft-limiting (SL-MQ) detector for frequency hopping spread spectrum multiple access (FH-SSMA) system is proposed and analyzed. Numerical and simulation results in both additive white Gaussian noise and frequency selective Rayleigh fading channels show that, as compared to the hard-limiting (HL) detector, the new SL-MQ with $M = 4$ can improve the system capacity by almost 10% at the bit error rate of $10^{-3}$. Furthermore, the performance of the SL-MQ is less sensitive to the optimum value of the amplitude threshold ($b$). Hence, the SL-MQ can tolerate an inaccurate estimate of this optimum value in practice. A practical maximum output SNR criterion is also discussed, which can provide a sub-optimum $b$ without knowing the system SNR and the number of interferers.

## 1 Introduction

Spread spectrum multiple access (SSMA) techniques have attracted considerable attention in personal and mobile communications due to their good anti-jamming capabilities and their potential for high capacity. In the SSMA system, the most common forms of the spread spectrum signals are *direct-sequence* (DS) and *frequency-hopping* (FH) [1]. Most studies on spread spectrum systems focused on a DS-SSMA because of its potential for higher capacity over the FH counterpart and its capability of offering diversity reception in a RAKE receiver [2]. However, in DS-SSMA, a stringent synchronization is required for demodulation and despreading, and a very good power-control algorithm is needed in order to minimize the multiple-access interference (MAI) and to reduce the "near/far" effect. As an alternative to DS-SSMA, FH-SSMA has its own advantages such as non-stringent timing requirement and good immunity to the "near-far" problem [2]. Recently, FH technique has been even adopted in a "Bluetooth" system, which is defined for the purpose of providing effortless service for mobile and business users by means of a small, short-range radio-based network. So far, various efforts have been devoted to studying the performance and system capacity of FH-SSMA in the presence of partial-band interference, co-channel interference, and fading detriments [3–8]. In particular, Goodman *et al.* [3] examined the system capacity of an FH system with MFSK in additive white Gaussian noise (AWGN) and Rayleigh multipath fading channel for digital mobile application. [4] studied the adjacent cell interference in FH-MFSK cellular mobile radio system. In [5–7], Goodman's analysis [3] was extended to evaluating the performance in Rician, shadowed Rician, Nakagami fading, or factory environment, while in [8], the efficiency of fast FH system was analyzed. For an SSMA with fast FH, hard-limiting (HL) detector is commonly used [3, 7] and its performance can be shown better than the square-law linear combining detector because the serious detriment to the system performance is the non-Gaussian interference incurred by the signals from multiple users [9, 10]. Such non-Gaussian interference may have an "impulsive" nature, i.e., the interference exhibits a large energy when several users simultaneously occupy the same frequency slot. In suppressing the "impulsive" noise, a soft-limiting (SL) detector could be more effective [11]. This idea has been noted by many researchers and various "soft" limiting (or other than hard limiting) schemes have been presented and studied, such as ratio-threshold with coding [12], nonparametric rank sum receiver [13], clipped-diversity combining [14, 15], and ratio-statistic combining [16]. In this paper, we focus on "clipped-diversity combining" like detector due to its computational simplicity and its robustness against partial band interference [14, 15]. In performance analysis, we adopt a multi-hit model as in [9, 17] to establish a direct performance comparison with the hard-limiting detector [3]. For the purpose of implementing a digital detector, we also consider a multi-level quantized detector.

This paper examines the performance of multi-level quantized soft-limiting (SL-MQ) detector in FH-SSMA system. The HL and SL are the two special cases of the SL-MQ when the number of levels ($M$) is set to 2 and $\infty$, respectively. Both AWGN and frequency selective Rayleigh fading channels are considered. Numerical and simulation results are presented and they show that the SL-MQ ($M > 2$) can offer a better bit error rate (BER) performance than the HL. In particular, at the bit error rate of $10^{-3}$, the system capacity can be improved by almost 10% by using the SL-4Q over the HL (i.e., SL-2Q). Smaller capacity improvement can be further obtained with more levels (i.e., $M > 4$). Moreover, the performance of the SL-MQ ($M \geq 4$) is less sensitive to the optimum value of the amplitude threshold ($b$) as compared to the HL. In other words, in practice, the SL-MQ can still work well even with an inaccurate estimate of the optimum value of $b$. The optimum value of $b$ can theoretically be obtained by minimizing the BER under the assumption of the knowledge of system SNR and the number of interferers. In practice, when such knowledge is unavailable *a priori*, a maximum output SNR criterion can be used and it can present a sub-optimum $b$ that is close to the optimum one.

This paper is organized as follows. In Section 2, an FH-SSMA system model with MFSK modulation and the corresponding mathematical model for the received signal are established. Also, the proposed SL-MQ is described. Section 3 derives the bit error probability for the SL-MQ in both AWGN and Rayleigh fading channels. Illustrative results, together with the maximum output SNR criterion for optimizing the amplitude threshold, are presented and discussed in Section 4. Finally, Section 5 concludes the paper.

## 2 System Description

Consider an FH-SSMA system with MFSK modulation over AWGN and frequency selective multipath Rayleigh fading channels [3]. A simplified block diagram of an FH-SSMA system is described in Fig. 1 (similar to that in [7]). In this system, each user is assigned a unique pseudo-random (PN) address, which is a sequence of $L$ $K$-bit code words. At the transmitter, the PN address is used to scramble (modulo-$2^K$ addition) the buffered $K$-bit message of a user. Each code word occupies a time slot of duration $\tau = T/L$ ($T$: symbol interval, $L$: number of time slots) and takes on a value between zero and $2^K - 1$. After passing through the channel, the signal at the receiver is demodulated by means of an MFSK demodulator. The resultant signal is then transformed to the $K$-bit code word and mixed with the address identical to that in the transmitter to remove the scrambled frequency translation. Next, each of the $2^K$ frequency slots is determined in a threshold detection manner and thus all detected tones are forming a $2^K \times L$ decision matrix. Due to the fact that the detected tones may come from both the desired user and other interfering users in the system, the majority logic decision rule [3] is used. It selects the row having the largest number of entries as the correct one.

The mathematical model is developed as follows. We consider $J$ users and assume the first user to be the user of interest and other users to be interferers. The PN address for user $u$ is $\bar{A} = [a_0^{(u)}, a_1^{(u)}, ..., a_{L-1}^{(u)}]$, where $a_l^{(u)} \in \{0, 1, ..., 2^K - 1\}$, $0 \leq l \leq L-1$. After dehopping, the output signal at the $t$-th slot and $l$-th tone is given by [9, 17]

$$r_{tl} = \delta_{ml}\sqrt{2\rho^{(0)}}\alpha_{tl}^{(0)} + \sum_{u=1}^{J-1} \gamma_{tl}^{(u)}\sqrt{2\rho^{(u)}}\alpha_{tl}^{(u)} + z_{tl} \tag{1}$$

where $m$ is the tone index transmitted by the first user and its presence ($l = m$) or absence ($l \neq m$) in the $l$-th tone is indicated by the Kronecker delta $\delta_{ml}$. $\{\rho^{(u)} = E_{\mathrm{W}}^{(u)}/(LN_o), u = 0, 1, ..., J-1\}$ are the normalized signal power per word. $E_{\mathrm{W}}^{(u)}$ is the word energy for user $u$ and $N_o$ is the noise spectral density. The sum term in (1) denotes the interference from multiple interferers, in which $\gamma_{tl}^{(u)}$ is an indicating function set to 1 if the $u$-th interferer (after dehopping) occupies the $l$-th tone at the $t$-th time slot and 0 otherwise. $z_{tl}$ is a complex-valued Gaussian noise with zero mean and unit variance, which is assumed to be independent and identically distributed (i.i.d.).

In (1), $\{\alpha_{tl}^{(u)}, u = 0, 1, ..., J-1\}$ for all users can be set to $e^{j\phi_{tl}^{(u)}}$ to reflect the AWGN channel or considered to be complex-valued fading parameters to represent the Rayleigh fading channel. In AWGN channel, $\{\phi_{tl}^{(u)}\}$ are the phase angles uniformly distributed over $[0, 2\pi)$ and they are assumed statistically independent for all $t$, $l$, and $u$. In Rayleigh fading channel, on the other hand, $\alpha_{tl}^{(u)}$ can be assumed Gaussian distributed (the amplitude of $\alpha_{tl}^{(u)}$ is Rayleigh distributed). Besides, we restrict

$a_k^{(u)} \neq a_l^{(u)}$ for $k \neq l$ so that the fading parameters are independent from slot to slot [9]. In addition, we assume that (i) fading parameters are independent for different users and (ii) the frequency spacing between the hops is larger than the coherence bandwidth of the Rayleigh fading channel [3, 9]. As a result, the fading parameters $\{\alpha_{tl}^{(u)}\}$ can be also assumed mutually independent for all $t$, $l$, and $u$ [9].

