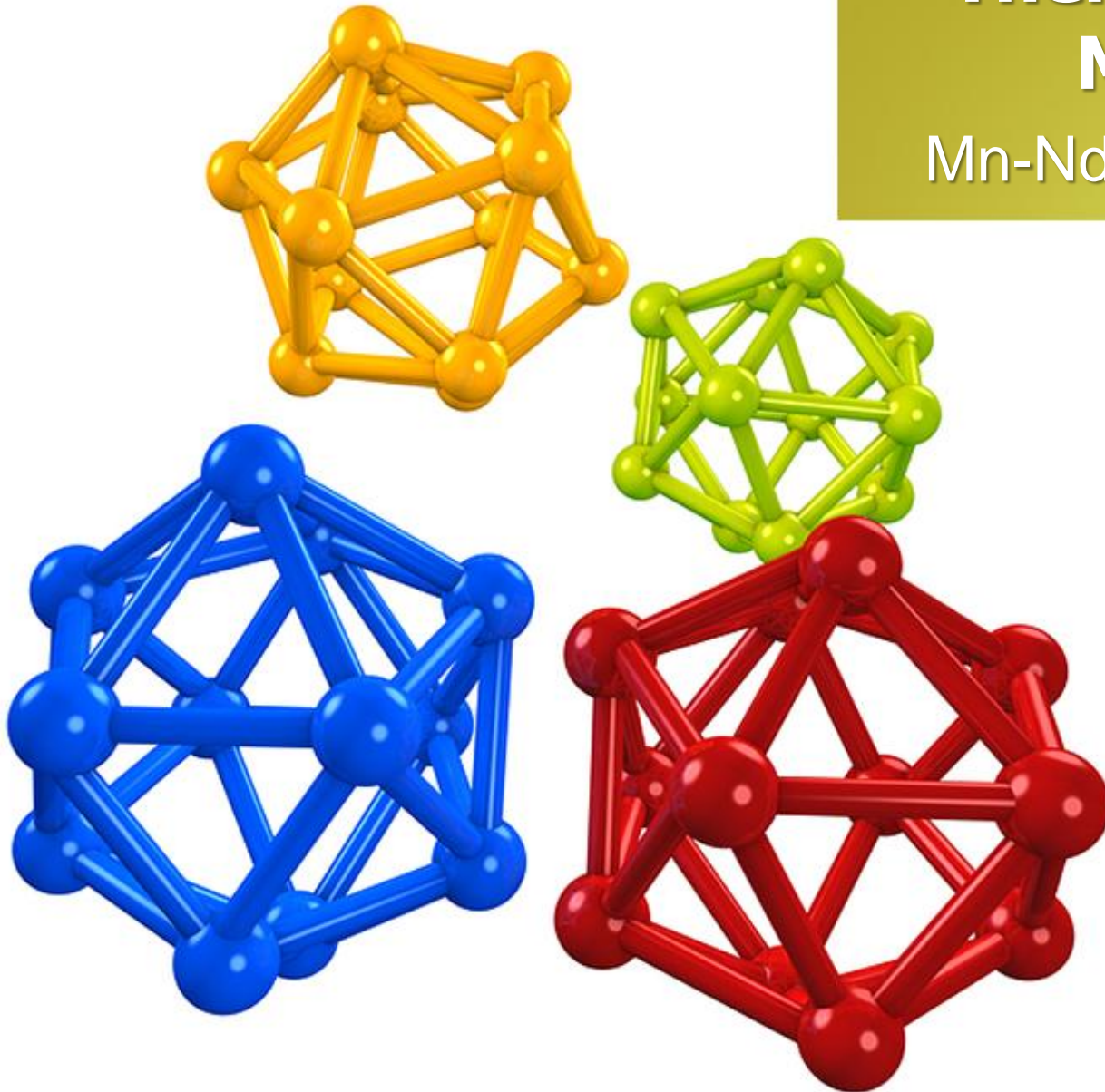


# Thermodynamic Modeling

Mn-Nd phase diagram



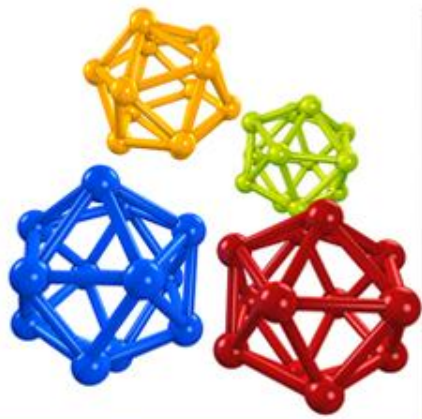
By  
**Ahmad Mostafa**

Presented to  
**Dr. Mamoun Medraj**

# Outlines

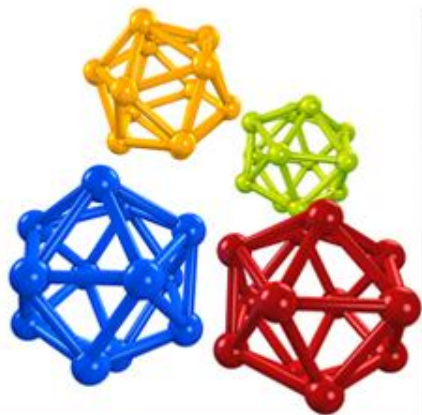


- Introduction
- Experimental data
- Gibbs energy models
- Thermodynamic modeling and experimental investigation of Mn-Nd phase diagram
- Conclusions



