An Analytical Model for Performance Evaluation of Multistage Parallel Interference Cancellation Detectors in CDMA Systems *

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Abstract

In multistage parallel interference cancellation (PIC), multiaccess interference from different users are mutually correlated. In this paper, an analytical model is proposed to refine the variance of the decision variable by taking the correlation into account. Price's theorem is used to evaluate the non-linear relations involved in some of the covariances. It is shown that the derived variance is very close to that obtained by simulation and improves the accuracy of BER evaluation.

1 Introduction

Multistage parallel interference cancellation (PIC) structures can offer better performance than linear detectors at a much lower complexity in wireless CDMA systems [1, 2, 3]. By introducing weights in conventional PIC scheme, the performance of PIC detectors can be further improved [4, 5, 6]. Performance analyses of various multistage PICs have been recently discussed in [1, 7, 8, 9] for conventional PIC, and in [5, 6] for partial PIC. In [7, 8], hard decisions obtained from the previous stage are used in reconstructing the multiaccess interference (MAI), while soft outputs from the correlators are employed in [6, 9] to estimate the interfering signal. As noticed in [7, 8], there is a large discrepancy between simulation and analytical results on the bit error rate (BER). The reasons for such a difference are twofold. One factor is the inaccuracy in variance due to the invalid assumption that the interference from different users and Gaussian noise are mutually uncorrelated. The other factor can be the assumption that the MAI is a Gaussian process. For a small number of users, the central limit theorem may not hold, thus the MAI may not be well approximated as a Gaussian process.

In this paper, we present an analytical model to evaluate the performance of multistage interference cancellation detectors. What differentiates our method from the previous ones is that the variance of the decision variable is refined by considering the correlation between the multiaccess interference and Gaussian noise. This approach needs the evaluation of the covariances that involve bit decisions made in the previous stage. The difficulty arises from the non-linear relation introduced by hard decision. By using Price's theorem [10] we show that a very accurate variance can be derived and improve the accuracy of the analytical results on the BER.

The rest of the paper is organized as follows. System models are presented in Section 2. Detailed evaluation of the refined variance of the decision variable using Price's theorem is given in Section 3. Analysis results are demonstrated in Section 4. Section 5 concludes the paper.

2 System Model

We consider a synchronous CDMA system with BPSK modulation. The received signal is given by

$$r(t) = \sum_{k=1}^{K} \alpha_k b_k(t) c_k(t) + \eta(t) \tag{1}$$

where α_k is the amplitude of the k-th user. $b_k(t)$ and $c_k(t)$ denote the data and PN waveform of the k-th user, respectively. $\eta(t)$ is an additive white Gaussian noise (AWGN) with double-sided density $N_0/2$.

The despreading for the i-th user at the base-station receiver is performed as follows

$$y_i = \frac{1}{T}\int_0^T r(t) c_i(t) dt = \alpha_i b_i + \sum_{k=1 k\neq i}^{K} \alpha_k \rho_{ik} b_k + z_i \tag{2}$$

where $\rho_{ik} = \frac{1}{T}\int_0^T c_i(t) c_k(t) dt$ denotes the cross-correlation coefficient between $c_i(t)$ and $c_k(t)$. $z_i = \frac{1}{T}\int_0^T c_i(t)\eta(t) dt$ is the noise component for the i-th user after despreading.

Without loss of generality we focus on the first user. A single-user receiver makes a decision as

$$\hat{b}_1^{(0)} = \mathrm{sgn}\{y_1\} \tag{3}$$

*This work is partially supported by Ericsson Research Canada

For PIC, the MAI is reconstructed with the decisions made in the previous stage and then removed from each user's received signal which can be expressed as

$$r_1^{(1)}(t) = r(t) - \sum_{k=2}^{K} \alpha_k \hat{b}_k^{(0)} c_k(t) \tag{4}$$

Then a decision is made based on the less interfered signal $r_1^{(1)}(t)$, i.e.,

$$\hat{b}_1^{(1)} = \mathrm{sgn}\{y_1^{(1)}\} \tag{5}$$

where

$$\begin{aligned} y_1^{(1)} &= \frac{1}{T}\int_0^T r_1^{(1)}(t) c_1(t) dt \\ &= \alpha_1 b_1 + \sum_{k=2}^{K} \alpha_k \rho_{1k} [b_k - \hat{b}_k^{(0)}] + z_1 \end{aligned} \tag{6}$$