Envelope detection has been commonly used to detect the frequency tone in each time slot as shown in [3, 9]. The test statistics $\{Y_l\}$ for all frequency tones can be obtained as follows.

$$y_{tl} = d(|r_{tl}|) \tag{2}$$

$$Y_l = \sum_{t=0}^{L-1} y_{tl} \quad , \quad l = 0, 1, ..., 2^K - 1 \tag{3}$$

where $d(x)$ is a nonlinear decision function, $|r_{tl}|$ is the amplitude of $r_{tl}$, and $y_{tl}$ is the functioned amplitude level.

In (2), both square law linear combining [1, 9] and hard-limiting (HL) detector [3, 7] can be used by setting respectively $d(x)$ to $x^2$ and $d_{HL}(x)$, where $d_{HL}(x)$ is defined as

$$d_{HL}(x) = \begin{cases} 1, & \text{if} \quad x > b \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where $b$ is the threshold.

Motivated by using a soft-limiting (SL) function to robustly suppress the non-Gaussian interference [11, 14, 15], we examine the performance of a multi-level quantized soft-limiting (SL-MQ) detector, whose decision function is given by

$$d_{SL-MQ}(x) = \begin{cases} M-1, & \text{if} \quad x > b \\ i, & \text{if} \quad \frac{i}{M-1}b < x \leq \frac{(i+1)}{M-1}b, \quad i = 0, 1, ..., M-2 \end{cases} \tag{5}$$

where $M$ is the number of levels.

One can readily see that by setting $M = 2$, the above function $d_{SL-MQ}(x)$ reduces to the HL function. On the other hand, the SL-MQ function in (5) can also be expressed as $\frac{1}{M-1}d_{SL-MQ}(x)$ to imitate the pure SL function. If $M \to \infty$, the normalized $\frac{1}{M-1}d_{SL-MQ}(x)$ would become a SL function, i.e., $d(x) = 1$ if $x > b$ and $d(x) = x/b$ otherwise. Here, we use the expression in (5) because the representation of the finite output values in binary bits is obvious.

## 3 Performance Analysis

In this section, we extend Goodman's analysis [3], which was for the hard-limiting function in (4) (or, equivalently, the SL-2Q), to analyzing the SL-MQ ($M > 2$) function. Besides, we also refine the analysis of [3], which limited the hits (more than one user occupying the same frequency tone) on the spurious rows and limited the maximum number of hits to be 1, so as to study the effects of multiple hits and hits on all rows on the performance.

To yield mathematically tractable analysis, we assume that all users have the same power, i.e., $\rho^{(0)} = \cdots = \rho^{(J-1)} = \rho$. The performance of the proposed SL-MQ technique is to be evaluated as follows.

## 3.1 Probability of $y_{tl} = q$

The probability of $y_{tl} = q$ is the probability of mapping the received signal $r_{tl}$ into level $q$ in the SL-MQ. To ease our discussion, we let $\Gamma_{tl} = \left[\gamma_{tl}^{(1)}, \gamma_{tl}^{(2)}, ..., \gamma_{tl}^{(J-1)}\right]^T$ be the state vector representing the presence or absence of the interferers in the $(t, l)$ position (the $l$-th tone at the $t$-th time slot). Each element in the vector $\Gamma_{tl}$ takes the value from the set $\{0, 1\}$, and the space of all possible $2^{J-1}$ state vectors is denoted by $S_\Gamma$ .

### 3.1.1 Conditional Probability $\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\}$

We first consider the probability of $y_{tl} = q$ conditioning on $\Gamma_{tl} = \mathcal{A}$, denoted by $\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\}$. Here, $\mathcal{A}$ is a particular state vector in the space $S_\Gamma$. We proceed the evaluation in both AWGN and Rayleigh fading channels.

**A. AWGN Channel**

In AWGN channel, we further denote a phase vector, i.e., $\Psi_{tl} = \left[\delta_{ml}^{(0)}\psi_{tl}^{(0)}, \gamma_{tl}^{(1)}\psi_{tl}^{(1)}, ..., \gamma_{tl}^{(J-1)}\psi_{tl}^{(J-1)}\right]^T$, to represent the phase angles of the users being present in the $(t, l)$ position. Thus, conditioning on $\Gamma_{tl} = \mathcal{A}$ and $\Psi_{tl}$, the probability density function (pdf) of $|r_{tl}|$ can be written in the form of a Rice distribution [1, 18], i.e.,

$$f(r) = re^{-(r_d^2+r^2)/2}\sum_{n=0}^{\infty}\frac{1}{n!\,n!}\left(\frac{r_d^2}{2}\right)^n\left(\frac{r^2}{2}\right)^n \tag{6}$$

where $r_d$ is the amplitude of the conditional mean of $r_{tl}$, i.e.,

$$r_d = \left|\delta_{ml}\sqrt{2\rho^{(0)}}e^{j\psi_{tl}^{(0)}} + \sum_{u=1}^{J-1}\gamma_{tl}^{(u)}\sqrt{2\rho^{(u)}}e^{j\psi_{tl}^{(u)}}\right| \tag{7}$$

Accordingly, it can be shown that the conditional probability of $y_{tl} = q$ is given by

$$\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}, \Psi_{tl}\} = \begin{cases} P((q+1)\beta) - P(q\beta), & q = 0, 1, ..., M-2 \\ 1 - P(q\beta), & q = M-1 \end{cases} \tag{8}$$

where $\beta := b/(M-1)$ is the duration for each sub-interval and $P(a)$ is the probability that $|r_{tl}| < a$, i.e.,

$$\begin{aligned} P(a) &= \int_0^a f(r)dr \\ &= e^{-r_d^2/2}\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{r_d^2}{2}\right)^n\left[1 - e^{-a^2/2}\sum_{i=0}^{n}\left(\frac{a^2}{2}\right)^i\frac{1}{i!}\right] \end{aligned} \tag{9}$$

As a result, the conditional probability of interest, $\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\}$, can be obtained by averaging (8) over $\Psi_{tl}$. We have

$$\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\} = \mathrm{E}_{\Psi_{tl}}\{\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}, \Psi_{tl}\}\} \tag{10}$$

Evidently, because of the averaging over $\Psi_{tl}$, the required computational complexity in (10) can be extremely large if the weight of $\mathcal{A}$, i.e., the number of element "1" in $\mathcal{A}$, is more than 1. However, when the number of frequency tones is very large, the probability of more than two interferers occupying the same frequency tone (hits) is negligibly small. As a result we may limit the maximum number of hits to 2 for simplicity in the evaluation. We will verify the validity of such evaluation later.

**B. Rayleigh Fading Channel**

Now, we consider the Rayleigh fading channel. We assume all fading parameters $\{\alpha_{tl}^{(u)}\}$ to be i.i.d. Gaussian with zero mean and unit variance. Conditioning on $\Gamma_{tl} = \mathcal{A}$, $r_{tl}$ in (1) is a zero-mean Gaussian random variable with variance $\sigma_{\mathcal{A}}^2$. It can be readily shown that $\sigma_{\mathcal{A}}^2 = 2\rho(w+1)+1$ for $l = m$ and $2\rho w + 1$ for $l \neq m$, where $w := W(\mathcal{A})$ is the weight of the vector $\mathcal{A}$. As a result, by using (2), the conditional probability of $y_{tl} = q$ is given by

$$\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\} = \begin{cases} \exp(-(q\beta)^2/2\sigma_{\mathcal{A}}^2) - \exp(-((q\beta+\beta)^2/2\sigma_{\mathcal{A}}^2), & q = 0, 1, ..., M-2 \\ \exp(-b^2/2\sigma_{\mathcal{A}}^2), & q = M-1 \end{cases} \tag{11}$$

where $\beta := b/(M-1)$ as previously mentioned.

### 3.1.2 Unconditional Probability $\Pr\{y_{tl} = q\}$

After obtaining $\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\}$, the unconditional probability can be evaluated by taking the expectation of $\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\}$ over all $\mathcal{A}$ in the space $S_\Gamma$. Here, from (10) and (11), we note that the probabilities $\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\}$ are equal for various vectors $\Gamma_{tl} = \mathcal{A}$ if those vectors have the equal weight. Accordingly, we have

$$\Pr\{y_{tl} = q\} = \sum_{w=0}^{J-1} \Pr\{y_{tl} = q|W(\mathcal{A}) = w\}\Pr\{W(\mathcal{A}) = w\} \tag{12}$$

where $\Pr\{y_{tl} = q|W(\mathcal{A}) = w\}$ denotes the probabilities of $\Pr\{y_{tl} = q|\Gamma_{tl} = \mathcal{A}\}$, in which $\{\Gamma_{tl} = \mathcal{A}\}$ have the same weight. $\Pr\{W(\mathcal{A}) = w\}$ is the probability of $\mathcal{A}$ having $w$ elements of "1". In other words, $\Pr\{W(\mathcal{A}) = w\}$ is the probability of $w$ users out of $J-1$ interferers sending tones to the $(t, l)$ position.