# Introduction

- Phase diagram can be calculated from Gibbs energy function of phases.
- The required Gibbs energy functions are usually obtained by a computer-assisted statistical procedure, using experimental thermochemical and constitutional data as input.
- CALPHAD (CALculation of PHAse Diagram).



# Experimental Data

- ❖ Steps of thermodynamic optimization of a phase diagram:

## First step

- Collecting and categorizing any experimental information linked to Gibbs energy.
  - ✓ Thermochemical data
  - ✓ Crystallographic data
  - ✓ Review articles



# Experimental Data

## Second step

- The critical evaluation of the collected data by discarding the bad or contradictory data, using the following methods:

### Recording the data

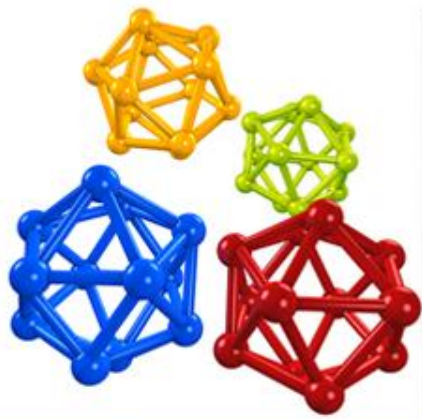
- ✓ The used technique
- ✓ Phases presented
- ✓ Purity of the samples
- ✓ Experimental conditions
- ✓ Quality and accuracy of the measurements

### Graphical comparison



## Experimental Data

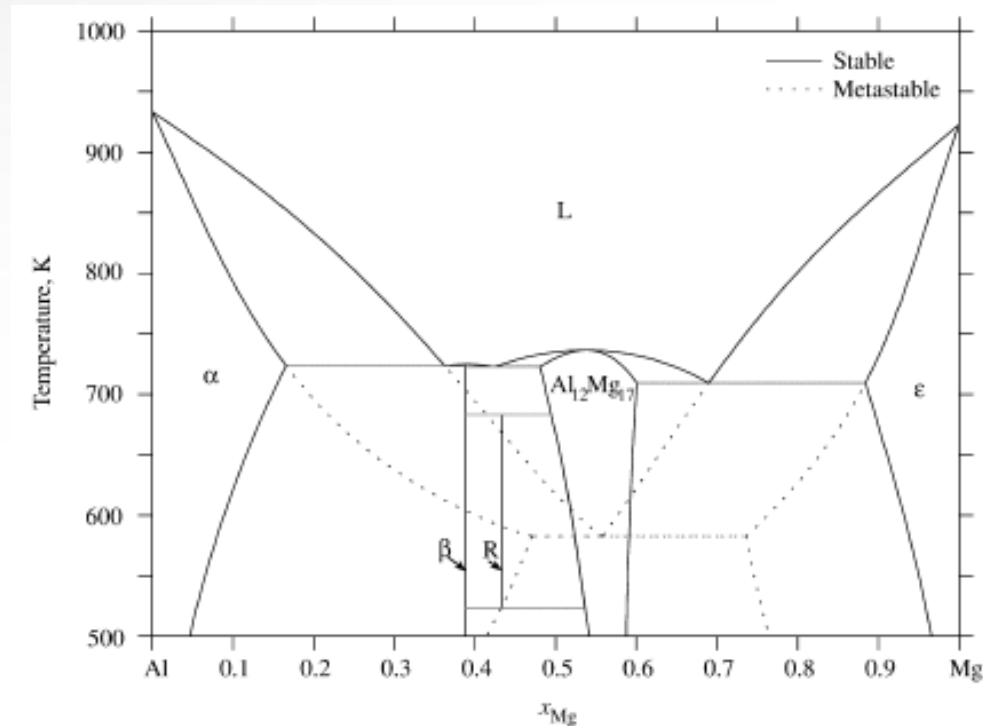
- The main difficulty in the phase diagram modeling is that the start values must be supplied for all model parameters.
  - ❖ If the initial experimental information are available, then:
    - i. It is more practical to begin optimization with minimal data set
- Example:**
- ☐ For binary systems, it is enough to start with data pertaining to:
    - ✓ Invariant points
    - ✓ Congruent melting
  - ☐ For ternary systems, the start begins with extrapolating the data from the constituent binaries.



# Experimental Data

ii. The experimental data can also be extrapolated from the metastable equilibrium PD

- This can reduce the number of phases that may optimized together



Stable and metastable phases of Al-Mg phase diagram





# Experimental Data

- Once the initial description is obtained by the minimal data set, more experimental data can be added.