3 Evaluation the Variance

Conditioned on the data vector $\boldsymbol{b} = (b_1, b_2, \cdots, b_K)$, the conditional variance $\mathrm{var}\{y_1^{(1)}|\boldsymbol{b}\}$ can be evaluated as follows

$$\begin{aligned} &\mathrm{var}\{y_1^{(1)}|\boldsymbol{b}\} = \mathrm{var}\left\{\sum_{k=2}^{K} \alpha_k \rho_{1k}[b_k - \hat{b}_k^{(0)}] + z_1 |\boldsymbol{b}\right\} \\ &= \sum_{k=2}^{K} 4|\alpha_k|^2 \rho_{1k}^2 P_e^{(0)} - 2\sum_{k=2}^{K} \alpha_k \rho_{1k} E\{z_1 \hat{b}_k^{(0)}|\boldsymbol{b}\} \\ &- \sum_{n=2}^{K} \sum_{\substack{m=2 \\ m\neq n}}^{K} \alpha_m \alpha_n \rho_{1m} \rho_{1n} E\{b_m \hat{b}_n^{(0)} + b_n \hat{b}_m^{(0)}|\boldsymbol{b}\} \\ &+ \sum_{n=2}^{K} \sum_{\substack{m=2 \\ m\neq n}}^{K} \alpha_m \alpha_n \rho_{1m} \rho_{1n} E\{\hat{b}_m^{(0)} \hat{b}_n^{(0)}|\boldsymbol{b}\} + \mathrm{var}\{z_1|\boldsymbol{b}\} \end{aligned} \tag{7}$$

To our best knowledge, in all existing analyses, only the first and the last terms of (7) are counted when evaluating the variance of $y_1^{(1)}$. Due to the non-linear relation in the remaining terms of (7), these terms are often assumed to be zero for simplicity [6, 7, 8]. However, our study shows this assumption is not valid, especially when the system load as well as the PN cross-coefficient are high. In the following, we will deal with the non-linear problem using Price's Theorem.

3.1 Price's Theorem [10]

Given two jointly normal RVs x and y, we form the mean

$$I = E\{g(x,y)\} = \int_{-\infty}^{\infty}\int_{\infty}^{\infty} g(x,y) f(x,y) dx dy \tag{8}$$

of some function $g(x,y)$ of (x,y). The above integral is a function $I(\mu)$ of the covariance μ of the RVs x and y and of four parameters specifying the joint density $f(x,y)$ of x and y. If $g(x,y)f(x,y) \to 0$ as $(x,y) \to \infty$, then

$$\frac{\partial^n I(\mu)}{\partial \mu^n} = E\left\{\frac{\partial^{2n} g(x,y)}{\partial x^n \partial y^n}\right\} \tag{9}$$

Price's Theorem provides us a tool to cope with the nonlinear relation encountered in (7).

3.2 Evaluation of $E\{z_1 \hat{b}_k^{(0)}|\boldsymbol{b}\}$

We define

$$I_1(\mu_1) = E\{z_1 \hat{b}_k^{(0)}|\boldsymbol{b}\} \tag{10}$$

where $\mu_1 = E\{z_1(y_k - m_{y_k|\boldsymbol{b}})|\boldsymbol{b}\}$, $m_{y_k|\boldsymbol{b}}$ is the mean of y_k conditioned on $\boldsymbol{b}$. By applying Price's Theorem, it follows that

$$\frac{\partial I_1}{\partial \mu_1} = E\left\{\frac{\partial z_1 \partial[\mathrm{sgn}(y_k)]|\boldsymbol{b}}{\partial z_1 \partial y_k}\right\} = E\{2\delta(y_k)|\boldsymbol{b}\} = 2 f_{y_k|\boldsymbol{b}}(0) \tag{11}$$

where

$$f_{y_k|\boldsymbol{b}}(y_k) = \frac{1}{\sqrt{2\pi}\sigma_{y_k|\boldsymbol{b}}} \exp\left\{-\left(\frac{y_k - m_{y_k|\boldsymbol{b}}}{\sqrt{2}\sigma_{y_k|\boldsymbol{b}}}\right)^2\right\} \tag{12}$$

is the probability density function (pdf) of y_k conditioned on $\boldsymbol{b}$. $\sigma_{y_k|\boldsymbol{b}}$ is the standard deviation of y_k conditioned on $\boldsymbol{b}$, and $\sigma_{y_k|\boldsymbol{b}} = \sigma_n$, where σ_n is the standard deviation of the Gaussian noise.

