On the Slopes of Phase Boundaries

ARTHUR D. PELTON

Several simple equations are derived which can be used to check binary phase diagrams for thermodynamic consistency. In many cases, the only thermodynamic data required are the entropies of fusion.

I. INTRODUCTION

PHASE diagram compilation and evaluation is a topic of great current interest. The alloy phase diagram compilation program of the American Society for Metals has been underway for several years. The American Ceramic Society is lauching a similar project for ceramic phase diagrams. Thousands of diagrams are being evaluated under these international programs.

In evaluating a phase diagram, it is important to check that the diagram is consistent with the thermodynamic properties of the system. Ideally, a complete computer-assisted optimization of all available thermodynamic and phase equilibrium data on a system should be performed with a view to obtaining equations for the Gibbs energies of the phases which can be used to calculate the phase diagram and the thermodynamic properties in a completely self-consistent manner. Many systems have already been fully optimized in this way.

However, there are also a number of simple equations which can be used for providing a quick check on the consistency of phase diagrams. These need only a hand calculator for their solution and often require no more thermodynamic data than the entropies of fusion of the components.

The best example of such an equation, which has been known for a century, but which is still not used regularly, relates the limiting slopes of the liquidus and solidus in a binary system when the mole fraction of one component is unity:

$$(dT/dX_A^L)_{x_A=1}^{-1} - (dT/dX_A^S)_{x_A=1}^{-1} = \Delta h_{f(A)}^{\circ} / R(T_{f(A)}^{\circ})^2$$
 [1]

where $(dT/dX_A^L)_{x_A=1}$ and $(dT/dX_A^S)_{x_A=1}$ are the slopes of the liquidus and solidus when the mole fraction, X_A , of component A equals unity, $\Delta h_{f(A)}^{\circ}$ is the molar enthalpy of fusion of A, and $T_{f(A)}^{\circ}$ is the melting point of A (in kelvins). The only requirement involved in Eq. [1] is that Raoult's law be obeyed in the limit for the solid and liquid phases.

In Figure 1 is shown the K-Na phase diagram. [1] The experimental points are from Reference 2. The experimental limiting liquidus and solidus slopes at $X_K = 1$ are drawn on the diagram. These are read off the diagram as $(dT/dX_K^L) = 270$ and $(dT/dX_K^S) = 1120$. From Eq. [1], with $T_{f(K)}^o = 336.34$ K (63.19 °C) we calculate $\Delta h_{f(K)}^o = 2.6$ kJ/mol which is very close to the value of 2.3 kJ/mol found in recent compilations. [3] Hence the limiting slopes of the experimental diagram are thermodynamically consistent. An example of a diagram which does not pass this test is provided by the Na-Sr diagram [4] shown in Figure 2(a). Limit-

Manuscript submitted August 12, 1987.

ing liquidus and solidus slopes at the melting point of Sr (774 °C), when substituted into Eq. [1], give $\Delta h_{f(Sr)}^{\circ} = 14.6 \text{ kJ/mol}$ which is about twice the correct value^[3] of 7.4 kJ/mol. A recent critical evaluation of the Na-Sr system^[5] has resulted in the revised diagram shown in Figure 2(b) in which the limiting liquidus slope is much steeper. (Probably Na losses by volatilization during the measurements resulted in the incorrect liquidus of Figure 2(a).)

In the present article, some other simple relations involving the slopes of phase boundaries of binary systems are derived and discussed.

II. RATIOS OF SLOPES AT INVARIANTS

In Figures 3(a) through (d) are shown four cases of invariants involving three phases: α , β , and γ in a binary system with components A and B. Let $\sigma_{\gamma\alpha}$ and $\sigma_{\gamma\beta}$ be the slopes of the γ -phase boundaries of the $(\gamma + \alpha)$ and $(\gamma + \beta)$ regions at the invariant temperature as shown in the figure:

$$\sigma_{\gamma\alpha} = (dT/dX_B)_{\gamma\alpha} \qquad \sigma_{\gamma\beta} = (dT/dX_B)_{\gamma\beta}$$
 [2]

where X_B is the mole fraction of B.