❖ **More experimental data can be obtained by:**

**i. Estimation of the experimental data using different methods**

**Example:**

- ❑  $\Delta H_{\text{formation}}$  of transition metal compounds can be obtained by Miedema's method
- ❑  $C_p$  (Molar) can be approximated by Kopp-Neumann rule
- ❑ Excess partial molar Gibbs energies can be calculated by 
$$\frac{{}^{\text{xs}}G_{\text{A}}}{RT} = \ln \gamma_{\text{A}} \text{ and } \frac{{}^{\text{xs}}G_{\text{B}}}{RT} = \ln \gamma_{\text{B}}$$
- ❑ Experimental slopes of the phase boundaries involving the terminal phases in binary systems can be checked by Van't Hoff's equation





# Experimental Data

- Application of Van't Hoff's equation

$$(dX/dT)_{\text{solidus}} - (dX/dT)_{\text{liquidus}} = \Delta H_{\text{fus}} / RT_m^2$$

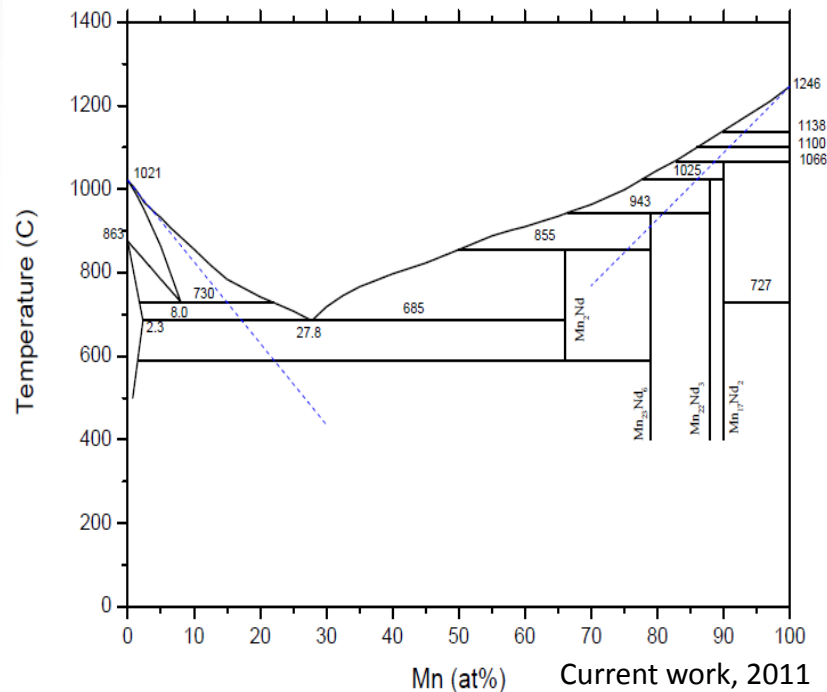
X: is the mole fraction of the solute

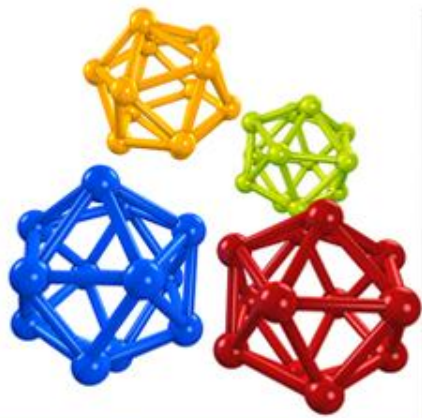
$\Delta H_{\text{fus}}$ : Enthalpy of fusion

$T_m$ : melting point of the pure element

The initial value of  $(dX/dT)_{\text{solidus}}$  for the terminal phases=0, then:

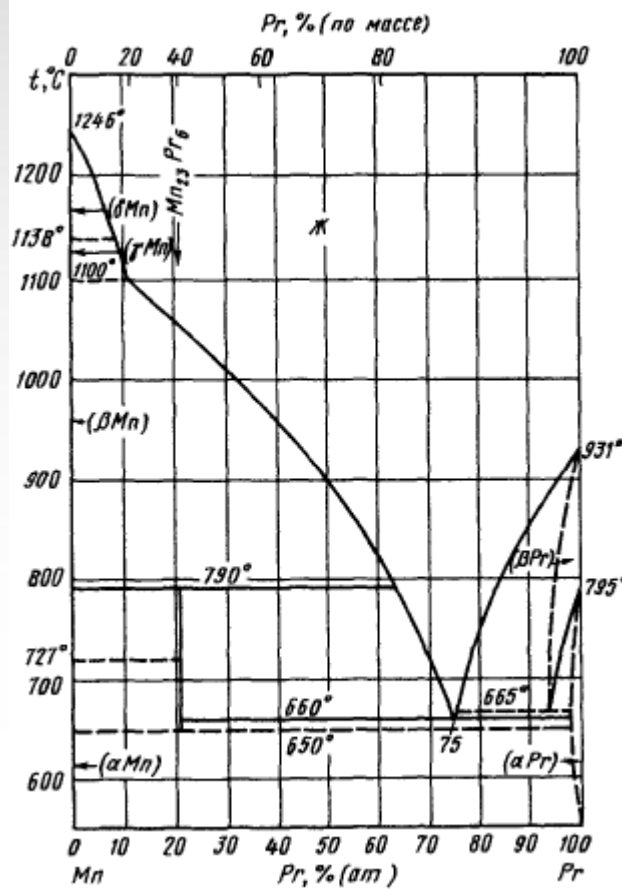
$$\Delta H_{\text{fus(Nd)}} = -(dX/dT)_{\text{liq(Nd)}} \times RT_m^2$$