Note that the probability of a user sending a tone to the $l$-th frequency is $1/2^K$. Hence, $\Pr\{W(\mathcal{A}) = w\}$ can be given by

$$\Pr\{W(\mathcal{A}) = w\} = \binom{J-1}{w}\left(\frac{1}{2^K}\right)^w\left(1 - \frac{1}{2^K}\right)^{J-1-w} \tag{13}$$

Compared with Goodman's analysis [3], where the maximum number of hits on the spurious rows ($l \neq m$) was limited to 1, $\Pr\{y_{tl} = q\}$ in (12) considers the number of hits from 0 to $J-1$. Furthermore, [3] did not consider the hits from interferers on the correct row ($l = m$), while here the hits on all (spurious and correct) rows are considered. For this reason, the analysis presented in this section (Section 3) is henceforth referred to as the *refined* analysis as compared with Goodman's analysis in [3]. The effects of multiple hits and hits on all rows on the performance will be investigated later in Section 4.

### 3.2 Probability of $Y_l = i$

After the combining of $y_{tl}$, $Y_l$ in (3) takes values from 0 to $(M-1) \times L$. For the HL ($M = 2$), we can calculate the probability of $Y_l = i$ by using the binomial expansion as shown in [3]. For a general SL-MQ and large $L$, however, such calculation requires tremendously computational effort. Here, we employ the characteristic function (CF) method [19]. It can be shown that

$$\Pr\{Y_l = i\} = \int_{-1/2}^{1/2} [\Phi_y(2\pi f)]^L \, e^{-j\,i\,2\pi f} df \tag{14}$$

where $[\Phi_y(2\pi f)]^L$ is the CF of $Y_l$, in which $\Phi_y(\omega = 2\pi f)$ is the CF of $y_{tl}$ given by

$$\Phi_y(\omega) = \sum_{q=0}^{M-1} \Pr\{y_{tl} = q\} e^{jq\omega} \tag{15}$$

Here, $\Pr\{y_{tl} = q\}$ can be evaluated using (12).

One can readily check that when $M = 2$, $\Pr\{Y_l = i\}$ in (14) can be easily expressed in a binomial expansion identical to that in [3].

### 3.3 Probability of $P(n, k)$

$P(n, k)$ denotes the probability that $n$ is the maximum number of $Y_l$ and that exactly $k$ unwanted (spurious) rows having the value of $n$ [3].

Here, we adopt the approach developed by Goodman *et al.* [3] to compute $P(n, k)$ after obtaining $\Pr\{Y_l = i\}$ evaluated by (14). For notational simplicity, we define $P_s(i) := \Pr\{Y_l = i | l \neq m\}$ as the probability of $Y_l = i$ in the spurious rows ($l \neq m$).

Over $2^K - 1$ incorrect (spurious) rows, we have

$$\begin{aligned} P(n,k) &= \binom{2^K-1}{k} [P_s(n)]^k \left[\sum_{i=0}^{n-1} P_s(i)\right]^{2^K-1-k}, && n = 1, 2, ..., (M-1)L \\ P(0, 2^K - 1) &= [P_s(0)]^{2^K-1}, && n = 0, k = 2^K - 1 \\ P(0, k) &= 0, && n = 0, k \neq 2^K - 1 \end{aligned} \tag{16}$$

Note that in the above $n \leq (M-1)L$ due to the use of the SL-MQ function, while in [3] $n \leq L$ for the HL function (i.e., the SL-2Q).

### 3.4 Error rate

The word error rate (WER) is given by [3] but with $(M-1)L$ instead of $L$

$$P_{\mathrm{W}} = 1 - \sum_{i=0}^{(M-1)L} P_c(i) \sum_{k=0}^{2^K-1} \frac{1}{k+1} P(i,k) \tag{17}$$

where $P_c(i) := \Pr\{Y_l = i | l = m\}$ denotes the probability in the correct row $(l = m)$.

The bit error rate (BER) is thus given by

$$P_b = \frac{2^{K-1}}{2^K - 1} P_{\mathrm{W}} \tag{18}$$

## 4 Illustrative Results

In this section, several numerical and simulation results are presented to illustrate the performance of the SL-MQ detector. The numerical results are computed by using the formulae in Section 3. For comparison, the simulation results are obtained by the Monte Carlo trials following the system model and decision procedures described in Section 2.

We consider the radio system with the same parameters as those in [3] (i.e., $K = 8$ and $L = 19$), which gives the maximum number of users under the condition of $P_b \leq 10^{-3}$ in an FH-SSMA system with bandwidth = 20MHz and bit rate = 32kb/s. Here, an AWGN channel and a frequency selective Rayleigh fading channel are considered and the corresponding signal-to-noise ratio per bit (SNR) are defined to be $\rho L/K$ and $2\rho L/K$, respectively, where the factor 2 in Rayleigh fading channel accounts for $E\{|\alpha_{tl}^{(u)}|^2\} = 2$, in which the variance of $\alpha_{tl}^{(u)}$ is normalized to 1 without loss of generality.

### 4.1 Performance of Hard-Limiting Detector (i.e. SL-2Q)

We first consider the performance of the hard-limiting (HL) detector, i.e., the SL-2Q. A comparison of Goodman's analysis [3] and our refined one evaluated via (18) and other formulae in Section 3 on the BERs of the HL (SL-2Q) versus the value of $b$ (threshold) in Rayleigh fading is shown in Fig. 2, where SNR = 20dB and $J = 70, 100, 120$. We can see that when $b$ is around 2.5, the HL detector can reach its minimum BER performance. Besides, we note that the BER evaluated by [3] is generally worse than that from the refined analysis, though the difference between the corresponding curves is not large. A possible explanation is as follows. Goodman's analysis [3] considered the hits on the spurious row while neglecting the hits on the correct row. It is well known that when there are hits on the spurious rows, the system performance will be degraded. However, in Rayleigh fading, the hits on the correct row will increase the energy of the signal and the resulting performance would be a little bit better than that without considering the hits on the correct row. That is why the BERs computed from [3] are worse than those from the refined ones.

| $J$ | Goodman [3] | Refined | Simulation |
|---|---|---|---|
| $J$=70 | 6.85e-5 | 3.65e-5 | 4.73e-5 |
| $J$=100 | 1.21e-3 | 6.03e-4 | 6.65e-4 |
| $J$=120 | 4.86e-3 | 2.39e-3 | 2.47e-3 |

Table 1: Comparison of simulation and theoretical results on the WER of the HL (SL-2Q) in an FH-SSMA system with SNR = 20dB over Rayleigh fading. $J$: number of users.

Next, computer simulation is employed to verify the theoretical results. Our simulation is conducted as follows. First, we generate a set of random sequences $\{\bar{A}\}$ as the PN addresses for all users. Then, we model the received signals according to (1), where all the fading parameters $\{\alpha_{tl}^{(u)}\}$ and the background noises $\{z_{tl}\}$ are all complex-valued Gaussian random variables with zero mean and unit variance. Each user can select randomly one out of $2^K$ available frequency tones to transmit. Suppose the selected tone for user $u$ is $l^{(u)}$. Thus, $\gamma_{tl}^{(u)}$ is set to one if $l^{(u)} + a_t^{(u)} - a_t^{(0)} \bmod 2^K$ equals $l^{(0)}$ and zero otherwise. After obtaining $r_{tl}$, we compute $y_{tl}$ and $Y_l$ using (2) and (3), respectively. Next, we make the word decision in favor of the row corresponding to the largest sum or the one chosen with equal probability from several rows having the largest sum. A counter is used to count the word errors whenever the decision row is different from the transmitted tone. Finally, $3 \times 10^6$ trials are carried out to estimate the word error rate (WER).

The simulation results on the WER of the HL (SL-2Q) in an FH-SSMA system over Rayleigh fading with SNR = 20dB, $b = 2.75$ (used in [3]), and $J = 70, 100, 120$ are listed in Table 1 in comparison with the theoretical ones computed via Goodman's analysis [3] and our refined one (derivation in Section 3). It can be seen that the simulation results on the WERs are close to the refined ones evaluated via (17), but deviated from those by [3].

The derivation in Section 3 considers a general case of more than one hit while [3] limits the maximum number of hits on a spurious row to be 1. In order to examine the effect of the number of hits on the performance, we assume $N_h$ to be the maximum number of hits being limited, and define

$$\Pr\{W(\mathcal{A}') = w\} = \begin{cases} \Pr\{W(\mathcal{A}) = w\}, & w < N_h \\ \sum_{i=N_h}^{J-1} \Pr\{W(\mathcal{A}) = i\}, & w = N_h \end{cases} \tag{19}$$

where $\mathcal{A}'$ is the state vector in $S_\Gamma$ but its weight is not larger than $N_h$ and $\Pr\{W(\mathcal{A}) = w\}$ can be evaluated from (13).