From (11) it is straightforward that

$$\begin{aligned} I_1(\mu) &= 2 f_{y_k|\boldsymbol{b}}(0) E\{z_1(y_k - m_{y_k|\boldsymbol{b}})|\boldsymbol{b}\} + I(0) \\ &= 2 f_{y_k|\boldsymbol{b}}(0) \rho_{1k} \sigma_n^2 \end{aligned} \tag{13}$$

3.3 Evaluation of $E\{b_m \hat{b}_n^{(0)}|\boldsymbol{b}\}$

Define

$$I_2 = E\{b_m \hat{b}_n^{(0)}|\boldsymbol{b}\} \tag{14}$$

I_2 can be evaluated from its definition, i.e.,

$$\begin{aligned} I_2 &= b_m \int_{-\infty}^{\infty} \mathrm{sgn}(y_n|\boldsymbol{b}) f_{y_n|\boldsymbol{b}}(y_n|\boldsymbol{b}) dy_n \\ &= b_m \left\{\int_0^{\infty} f_{y_n|\boldsymbol{b}}(y_n|\boldsymbol{b}) dy_n - \int_{-\infty}^{0} f_{y_n|\boldsymbol{b}}(y_n|\boldsymbol{b}) dy_n\right\} \\ &= b_m \left\{\mathrm{erfc}\left(-\frac{m_{y_k|\boldsymbol{b}}}{\sqrt{2}\sigma_n}\right)\right\} \end{aligned} \tag{15}$$

3.4 Evaluation of $E\{\hat{b}_m^{(0)} \hat{b}_n^{(0)}|\boldsymbol{b}\}$

Let

$$I_3(\mu_3) = E\{\hat{b}_m^{(0)} \hat{b}_n^{(0)}|\boldsymbol{b}\} = E\{\mathrm{sgn}(y_m)\mathrm{sgn}(y_n)|\boldsymbol{b}\} \tag{16}$$

where

$$\mu_3 = E\{y_m y_n|\boldsymbol{b}\} - E\{y_m|\boldsymbol{b}\}E\{y_n|\boldsymbol{b}\} \tag{17}$$

As y_m and y_n are jointly normal when conditioned on $\boldsymbol{b}$, we can apply Price's Theorem

$$\begin{aligned}\frac{\partial I(\mu_3)}{\partial \mu_3} &= E\left\{\frac{\partial^2[\mathrm{sgn}(y_m)\mathrm{sgn}(y_n)]}{\partial y_m \partial y_n}|\boldsymbol{b}\right\} \\ &= E\{4\delta(y_m)\delta(y_n)|\boldsymbol{b}\} = 4f_{y_m,y_n|\boldsymbol{b}}(0,0)\end{aligned} \tag{18}$$

where $f_{y_m,y_n}(y_m,y_n)$ is the joint pdf of y_m and y_n.

$$\begin{aligned}&f_{y_m,y_n|\boldsymbol{b}}(0,0) = \frac{1}{2\pi\sigma_{y_m|\boldsymbol{b}}\sigma_{y_n|\boldsymbol{b}}\sqrt{1-r^2}} \\ &\exp\left\{-\frac{1}{2(1-r^2)}\left[\frac{\mu^2_{y_m|\boldsymbol{b}}}{\sigma^2_{y_m|\boldsymbol{b}}} - 2r\frac{u_{y_m|\boldsymbol{b}}\mu_{y_n|\boldsymbol{b}}}{\sigma_{y_m|\boldsymbol{b}}\sigma_{y_n|\boldsymbol{b}}} + \frac{\mu^2_{y_n|\boldsymbol{b}}}{\sigma^2_{y_n|\boldsymbol{b}}}\right]\right\}\end{aligned} \tag{19}$$

σ_{y_m} and σ_{y_n} are the standard deviations of y_m and y_n, respectively. r denotes the correlation coefficient between y_m and y_n, i.e.,

$$r = \frac{E\{y_m y_n|\boldsymbol{b}\} - E\{y_m|\boldsymbol{b}\}E\{y_n|\boldsymbol{b}\}}{\sigma_{y_m|\boldsymbol{b}}\sigma_{y_n|\boldsymbol{b}}} \tag{20}$$