The following completely general expression for the ratio $\sigma_{\gamma\beta}/\sigma_{\gamma\alpha}$ is derived in the Appendix by applying the Gibbs-Duhem and Gibbs-Helmholtz equations. The derivation involves no assumptions.

$$\frac{\sigma_{\gamma\beta}}{\sigma_{\gamma\alpha}} = \frac{\left[X_A^a(S_A^{\gamma} - S_A^{\alpha}) + X_B^{\alpha}(S_B^{\gamma} - S_B^{\alpha})\right] \left[X_B^{\beta} - X_B^{\gamma}\right]}{\left[X_A^{\beta}(S_A^{\gamma} - S_A^{\beta}) + X_B^{\beta}(S_B^{\gamma} - S_B^{\beta})\right] \left[X_B^{\alpha} - X_B^{\gamma}\right]}$$
[3]

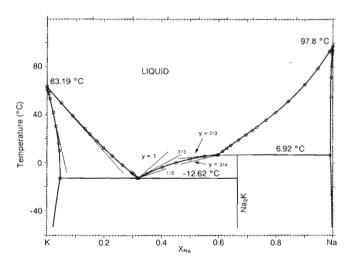
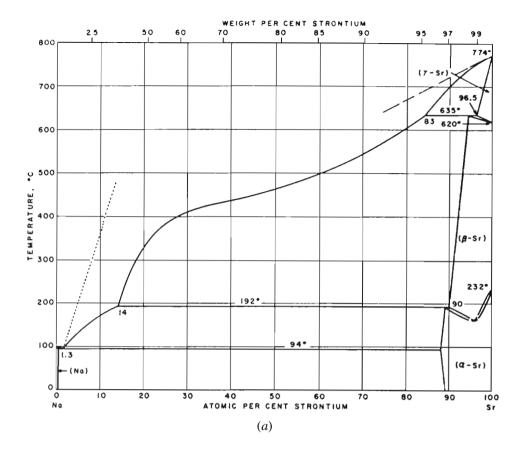


Fig. 1 — K-Na phase diagram reproduced from Ref. 1. Experimental points are from Ref. 2. Calculated phase boundary slopes at eutectic and peritectic are shown for different assumed stoichiometries of the compound as described in the text.

ARTHUR D. PELTON is Co-Director, Centre for Research in Computational Thermochemistry, Ecole Polytechnique de Montreal, P.O. Box 6079, Station A, Montreal, PQ, Canada H3C 3A7.



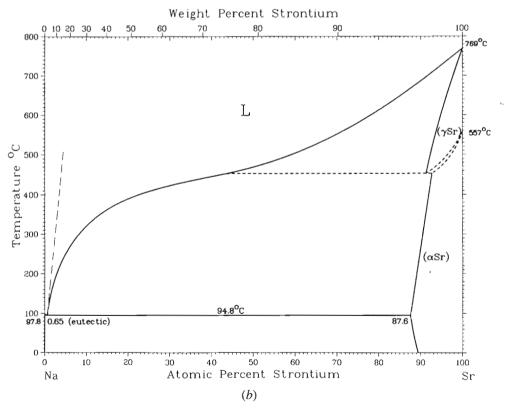


Fig. 2—(a) Na-Sr phase diagram as reported by Ref. 4 (reproduced from Constitution of Binary Alloys). Dashed line is reported limiting liquidus slope at $X_{\rm Sr}=1$. Dotted line is liquidus slope at the eutectic calculated as described in text. (b) Na-Sr phase diagram reproduced from Ref. 5. Liquidus is based on data of Ref. 7. Dashed line is liquidus slope at the eutectic calculated as described in text.