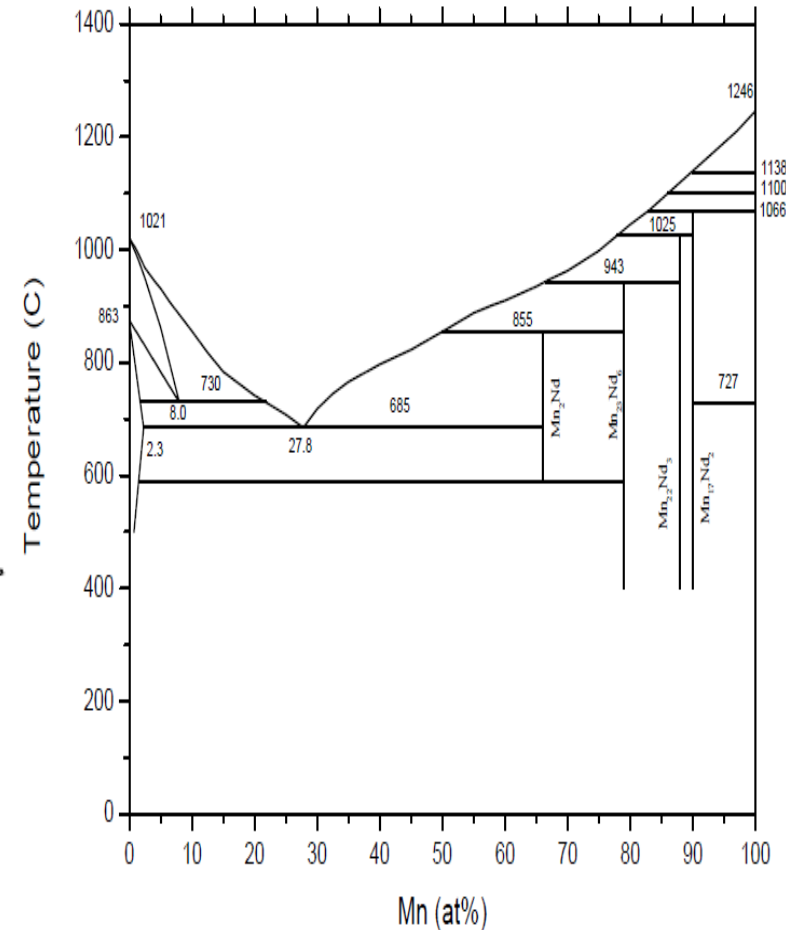
# Experimental Data

ii. Data of similar systems can also offer a great deal of information

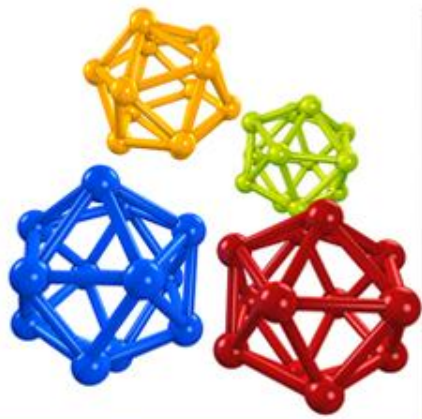


Mn-Pr phase diagram  
from literature

Saccone et al., 1985



Mn-Nd phase diagram  
recently studied



# Gibbs energy models

$$G = G^{\text{ref}} + G^{\text{ideal}} + G^{\text{xs}} + G^{\text{phys}}$$

$G^{\text{ref}}$ : Reference surface for Gibbs energy

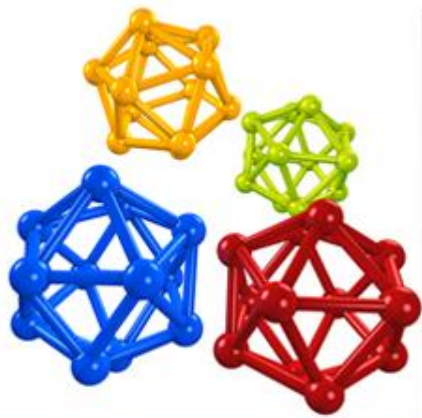
$G^{\text{ideal}}$ : Gibbs energy for ideal mixing

$G^{\text{xs}}$ : is the excess Gibbs free energy

$G^{\text{phys}}$ : physical contribution energy

## ❖ Types of thermodynamic models:

- ☐ Regular Solutions Models
- ☐ Sublattice Model
- ☐ Associated Solutions
- ☐ Cluster Variation Model
- ☐ Quasi-chemical Model
- ☐ Cluster Expansion Model



# Gibbs energy models

- Choice of Gibbs energy model for a phase should be physically adequate for the P-X-T domain, in which the phase is stable.
- The model should have extrapolating capability in the higher-order systems.
- The good strategy to start the optimization process with simple Gibbs free energy models which have few model parameters.