From (18) it is easy to show that

$$I_3(\mu_3) = 4f_{y_m,y_n|\boldsymbol{b}}(0,0)\mu_3 + I_3(0) \tag{21}$$

In order to evaluate $I_3(\mu_3)$, we need to know μ_3 and $I_3(0)$. μ_3 can be computed as

$$\begin{aligned}\mu_3 &= E\{y_m y_n|\boldsymbol{b}\} - E\{y_m|\boldsymbol{b}\}E\{y_n|\boldsymbol{b}\} \\ &= E\{z_m\}E\{z_n\} = \rho_{mn}\sigma_n^2\end{aligned} \tag{22}$$

For the evaluation of $I_3(0)$, if $\mu_3 = 0$, y_m and y_n are independent, hence

$$\begin{aligned}&I_3(0) = E\{\mathrm{sgn}(y_m)|\boldsymbol{b}\}E\{\mathrm{sgn}(y_n)|\boldsymbol{b}\} \\ &= \int_{-\infty}^{\infty}\mathrm{sgn}(y_m|\boldsymbol{b})f_{y_m|\boldsymbol{b}}(y_m)dy_m \\ &\int_{-\infty}^{\infty}\mathrm{sgn}(y_n|\boldsymbol{b})f_{y_n|\boldsymbol{b}}(y_n)dy_n \\ &= \left[\mathrm{erfc}\left(-\frac{m_{y_m|\boldsymbol{b}}}{\sqrt{2}\sigma_{y_m|\boldsymbol{b}}} - 1\right)\right]\left[\mathrm{erfc}\left(-\frac{m_{y_n|\boldsymbol{b}}}{\sqrt{2}\sigma_{y_n|\boldsymbol{b}}} - 1\right)\right]\end{aligned} \tag{23}$$

3.5 BER Evaluation

Define the variance of $y_1^{(1)}$ as Ψ^2, from (7) it follows that

$$\Psi^2 = \sum_i \mathrm{var}\{y_1^{(1)}|\boldsymbol{b}_i\}p(\boldsymbol{b}_i) \tag{24}$$

where $\boldsymbol{b}_i$ denotes a specific combination of $\boldsymbol{b}$.

If $y_1^{(1)}$ is Gaussian, then BER of the system is given by

$$P_e = Q\left(\frac{|\alpha_1|}{\Psi}\right) \tag{25}$$

4 Analytical Results

Analytical results are presented in Tables 1-3 and Figures 1-2 for variances and BER, respectively. Various ρ and number of users are considered, where ρ denotes the cross-correlation coefficient between two PN sequences. We assume equal cross-correlation among users, and all users have identical and constant amplitude. The carrier phase is not considered.

As shown in Tables 1-3, the variances obtained by the refined analysis are very close to those measured by simulation for all circumstances. The conventional analysis tends to underestimate the variance, especially when the number of users is large. For example, in Table 3, when E_b/N_0=10dB and $K = 20$, the variance provided by the conventional method is only 1/5 the true value measured by simulation.

When the refined variance is used in (25), the resulting BER accuracy is also improved as indicated in Figures 1-2.

Figures 1(a)-(c) show the performance comparison of the three approaches: simulation, conventional and refined for the case of 10 users and ρ =0.1, 0.15, and 0.20. When the MAI is small (i.e., ρ =0.1) the Gaussian noise becomes dominant and the three approaches provide almost the same results as shown in Figure 1(a). When the MAI gets larger, the refined analysis provides results closer to the simulation ones than the conventional analysis as indicated in Figures 1(b) and (c). However, it is noted that there is still a gap between the BER curves obtained by the refined analysis and simulation. This may be due to the invalid assumption of Gaussian MAI model when the number of users is small (e.g., 10). To further investigate the effects of number of users on the validity of a Gaussian MAI model, we examine the case of 20 users. Results plotted in Figures 2 (a)-(c) show that the refined analysis offers the BER performance much closer to that measured by simulation than the conventional analysis as ρ increases (i.e., larger MAI). The gap between the BER curves obtained by the refined analysis and simulation in Figures 2 (a)-(c) is smaller than that in Figures 1 (a)-(c). This indicates that the assumption of Gaussian MAI model is more acceptable as the number of users increases.