Subsequently, we evaluate the BER as a function of $N_h$ following the procedures in the previous section but using $\Pr\{W(\mathcal{A}') = w\}$ in (19) instead of (13). Furthermore, to compare with Goodman's method [3], where the hit on the correct row was not considered, we proceed our evaluation into two cases. In case 1, we consider hits on all rows, while in case 2, we limit hits on the spurious rows but no hit on the correct row (this is the worse case as previously mentioned). The resulting BERs of the HL (SL-2Q) versus the number of users are plotted in Fig. 3, where SNR = 20dB and $N_h = 1, 2, J-1$. The results, once again, show that in Rayleigh fading by considering the hits on the correct row (case 1), the performance is better than that with no hits on the correct row (case 2). Also, as predicted, the curve corresponding to the consideration of 1 hit in spurious rows (case 2, $N_h = 1$) is in agreement with that from Goodman's analysis [3]. Furthermore, by considering more hits (2 or $J-1$ hits), the performance becomes worse (the corresponding two curves are overlapping). However, the performance degradation in comparison with that for $N_h = 1$ is almost negligible due to the fact that there is a large number of available frequency tones ($2^K = 256$) and the probability of many hits (many interferers occupying the same frequency tone) is negligibly small. This suggests that for practical evaluation, we can limit the maximum number of hits to 1 or 2.

Now, we consider the performance of the SL-2Q in AWGN channel. In Fig. 4, performance comparison of analytical and simulation results (sim) for the BERs of the SL-2Q and the pure SL detector in an FH-SSMA system with SNR = 14dB and $J = 100$. The simulation model is similar to that for the Rayleigh fading but the parameters $\{\alpha_{tl}^{(u)}\}$ are set to $\{e^{j\phi_{tl}^{(u)}}\}$ with phase angles $\{\phi_{tl}^{(u)}\}$ uniformly distributed over $[0, 2\pi)$. In the simulation, the pure SL has a decision function $d_{SL}(x)$ being set to $x/b$ if $x < b$ and 1 otherwise (As mentioned in Section 2, the SL can be regarded as an asymptotic case of $\frac{1}{M-1}d_{SL-MQ}(x)$ in (5) as $M \to \infty$). The simulation results on BER of the SL indicate the best performance that the SL-MQ could achieve. In this figure, two sets of analytical results for the SL-2Q are plotted and they are evaluated by using Goodman's analysis [3] and our refined analysis presented in Section 3, respectively. Note that for the refined analysis, multi-dimensional integration is needed as shown in (10) whenever the weight of $\mathcal{A}$ is larger than 1, and the required computational effort will become extremely large. We computed the BERs by limiting the maximum number of hits to 1 and 2. As shown in Fig. 4, both results based on the refined analysis, i.e., SL-2Q (refined, 1 hit) and SL-2Q (refined, 2 hits) are much closer to the simulation ones, SL-2Q (sim), than those based on the Goodman's analysis [3]. Furthermore, with the maximum number of hits being limited to 2, the analytical results, SL-2Q (refined, 2 hits), are in an excellent agreement with the simulation ones, SL-2Q (sim). Unlike in the Rayleigh fading, the Goodman's analysis [3] yields an overestimated BER performance of the SL-2Q in AWGN channel, i.e., the estimated BERs are generally better than the simulation ones. Again, this is due to the fact that the Goodman's analysis [3] ignored the possible hit on the correct row so that the information-bearing signal $r_d$ in (7) was reduced to $\sqrt{2\rho^{(0)}}$, which was a constant. In contrast, the possible hits on the correct row considered in the refined analysis in Section 3 may render $r_d$ fluctuated and thus degrade the BER performance. Apart from these findings, the lower BER achieved by the pure SL at $b \approx 3.3$ over that by the SL-2Q at $b \approx 2.5$ indicates that by using the SL instead of the SL-2Q, the performance can be improved. By noting that the SL is an asymptotic case of the SL-MQ when $M \to \infty$, the performance of the SL-MQ for $M > 2$ is of interest as to be evaluated next.

## 4.2 Performance of SL-MQ ($M > 2$)

We examine the performance of the SL-MQ detector for multiple levels ($M > 2$). In Fig. 5, we compare the BERs of the SL-2Q (1 bit), SL-4Q (2 bits), SL-8Q (3 bits), and SL-16Q (4 bits) in an FH-SSMA system over an AWGN channel (Fig. 5(a)) and a Rayleigh fading channel (Fig. 5(b)) versus the amplitude threshold $b$. All BERs are analytically evaluated by using formula of (18) in Section 3. For the BERs in AWGN channel, we limit the maximum number of hits to 2, while for those in Rayleigh fading, the maximum number of $J-1=99$ hits is considered. The SL-2Q is the HL detector as considered in [3, 7, 9]. The results clearly show that for the SL-4,8,16Q, the minimum BERs are shown smaller than that of the SL-2Q and the corresponding optimum values of $b$ for threshold become larger than that for the HL (SL-2Q). Besides, the BER performance of the SL-MQ ($M > 2$) is less sensitive to the optimum value of $b$ than that of the SL-2Q. This property is very useful in practice when it is difficult to accurately estimate the optimum $b$.

To further study the performance of the SL-MQ, we focus on Rayleigh fading for simplicity in performance evaluation. Fig. 6 and Fig. 7 plot the BERs of the SL-MQ ($M = 2, 4, 8, 16$) in an FH-SSMA system over Rayleigh fading versus the SNR and the number of users ($J$), respectively. In Fig. 6, $J = 100$ and the value of $b$ for each SNR is set to the corresponding optimum one, which minimizes the BER. In Fig. 7, SNR = 20dB and the values of $b$ for the SL-2,4,8,16Q are set to 2.5, 3.25, 3.5 and 3.75, respectively, which are roughly estimated from Fig. 5(b). Fig. 6 indicates that the SL-MQ ($M > 2$) can outperform the SL-2Q. In particular, at the BER of $10^{-3}$, there is about 1.5dB improvement in SNR by using the SL-4Q over the SL-2Q. More improvement can be obtained by using the SL-8Q and SL-16Q. On the other hand, Fig. 7 shows that the SL-MQ ($M = 4, 8, 16$) provide a better system capacity than the SL-2Q. For example, at the BER of $10^{-3}$, the SL-4Q can improve the system capacity by almost 10%, i.e., from 118 users to 130 users. The penalty paid for this is the increase in the hardware complexity. For the SL-4Q (2 bits), the memory required is doubled as compared to that for the SL-2Q. The SL-8Q and SL-16Q provide further capacity improvement. However, the capacity increase becomes smaller while the required memory would be quickly increased. It appears that in an FH-SSMA system the SL-4Q is an appropriate detector with a better performance than the HL and a moderately increased complexity.

In Fig. 8, analytical and simulation results for the BERs of the SL-4Q and pure SL in an FH-SSMA system over AWGN channel (Fig. 8(a)) and Rayleigh fading (Fig. 8(b)) are compared. In Fig. 8(a), the analytical BERs for the SL-4Q are evaluated by using the formulae in Section 3 with the maximum number of hits of 1 and 2. The analytical results in the case of 2 hits are much closer to the simulation ones than those in the case of 1 hit. In Fig. 8(b), we also plot the analytical and simulation results for the SL-2Q in Rayleigh fading. Here, two sets of analytical results, evaluated respectively by the Goodman's method [3] and the refined one in Section 3, are shown. The results based on the refined analysis for both the SL-2Q and SL-4Q, once again, are in a good agreement with the simulation results. In addition, both figures show that the BER of the SL-4Q is close to the achievable best performance of the pure SL.

Finally, we address the optimization of the threshold $b$. As shown in the above evaluation, the value of $b$ played an important role on the performance of the SL-MQ and a meaningful method to evaluate an optimum $b$ was to minimize the BER performance under the assumption of the knowledge of system SNR and the number of interferers as in [3, 7, 9]. However, in practice, such system knowledge might be unknown *a priori* to the receiver and therefore, the usage of the minimum BER criterion (Min-BER) would be limited. In the following, an alternative criterion of practical interest based on the maximum output SNR is introduced to evaluate a sub-optimum threshold $b$.

### 4.3 Optimizing Threshold in Maximizing Output SNR

Assume that the number of time slots $L$ is large enough so that the test statistics $\{Y_l, l = 0, 1, ..., 2^K - 1\}$ can be approximated to be Gaussian distributed according to the Central Limit Theorem [19]. Let $Y_m$ be the test statistic in the correct row $m$ and $\{Y_l|m\}$ a set of independent and identically distributed (i.i.d.) test statistics in the spurious rows, where the condition $m$ indicates the $m$-th row to be the correct row. It can be shown that the system BER is mainly determined by $\Pr\{Y_m > Y_l|m\}$, i.e., the probability that $Y_m$ exceeds $Y_l$ [1] (see also the union bound in eq.(25) of [9]). If $\Pr\{Y_m > Y_l|m\}$ increases, the corresponding BER will decrease. Therefore, the Min-BER criterion is equivalent to the maximum $\Pr\{Y_m > Y_l|m\}$ and hence further equivalent, for Gaussian distributed $Y_m$ and $\{Y_l|m\}$, to the maximization of $E^2\{Y_m - Y_l|m\}/Var\{Y_m - Y_l|m\}$, i.e.,

$$b_{opt} = arg\left\{\mathrm{MAX}_b\left[\frac{E^2\{Y_m - Y_l|m\}}{Var\{Y_m - Y_l|m\}}\right]\right\} \tag{20}$$

where $E\{Y_m - Y_l|m\}$ and $Var\{Y_m - Y_l|m\}$ denote the conditional mean and variance of $Y_m - Y_l$, respectively.