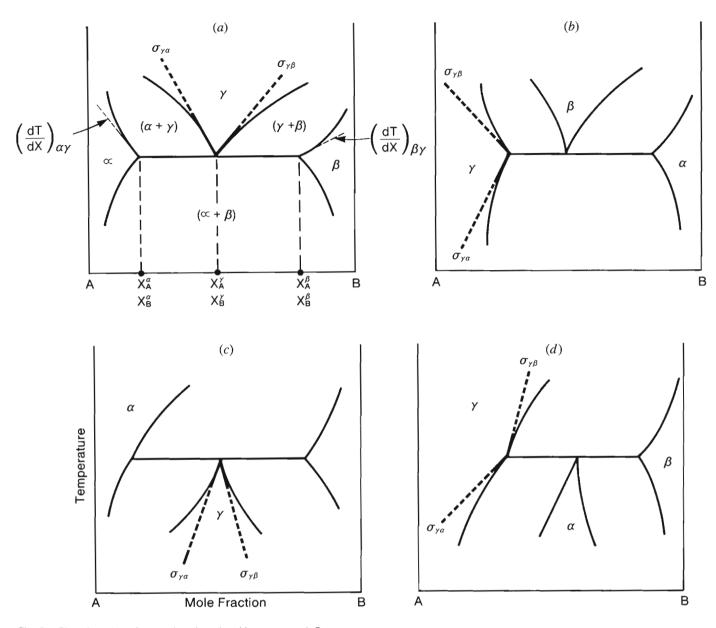


Fig. 3—Phase boundary slopes at invariants in a binary system A-B.

where X_A^{α} , X_B^{β} , etc. are the mole fractions of the components in the phases at the equilibrium invariant compositions as shown in Figure 3, and S_A^{α} , S_B^{β} , etc. are the partial molar entropies at these same invariant compositions.

Equation [3] may be converted into an expression involving partial molar enthalpies rather than entropies by noting that, at equilibrium, $g_A^{\gamma} = g_A^{\alpha}$. Hence: $(S_A^{\gamma} - S_A^{\alpha}) = (h_A^{\gamma} - h_A^{\alpha})/T$. Similar expressions for the other entropy terms in Eq. [3] can be written. Substituting these into Eq. [3] yields:

$$\frac{\sigma_{\gamma\beta}}{\sigma_{\gamma\alpha}} = \frac{\left[X_A^{\alpha}(h_A^{\gamma} - h_A^{\alpha}) + X_B^{\alpha}(h_B^{\gamma} - h_B^{\alpha})\right]\left[X_B^{\beta} - X_B^{\gamma}\right]}{\left[X_A^{\beta}(h_A^{\gamma} - h_A^{\beta}) + X_B^{\beta}(h_B^{\gamma} - h_B^{\beta})\right]\left[X_B^{\alpha} - X_B^{\gamma}\right]}$$
[4]

Equations [3] and [4] are identical. However, Eq. [3] is more useful than Eq. [4] in applications involving approximations since it is usually easier to approximate entropies than enthalpies. For example, the entropies of fusion of most metals obey Richard's rule reasonably closely. [6] Also, entropies of formation of intermetallic compounds are gener-

ally small.^[6] Furthermore, assuming that the entropy of mixing of a liquid phase is ideal usually gives rise to smaller errors than assuming that the enthalpy is ideal.

Some examples of applications of Eq. [3] to binary phase diagram evaluations are given below.

Case 1—Eutectic with No Intermediate Compounds

Consider a eutectic as in Figure 3(a), where α and β are the solid end-member phases and γ is the liquid. The term $(S_A^L - S_A^{\alpha})$ in Eq. [3] may be expanded as:

$$(S_A^L - S_A^\alpha) = (S_A^L - S_A^{OL}) - (S_A^\alpha - S_A^{O\alpha}) + (S_A^{OL} - S_A^{O\alpha})$$

$$\approx -R \ln(X_A^L/X_A^\alpha) + \Delta S_{f(A)}^\alpha$$
 [5]

where S_A^{OL} and $S_A^{O\alpha}$ are standard molar entropies of pure liquid and solid A, where $\Delta S_{f(A)}^{\circ}$ is the standard molar entropy of fusion of pure A, and where it has been assumed that the partial entropies of mixing are ideal.