5 Conclusions

In this paper, we suggested a refined analysis to evaluate the performance of PIC detectors. Unlike previous performance analysis, the correlation among the MAIs and Gaussian noise is considered and Price's theorem is applied to evaluate various covariances in order to derive a more accurate variance of the decision variable. This improves the accuracy of BER. However, we find in some cases, even with the refined variance, the BER curve obtained on the basis of the approximated Gaussian MAI model is not close enough to the simulation results. To further improve the BER accuracy, modified Gaussian approximation like the one suggested in [11] can be considered.

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Table 1: Variance of Decision Variable $y_1^{(1)}$ ($\rho = 0.1$)

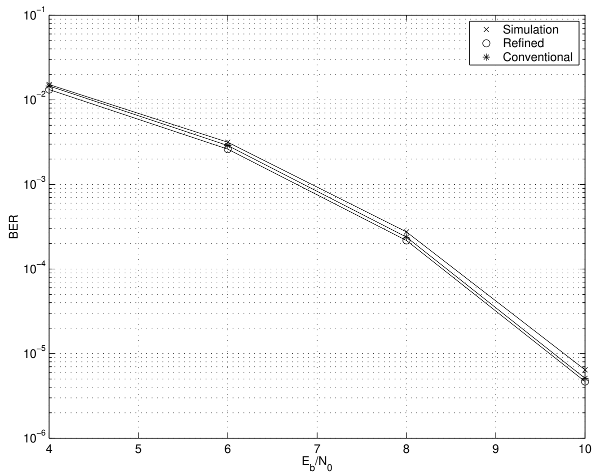
E_b/N_0	K	Sim	Anal(Ref)	Anal(Conv.)
6	5	0.125753	0.125764	0.126714
6	10	0.128127	0.128222	0.131222
6	15	0.14069	0.140849	0.140247
6	20	0.180242	0.180514	0.154119
8	5	0.0793618	0.0793397	0.0795471
8	10	0.0806572	0.0807954	0.0819567
8	15	0.0895937	0.0898944	0.0884023
8	20	0.123137	0.123471	0.0997475
10	5	0.0500349	0.0500381	0.0500686
10	10	0.0507057	0.0509197	0.0513547
10	15	0.0574501	0.05784	0.0560988
10	20	0.0873697	0.087767	0.0656662

Table 2: Variance of Decision Variable $y_1^{(1)}$ ($\rho = 0.15$)

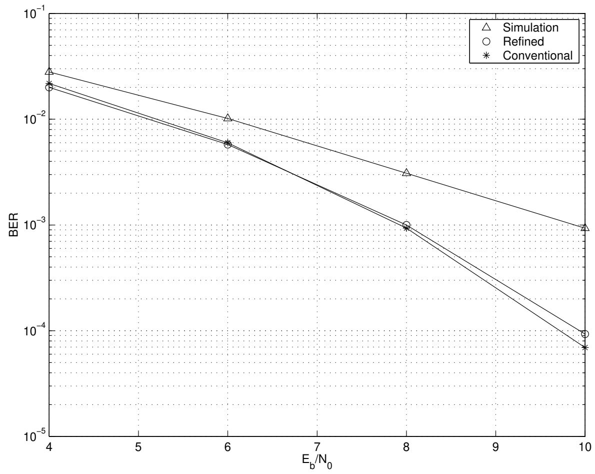
E_b/N_0	K	Sim	Anal(Ref)	Anal(Conv)
6	5	0.12743	0.127444	0.131222
6	10	0.156363	0.156642	0.158335
6	15	0.299469	0.299896	0.208709
6	20	0.673982	0.674381	0.278423
8	5	0.0802626	0.0805591	0.0819567
8	10	0.104273	0.104691	0.104691
8	15	0.241284	0.241507	0.149327
8	20	0.618537	0.618652	0.216121
10	5	0.0504632	0.0508895	0.0513547
10	10	0.0709711	0.0715974	0.068866
10	15	0.205454	0.205626	0.111665
10	20	0.590491	0.590216	0.176416

Table 3: Variance of Decision Variable $y_1^{(1)}$ ($\rho = 0.2$)