If we regard $Y_m - Y_l$ as the information bearing signal for decision, the measurement of (20) used to find the optimum threshold $b$ can be named as the maximum output SNR criterion (Max-SNR$_o$). Since the mean and variance of $Y_m - Y_l$ can be easily estimated in practice (i.e., the condition of the transmitted row $m$ can be replaced by that of the decision row), as opposed to the Min-BER criterion, the Max-SNR$_o$ in (20) does not need the knowledge of the system SNR and the number of interferers. This is desired in practice.

The ratios $E^2\{Y_m - Y_l|m\}/Var\{Y_m - Y_l|m\}$, i.e., output SNR (SNR$_o$), versus the threshold $b$ for SL-MQ ($M = 2, 4, 8, 16$) and $J = 100$ in an FH-SSMA system over AWGN channel (SNR $= 14$dB) and Rayleigh fading (SNR $= 20$dB) are plotted in Fig. 9(a) and Fig. 9(b), respectively. In the evaluation, we limit the maximum number of hits to 2 in the AWGN channel and consider it to be $J - 1 = 99$ in the Rayleigh fading. We still consider $K = 8$ and $L = 19$. The results in both figures show that by

| Channel | threshold | SL-2Q | SL-4Q | SL-8Q | SL-16Q |
|---|---|---|---|---|---|
| AWGN | $b_{opt}$, Min-BER | 2.525 | 2.975 | 3.425 | 3.425 |
| AWGN | $b_{opt}$, Max-SNR$_o$ | 2.525 | 3.425 | 3.650 | 3.875 |
| Rayleigh | $b_{opt}$, Min-BER | 2.525 | 3.200 | 3.425 | 3.650 |
| Rayleigh | $b_{opt}$, Max-SNR$_o$ | 2.750 | 3.875 | 4.100 | 4.325 |

Table 2: Optimum $b_{opt}$ in the Max-SNR$_o$ in comparison with those in the Min-BER for the SL-MQ ($M = 2, 4, 8, 16$) in an FH-SSMA system over AWGN channel (SNR = 14dB) and Rayleigh fading channel (SNR = 20dB). $J = 100$.

tuning the threshold $b$, a maximum SNR$_o$ can be obtained. Note that the resultant maximum SNR$_o$'s for the SL-2Q and SL-MQ ($M > 2$) are almost the same. This observation is not contradictory to the BER performance improvement of the SL-MQ ($M > 2$) over the SL-2Q because the nonlinear decision function $d(x)$ used in (2) will render the distribution of $Y_l$ deviated a little bit from the Gaussian case. In the SL-2Q, because of using the hard-limiting function, the deviation might be larger than those in the SL-MQ ($M > 2$) and that will enhance the SNR$_o$ for the SL-2Q. Hence, although the maximum SNR$_o$ of the SL-2Q seems to be the same to those of the others, its BER performance might be worse. Accordingly, the value of $b$ that maximizes the SNR$_o$ is also sub-optimum while the optimum one is corresponding to the minimum BER. It is interesting to compare the sub-optimum $b$ with the optimum value. Such comparison in an FH-SSMA system over AWGN (SNR = 14dB) and Rayleigh fading (SNR = 20dB) is made and the results are given in Table 2, where $J = 100$. In each case, for simplicity, the optimum value of $b$ ($b_{opt}$) is chosen from 20 values of $b$ equally distributed between 0.5 and 5.0. It can be seen from Table 2 that the optimum values of $b$ obtained in the Max-SNR$_o$ are close to those obtained in the Min-BER. Recalling that the BER performance of the SL-MQ ($M > 2$) has low sensitivity to the value of $b$, we can infer that the use of the sub-optimum $b$ ($b_{opt}$ in the Max-SNR$_o$) causes only a small performance degradation and the Max-SNR$_o$ is a practical measurement to evaluate the sub-optimum $b$. A lot of adaptive algorithms or search methods [11, 20] can be designed to implement (20) in the Max-SNR$_o$ and therefore, further discussion on this issue is omitted.

## 5 Conclusions

In this paper we proposed a multi-level quantized soft-limiting (SL-MQ) detector for an SSMA system with fast FH, and presented the corresponding performance analysis in both AWGN and Rayleigh fading channels. Besides, a practical maximum output SNR criterion was introduced to provide a sub-optimum $b$, which was shown to be close to the optimum corresponding to the minimum BER.

As compared with the HL detector, the SL-MQ ($M = 4$) can improve the system capacity by 10% at the BER level of $10^{-3}$ and its performance is less sensitive to the optimum value of the amplitude threshold $b$. In other words, the tolerance on $b$ can be relaxed in practice. The penalty paid for the capacity improvement is that the required memory for the SL-MQ is increased by $\log_2 M$ times as compared with that for the HL. Hence, there should be a compromise between the system capacity improvement and the increase in hardware complexity. In this context, the scheme SL-4Q appears to be a good candidate.

## Acknowledgment

This work is partially supported by Ericsson Research Canada. The authors are grateful to the reviewers and editor and acknowledge with pleasure their constructive comments.

## References

[1] J. G. Proakis, *Digital communications.* New York: McGraw-Hill, 3rd Ed, 1995.

[2] C. D'Amours and A. Yongacoglu, "MFSK based modulations for hybrid DS/FH CDMA systems operating in slowly Rayleigh fading channels," *IEEE Trans. Vehi. Tech.*, vol. VT-46, pp. 615–624, Aug. 1997.

[3] D. J. Goodman, P. S. Henry, and V. K. Prabhu, "Frequency hopped multilevel FSK for mobile radio," *Bell System Technical J.*, pp. 1257–1275, Sept. 1980.

[4] R. Viswanathan and S. C. Gupta, "Adjacent cell interference in FH-MFSK cellular mobile radio system," *IEEE Trans. Vehi. Tech.*, vol. VT-32, pp. 191–200, May 1983.

[5] R. H. Muammar, "Degradation in FH-MFSK mobile radio system capacity due to Rayleigh fading and Log-Normal shadowing," *IEEE Trans. Vehi. Tech.*, vol. VT-37, pp. 130–133, Aug. 1988.

[6] P. Yegani and C. D. McGillem, "FH-MFSK multiple-access communications systems performance in the factory environment," *IEEE Trans. Vehi. Tech.*, vol. VT-42, pp. 148–155, May 1993.

[7] U. Svasti-Xuto, Q. Wang, and V. K. Bhargava, "Capacity of an FH-SSMA system in different fading environments," *IEEE Trans. Vehi. Tech.*, vol. VT-47, pp. 75–82, Feb. 1998.

[8] U. Fiebig, "On the efficiency of fast frequency hopping multiple-access systems," in *Proc. IEEE Int. Commun. Conf., ICC92*, (Chicago, United States), pp. 302.2.1–302.2.5, 1992.

[9] O. C. Yue, "Performance of frequency hopping multiple access multilevel FSK systems with hard limited and linear combining," *IEEE Trans. Commun.*, vol. COM-29, pp. 1687–1694, Nov. 1981.

[10] J. Ilow, D. Hatzinakos, and A. N. Venetsanopoulos, "Performance of FH SS radio networks with interference modeled as a mixture of Gaussian and Alpha-Stable noise," *IEEE Trans. Commun.*, vol. COM-46, pp. 509–519, April 1998.

[11] J. F. Weng, *Performance of two-stage nonlinear detector in DS/CDMA system with impulse noise.* PhD thesis, Dept. of Electronic Eng., City Univ. of Hong Kong, Hong Kong, Aug. 1997.

[12] A. J. Viterbi, "A robust ratio-threshld technique to mitigate tone and partial band jamming in coded MFSK systems," in *Proc. IEEE Military Commun. Conf., MILCOM82*, pp. 22.4.1–22.4.5, 1982.

[13] R. Viswanathan and S. C. Gupta, "Nonparametric receiver for FH-MFSK mobile radio," *IEEE Trans. Commun.*, vol. COM-33, pp. 178–184, Feb. 1985.

[14] C. M. Keller and M. B. Pursley, "Clipped diversity combining for channels with partial band interference-Part I: clipped linear combining," *IEEE Trans. Commun.*, vol. COM-35, pp. 1320–1328, Dec. 1987.

[15] J. J. Chang and L. S. Lee, "An exact performance analysis of the clipped diversity combining receiver for FH/MFSK systems against a band multitone jammer," *IEEE Trans. Commun.*, vol. COM-42, pp. 700–710, Feb./Mar./April 1994.

[16] C. M. Keller and M. B. Pursley, "Clipped diversity combining for channels with partial band interference-Part II:ratio-statistic combining," *IEEE Trans. Commun.*, vol. COM-37, pp. 145–151, Feb. 1989.

[17] T. Y. Yan and C. C. Wang, "Mathematical models for cochannel interference in FH/MFSK multiple access systems," *IEEE Trans. Commun.*, vol. COM-32, pp. 670–678, June 1984.