Similarly, for the term $(S_A^L - S_A^\beta)$:

$$(S_A^L - S_A^\beta) \approx -R \ln(X_A^L/X_A^\beta) + (S_A^{OL} - S_A^{O\beta})$$
 [6]

The term $(S_A^{OL} - S_A^{O\beta})$ in Eq. [6] is the entropy of fusion of pure solid A with the β -phase structure. In light of Richard's rule, we can assume that this term will be approximately equal to $\Delta S_{(A)}^{\circ}$.

Similar expressions can be written for the other entropy terms. Substitution into Eq. [3] then gives the approximate equation:

$$(S_B^L - S_B^{\alpha}) = (S_B^L - S_B^{OL}) - (S_B^{\alpha} - S_B^{O\alpha}) + (S_B^{OL} - S_B^{O\beta})$$

$$+ (S_B^{O\beta} - S_B^{O\alpha}) \approx -R \ln(X_B^L/X_B^{\alpha}) + \Delta S_{f(B)}^{\alpha}$$

$$+ \Delta S_{\alpha \to \beta(B)}^{\alpha}$$
[9]

where $\Delta S_{\alpha \to \beta(B)}^{\circ}$ is the standard molar entropy change for the $\alpha \to \beta$ transformation of pure B. Substitution into

$$\frac{\sigma_{L\beta}}{\sigma_{L\alpha}} = \frac{\left[X_A^{\alpha} (R \ln(X_A^{\alpha}/X_A^L) + \Delta S_{f(A)}^{\circ}) + X_B^{\alpha} (R \ln(X_B^{\alpha}/X_B^L) + \Delta S_{f(B)}^{\circ}) \right] \left[X_B^{\beta} - X_B^L \right]}{\left[X_A^{\beta} (R \ln(X_A^{\alpha}/X_A^L) + \Delta S_{f(A)}^{\circ}) + X_B^{\beta} (R \ln(X_B^{\alpha}/X_B^L) + \Delta S_{f(B)}^{\circ}) \right] \left[X_B^{\alpha} - X_B^L \right]}$$
[7]

Eq. [3] yields:

$$\frac{\sigma_{L\beta}}{\sigma_{L\alpha}} \approx \frac{\left[X_A^{\alpha}(R \ln(X_A^{\alpha}/X_A^L) + \Delta S_{f(A)}^{\circ}) + X_B^{\alpha}(R \ln(X_B^{\alpha}/X_B^L) + \Delta S_{f(B)}^{\circ} + \Delta S_{\alpha \to \beta(B)}^{\circ})\right] \left[X_B^{\beta} - X_B^L\right]}{\left[X_A^{\beta}(R \ln(X_A^{\beta}/X_A^L) + \Delta S_{f(A)}^{\circ}) + X_B^{\beta}(R \ln(X_B^{\beta}/X_B^L) + \Delta S_{f(B)}^{\circ})\right] \left[X_B^{\alpha} - X_B^L\right]}$$
[10]

Equation [7] contains only the compositions of the three phases at the eutectic temperature as well as the entropies of fusion of A and B which are generally well known. In the case of negligibly small solid solubility $(X_A^{\alpha} \approx 1, X_B^{\beta} \approx 1)$, Eq. [7] takes on the simplified form:

$$\frac{\sigma_{L\beta}}{\sigma_{L\alpha}} \approx \frac{X_A^L(\Delta S_{f(A)}^\circ - R \ln X_A^L)}{X_B^L(\Delta S_{f(B)}^\circ - R \ln X_B^L)}$$
[8]

An example of the application of Eq. [7] to phase diagram analysis is provided by the Na-Sr system. One version of this diagram^[4] is shown in Figure 2(a) while another version^[5] based principally upon the liquidus of Reference 7 is shown in Figure 2(b). The liquidus curves between 100° and 200 °C are very different in the two versions.