## Example:

- ✓ As a first approximation, all intermediate phases in a binary system can be treated as stoichiometric compounds.



# Mn-Nd Phase diagram

- The Mn-Nd phase diagram is reconstructed experimentally using ICP, DSC, SEM/EPMA, XRD and microscopic analysis.
- No thermodynamic description was carried out on this system.
- The thermodynamic data of the system are obtained from similar systems as mentioned in the literature
- In this work, the liquid solution phases will be modeled with the quasi-chemical model (QCM).
- Random solution model is used for terminal solid solutions.



# Mn-Nd Phase diagram

- The Gibbs energy for the terminal solid solution phases is described by the following equation

$$G^\phi = x_i {}^0G_i^\phi + x_j {}^0G_j^\phi + RT[x_i \ln x_i + x_j \ln x_j] + {}^{ex}G^\phi$$

$\phi$ : is the phase of interest

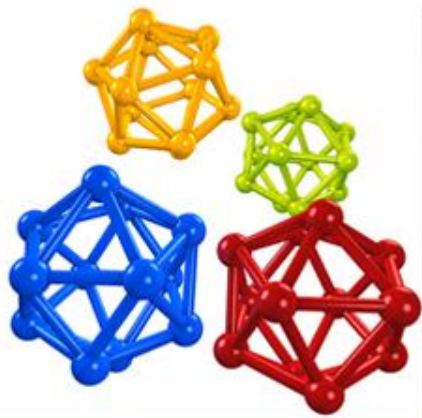
$x_i$  and  $x_j$ : are the mole fractions of components  $i$  and  $j$  respectively

- The excess Gibbs energy is described by the *Redlich-Kister* polynomial model

$${}^{ex}G^\phi = x_i x_j \sum_{n=0}^{n=m} {}^nL_{i,j}^\phi (x_i - x_j)^n$$

$${}^nL_{i,j}^\phi = a_n + b_n \times T$$

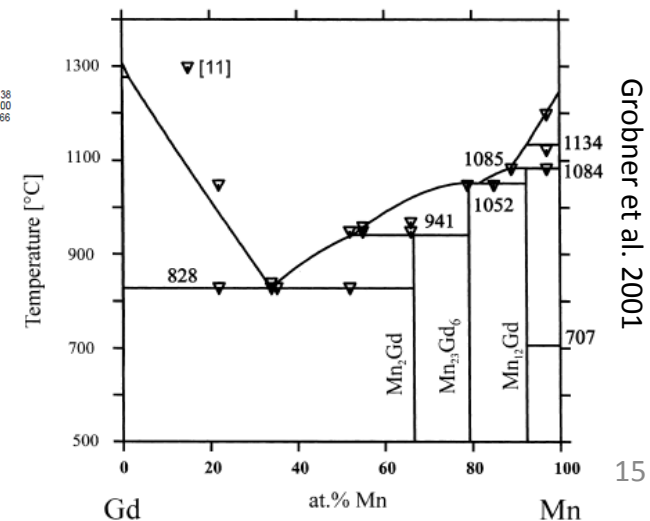
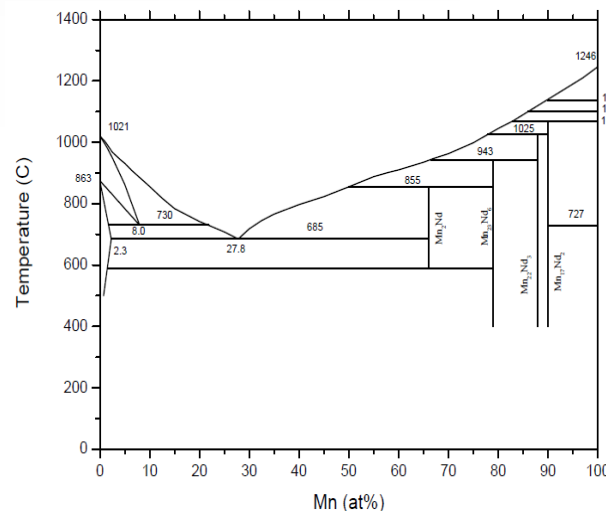
- Where,  $a$  and  $b$ : are the parameters of the model need to be optimized.