[18] D. Parsons, *The mobile radio propagation channel.* London: Pentech Press, 1992.

[19] A. Papoulis, *Probability, random variables, and stochastic processes.* New York: McGraw-Hill, 1991.

[20] S. Haykin, *Adaptive filter theory.* New Jersey: Prentice-Hall Int., 3rd ed., 1996.

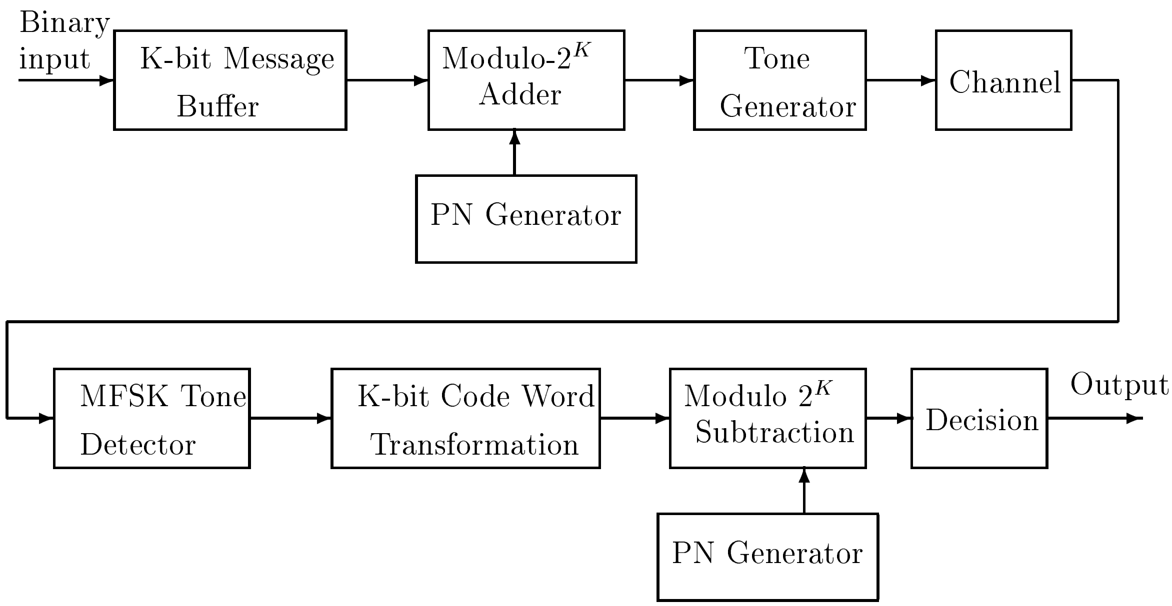



Fig. 1: A simplified block diagram of an FH-SSMA system

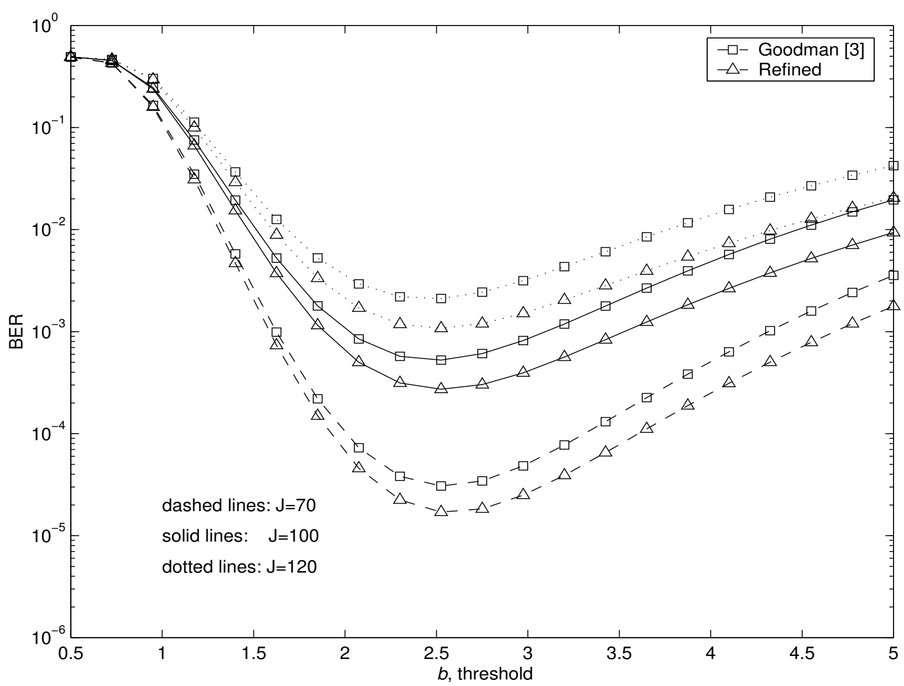



Fig. 2: Comparison of Goodman's [3] and our refined analysis for the BER of the SL-2Q in an FH SSMA system over a Rayleigh fading channel. SNR = 20dB.

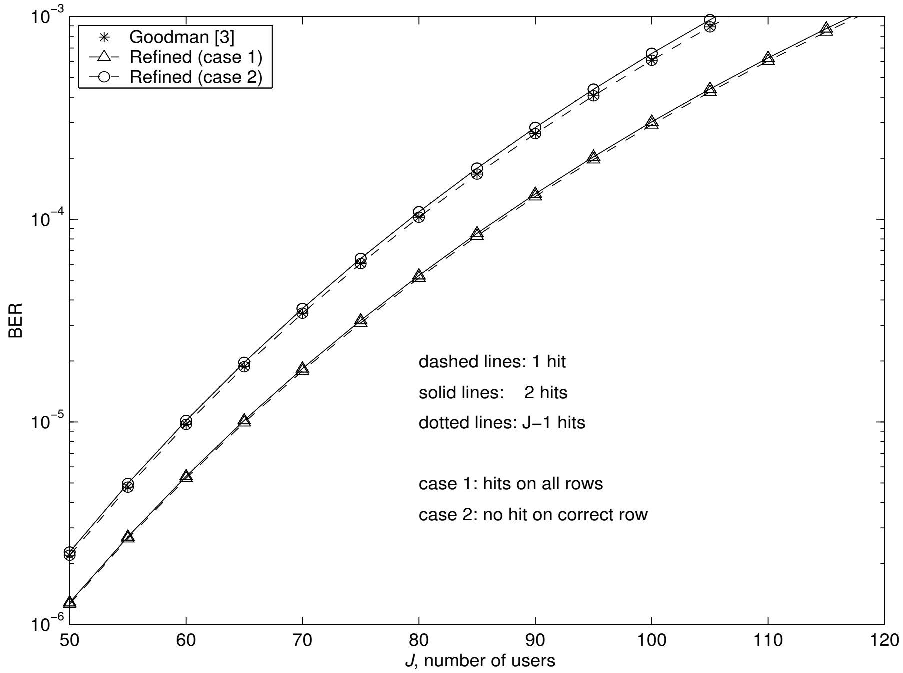



Fig. 3: Effect of the maximum number of hits on the BER of the HL (SL-2Q) in an FH SSMA system over Rayleigh fading. SNR = 20dB.