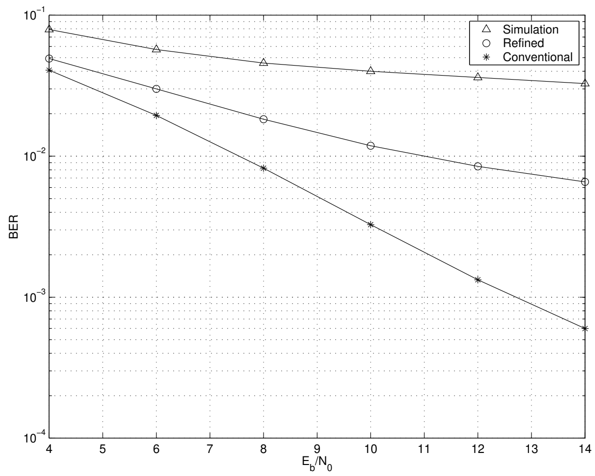
E_b/N_0	K	Sim	Anal(Ref)	Anal(Conv)
6	5	0.134465	0.134925	0.145215
6	10	0.283549	0.282886	0.234513
6	15	0.883452	0.88388	0.380008
6	20	2.21445	2.21559	0.563277
8	5	0.086168	0.0870622	0.0923352
8	10	0.229595	0.228798	0.173807
8	15	0.841166	0.840677	0.315598
8	20	2.214	2.21369	0.497271
10	5	0.055205	0.0565392	0.0593108
10	10	0.196531	0.195465	0.135212
10	15	0.822619	0.822135	0.274465
10	20	2.23417	2.23157	0.455111



(a) 10 users, $\rho = 0.1$

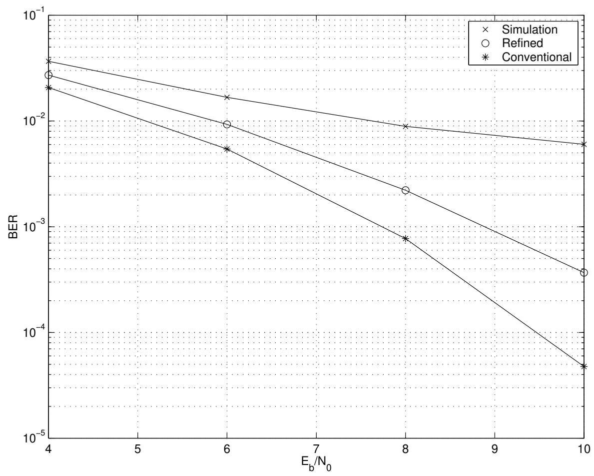


(b) 10 users, $\rho = 0.15$

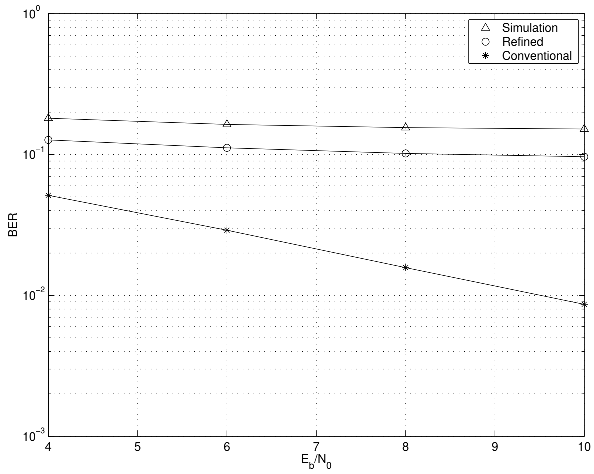


(c) 10 users, $\rho = 0.2$

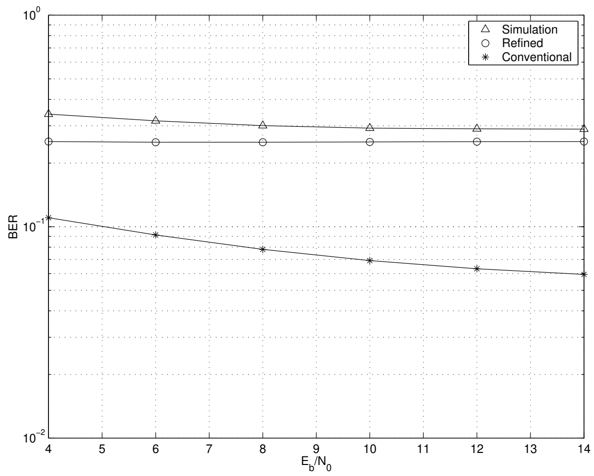
Figure 1: Performance of 1-stage PIC in AWGN channel, 10 users



(a) 20 users, $\rho = 0.1$



(b) 20 users, $\rho = 0.15$



(c) 20 users, $\rho = 0.2$

Figure 2: Performance of 1-stage PIC in AWGN channel, 20 users