# Mn-Nd Phase diagram

## ❖ Why quasi-chemical model?

- ✓ It offers greater flexibility in optimizing the parameters for the systems show large degree of SRO in the liquid.
  - ✓ It is more realistic because it considers the preferential formation of the nearest neighbor A-B pairs for SRO.
  - ✓ Choosing the QCM for liquid phase leads to the consistency with the other existing databases developed with the same model.
- Similarity between Mn-Gd system and Mn-Nd system is found, as shown.



Grobner et al. 2001





# Mn-Nd Phase diagram

- From similarity with Mn-Gd system, the parameters ( $\text{Jmol}^{-1}$ ) can be concluded :

$$L(\text{LIQUID}, \text{Gd}, \text{Mn}; 0) = -5020.871 + 0.56166 * T$$

$$L(\text{LIQUID}, \text{Gd}, \text{Mn}; 1) = 1041.42$$

$$G(\text{Mn}_{12}\text{Gd}, \text{Mn}:\text{Gd}; 0) = -52000 + 12.1 * T + 12 * G_{\text{Mn}} + G_{\text{Gd}}$$

$$G(\text{Mn}_{23}\text{Gd}_6, \text{Mn}:\text{Gd}; 0) = -185000 + 27.6 * T + 23 * G_{\text{Mn}} + 6 * G_{\text{Gd}}$$

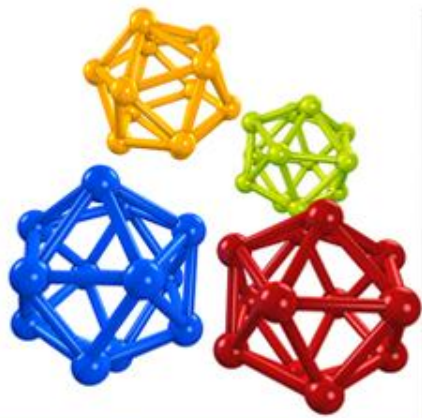
$$G(\text{Mn}_2\text{Gd}, \text{Mn}:\text{Gd}; 0) = -19242 + 2.87 * T + 2 * G_{\text{Mn}} + G_{\text{Gd}}$$

$$G(\text{FCC A1}, \text{Gd}; 0) = 10000 + G_{\text{Gd}}$$

$$L(\text{BCC A2}, \text{Gd}, \text{Mn}; 0) = 50000$$

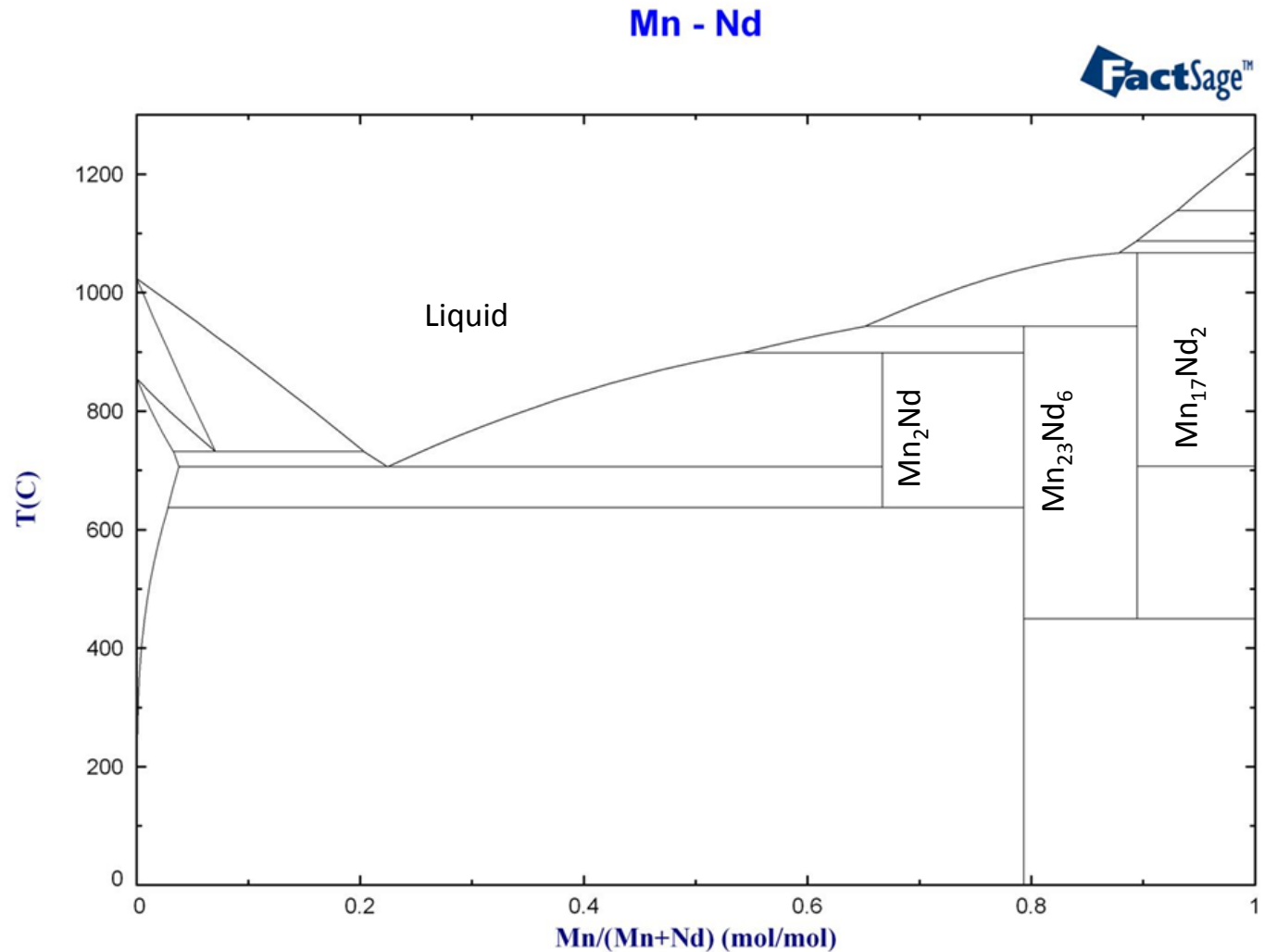
$$L(\text{HCP A3}, \text{Gd}, \text{Mn}; 0) = 50000$$