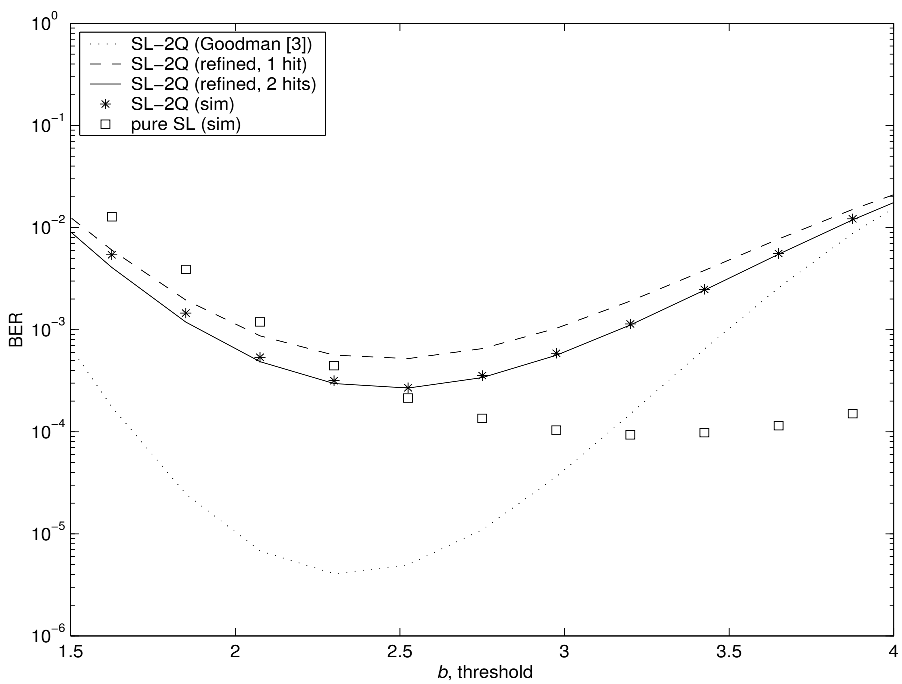
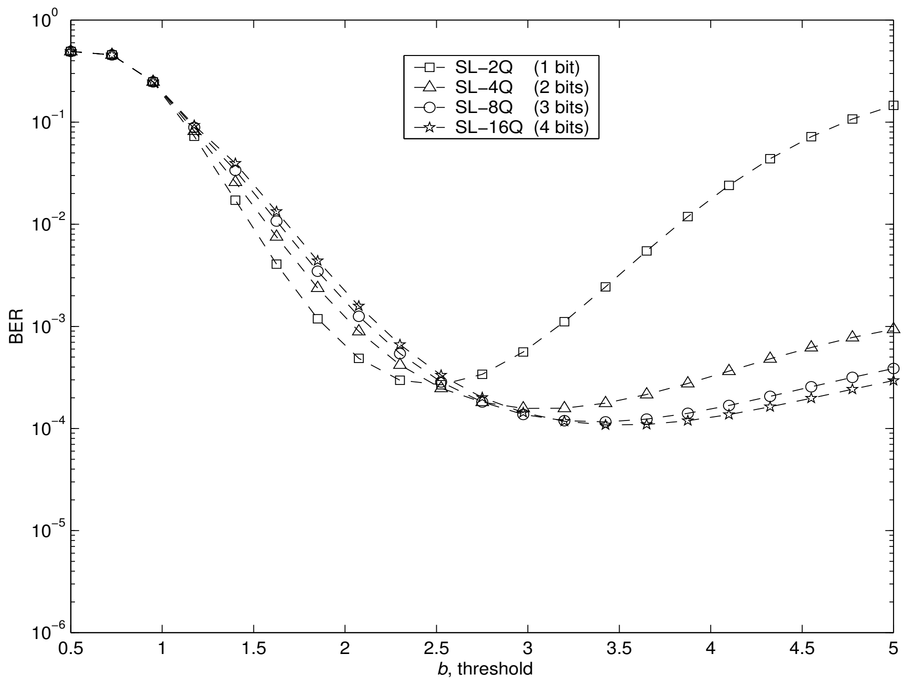
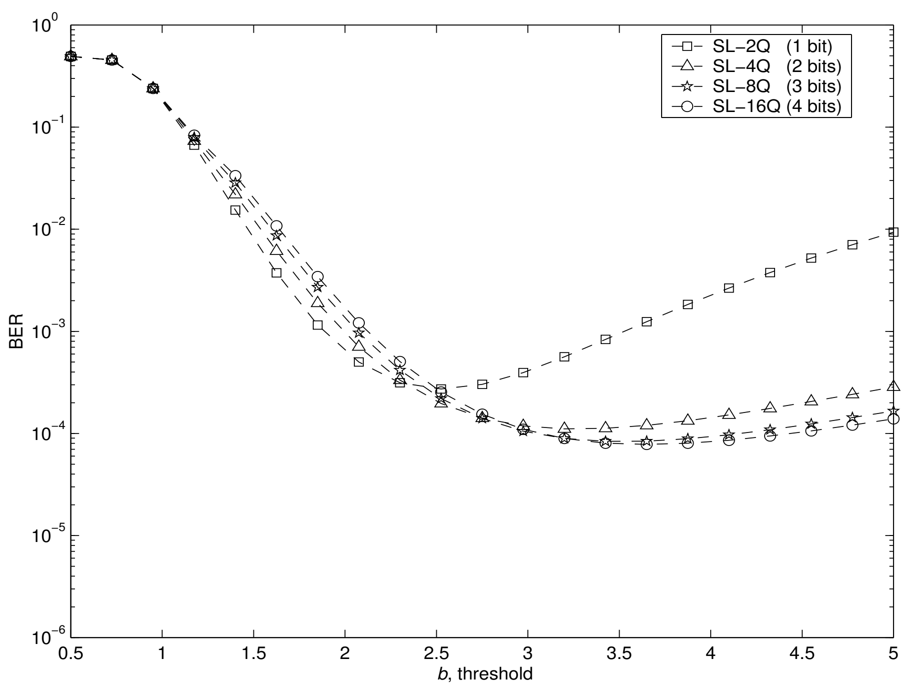



Fig. 4: Comparison of analytical and simulation results for the BERs of the HL (SL-2Q) and pure SL in an FH SSMA system over an AWGN channel. SNR = 14dB and $J = 100$.




(a) AWGN channel, SNR = 14dB, $J = 100$, 2 hits are considered.




(b) Rayleigh fading, SNR = 20dB, $J = 100$, $J - 1$ hits are considered.

Fig. 5: Effect of the threshold on the BERs of the SL-MQ in an FH-SSMA system over AWGN channel and Rayleigh fading.

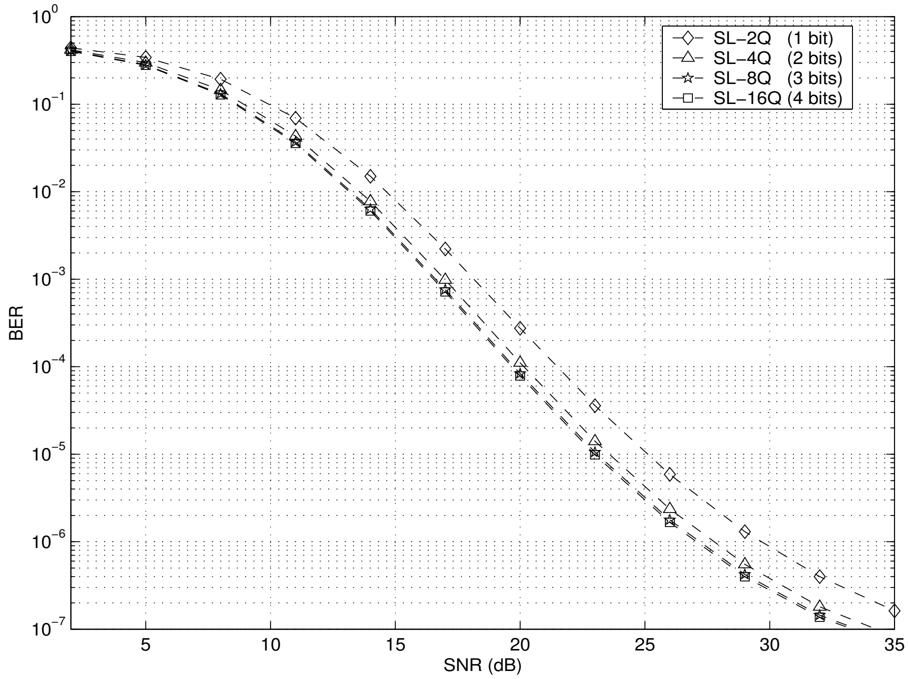



Fig. 6: BERs of the SL-MQ versus SNR in an FH-SSMA system over Rayleigh fading. $J = 100$.