$$L(\text{FCC A1}, \text{Gd}, \text{Mn}; 0) = 50000$$



- Applying the obtained parameters for Mn-Nd phase diagram optimization:

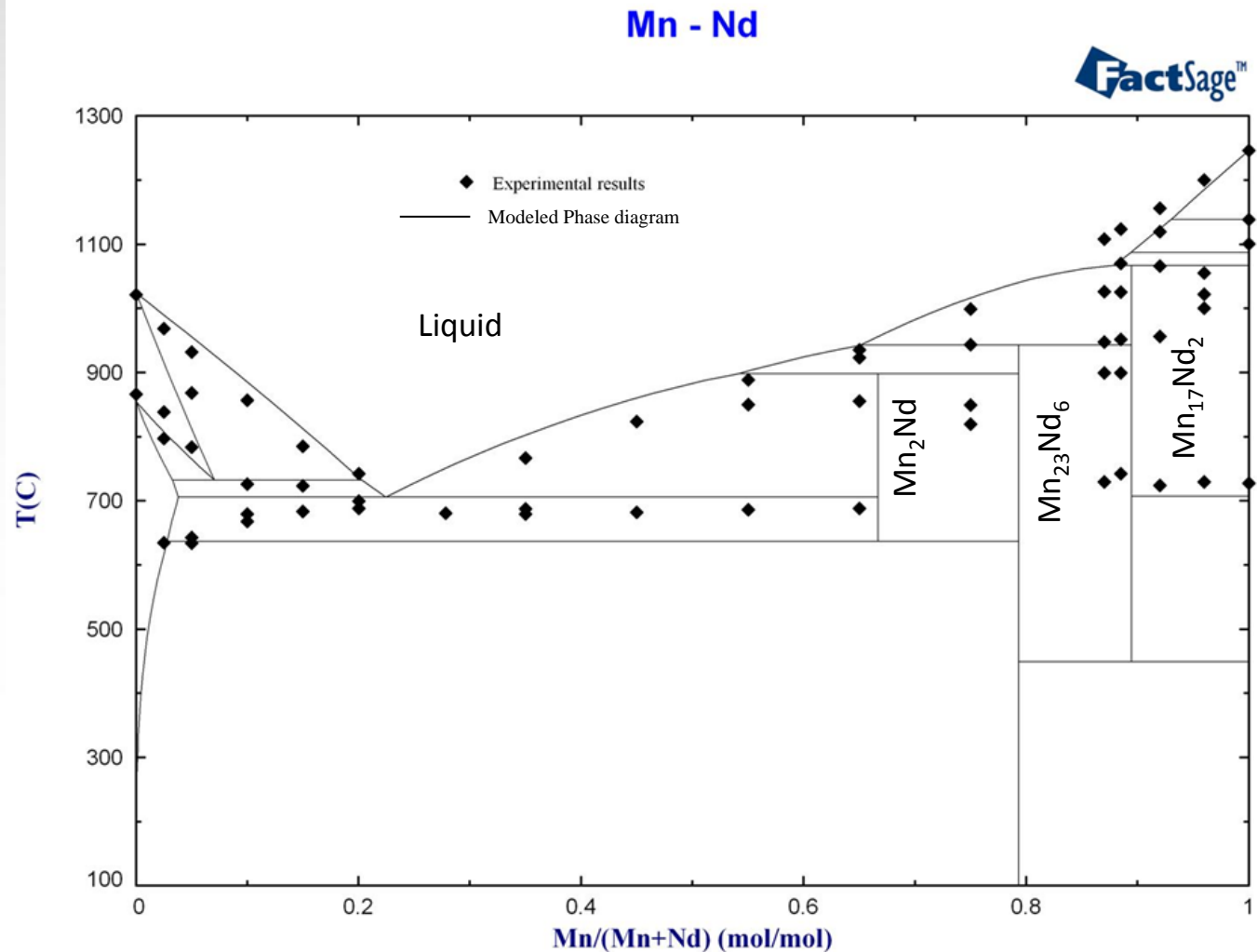
# Mn-Nd Phase diagram

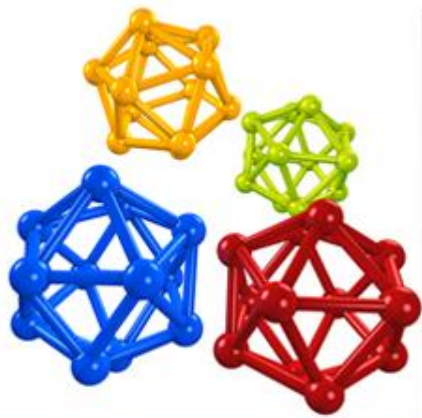




# Mn-Nd Phase diagram

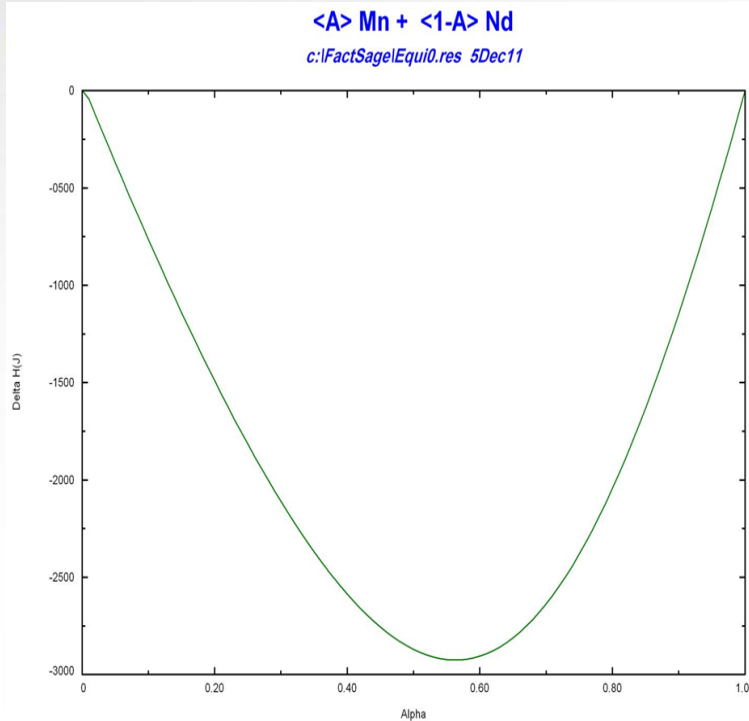
- Comparison between the experimental results and the thermodynamic model.



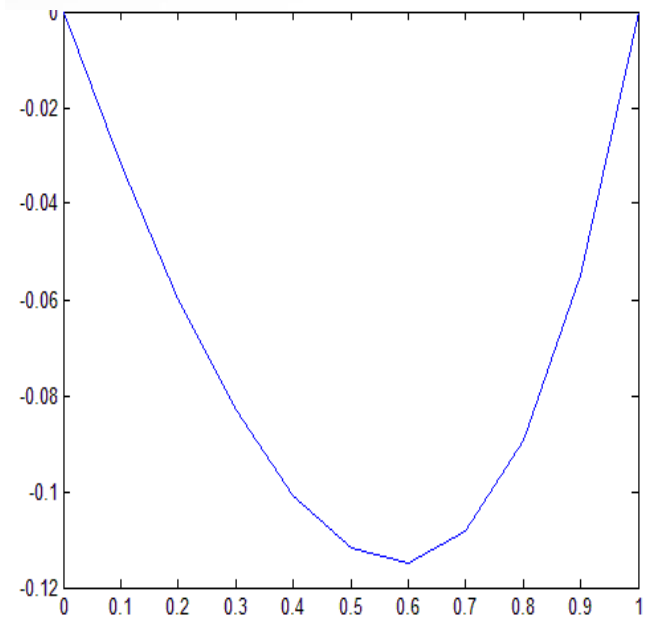


- Comparison between the Miedema's value and the thermodynamic model of the heat of mixing for the liquid phase.

# Mn-Nd Phase diagram



Calculated heat of mixing



Miedema's heat of mixing

- Yet, the two curves are not corresponding. The parameters should be modified.

# Conclusions



- ❖ **Success of optimization depends on:**
  - ✓ The selected model
  - ✓ The selected experimental data
  - ✓ The number of models parameters
  - ✓ Starting values for the model parameters
  - ✓ The order in which the parameters are arranged
- ❖ **Thermodynamic properties of the similar systems are the easiest way to start with.**
- ❖ **Many improvements are required to justify the Mn-Nd phase diagram model**
- ❖ **The final model can be used in extrapolating the Mg-Mn-Nd ternary phase diagram.**

# Thank you