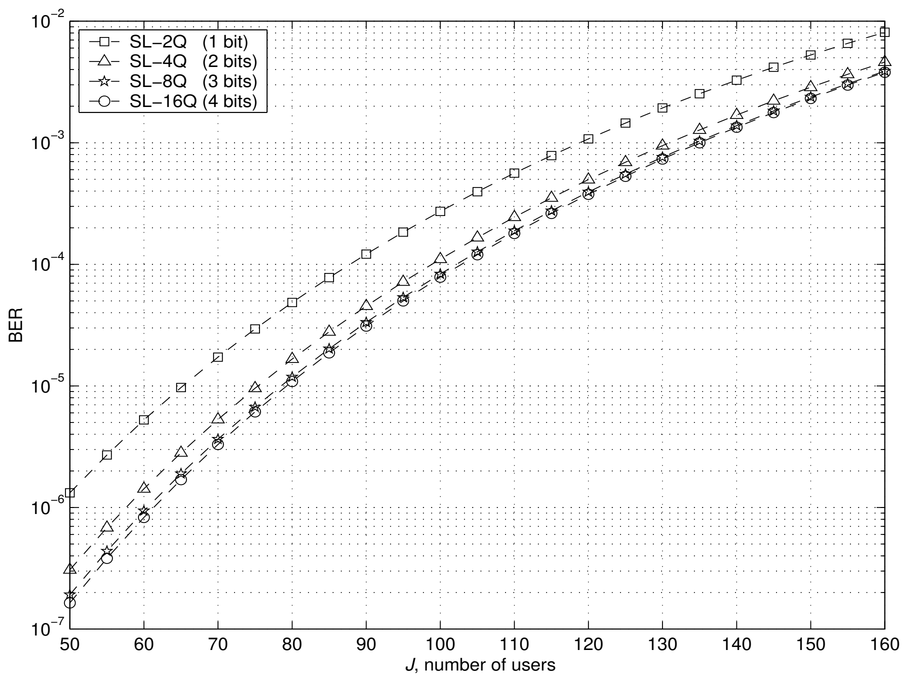
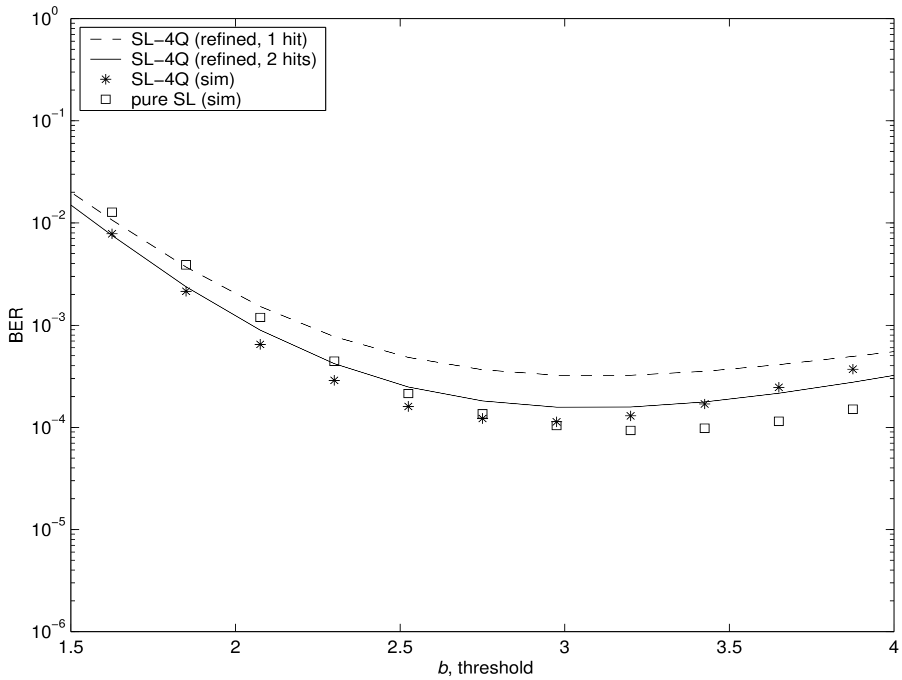
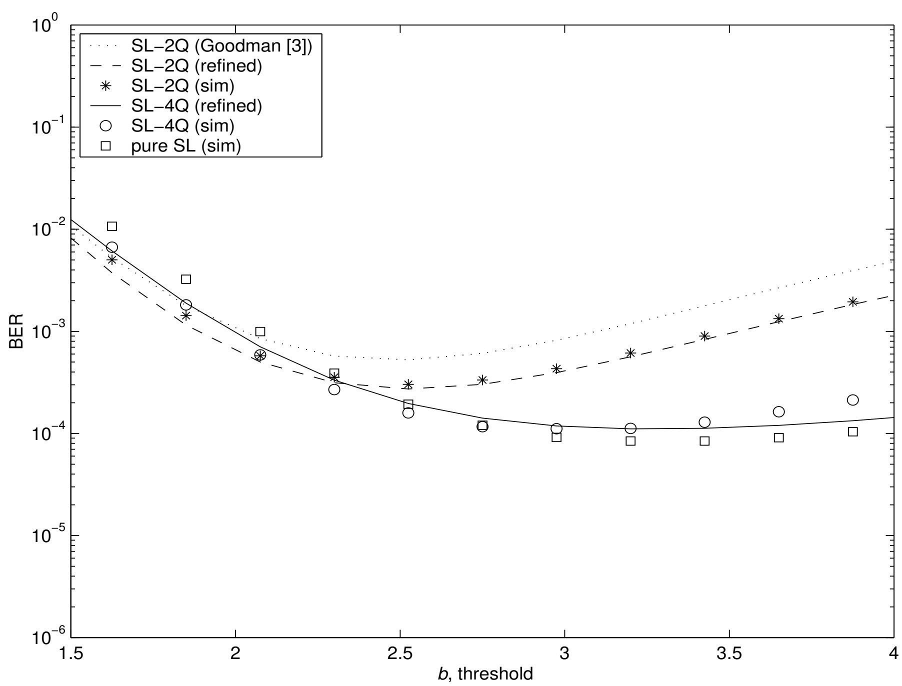



Fig. 7: BERs of the SL-MQ versus the number of users ($J$) in an FH-SSMA system over Rayleigh fading. SNR = 20dB.

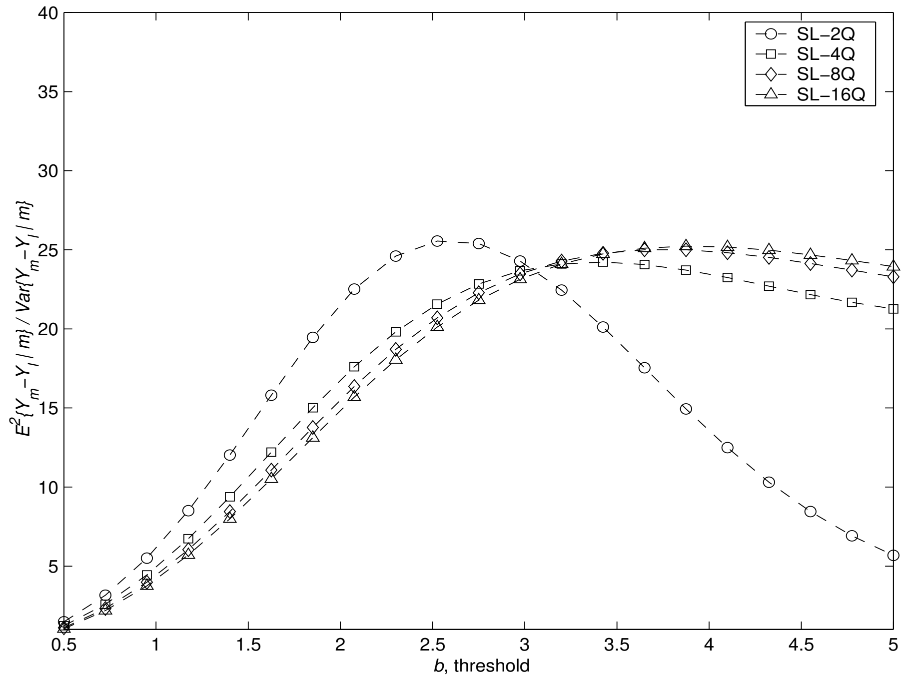



(a) AWGN channel, SNR = 14dB, $J = 100$, 1 or 2 hits are considered.

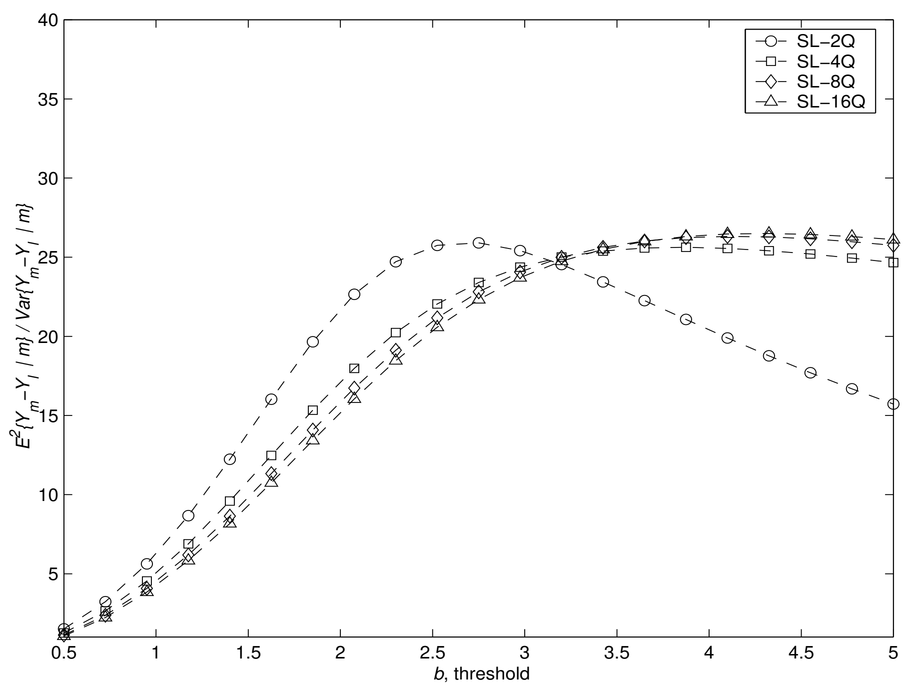



(b) Rayleigh fading, SNR = 20dB, $J = 100$, $J - 1$ hits are considered.

Fig. 8: Comparison of analytical and simulation results for the SL-4Q and pure SL in an FH-SSMA system over AWGN channel and Rayleigh fading.




(a) AWGN channel, SNR = 14dB, 2 hits considered




(b) Rayleigh fading channel, SNR = 20dB, $J-1$ hits considered

Fig. 9: $E^2\{Y_m - Y_l|m\}/Var\{Y_m - Y_l|m\}$ versus the threshold $b$ in an FH-SSMA system over AWGN and Rayleigh fading. $J = 100$.