In the case of the diagram in Figure 2(b), we apply Eq. [7] at the eutectic at 94.8 °C where " α " is now solid Na (in which Sr is negligibly soluble) and " β " is solid Sr. Hence: $X_{Na}^{\alpha} = 1$, $X_{Sr}^{\alpha} = 0$, $X_{Na}^{\beta} = 0.124$, $X_{Sr}^{\beta} = 0.876$, $X_{Na}^{L} = 0.9935$, $X_{Sr}^{L} = 0.0065$. Entropies of fusion of Na and Sr are taken from Reference 3. The slope of the Naliquidus taken from the diagram is $\sigma_{L\alpha} = (97.86 - 94.8)/(-0.0065) = -471$. From Eq. [7], the slope of the Srliquidus ($\sigma_{L\beta}$) is then calculated as 10700. (From the simplified Eq. [8] we calculate 10200.) The calculated slope is plotted on Figure 2(b). Agreement with the experimental liquidus slope is excellent. Similar calculations performed for the diagram in Figure 2(a) yield the calculated liquidus slope shown. The reported liquidus is not in agreement with the calculations.

Case 2—Invariant Associated with an Allotropic Transformation

Consider an invariant associated with the $\alpha \to \beta$ allotropic transformation of component B as shown in Figure 4. In this figure, the invariant is similar to Figure 3(b), since $X_B^{\alpha} > X_B^{\beta}$, but the following derivation also applies for the case (like Figure 3(d)) where $X_B^{\beta} > X_B^{\alpha}$.

The entropy terms of Eq. [3] are once again expanded as in Eq. [5], and the assumption of ideal mixing entropies is made. We note now that:

If the solid solubilities are small $(X_B^{\beta} \approx 1, X_B^{\alpha} \approx 1)$, then Eq. [10] simplifies to

$$\frac{\sigma_{L\beta}}{\sigma_{L\alpha}} \approx \frac{\Delta S_{f(B)}^{\circ} + \Delta S_{\alpha \to \beta(B)}^{\circ} - R \ln X_{B}^{L}}{\Delta S_{f(B)}^{\circ} - R \ln X_{B}^{L}}$$
[11]

An example of the application of Eq. [10] is provided by the Na-Sr phase diagram^[4] shown in Figure 2(a). At 192 °C there is an invariant associated with the supposed $\alpha \to \beta$ transformation of Sr. In Eq. [10] we set: $X_B^L = (1 - X_A^L) = 0.14$, $X_B^{\alpha} = 0.88$, $X_B^{\beta} = 0.90$, and entropies of fusion are taken from Reference 3. The liquidus slope ratio is read graphically from the diagram as $\sigma_{L\beta}/\sigma_{L\alpha} \approx 6.5$. Substitution into Eq. [10] then gives $\Delta S_{\alpha\to\beta(B)}^{\alpha} = 123$ J/mol·K. (The simplified Eq. [11] gives 134.) This is impossibly large for the entropy of an allotropic transformation by a factor of approximately 50. Hence, the liquidus slope ratio at 192 °C shown in Figure 2(a) is also impossibly large.

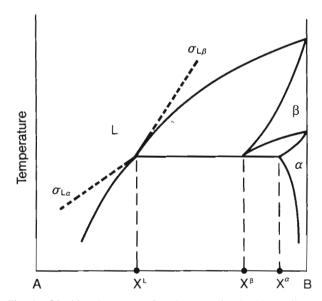


Fig. 4—Liquidus slopes at an invariant associated with an allotropic transformation.

We consider the liquidus slopes $\sigma_{L\alpha}$ and $\sigma_{L\beta}$ at a peritectic associated with the melting of an incongruent compound $A_{1-y}B_y$ as shown in Figure 5. In this example, $X_A^{\alpha} = (1-y)$ and $X_B^{\alpha} = y$.

The entropy of fusion of completely ordered intermetallic compounds may be approximated as the average of the entropies of fusion of the elements plus the entropy of mixing of the liquid.^[6] This is equivalent to saying that the entropy of formation of the compound from the solid elements is zero, or that the partial molar entropies of A and B in the compound are equal to the standard molar entropies. Hence, in Eq. [3] we may make the substitution:

$$(S_A^L - S_A^{\alpha}) \approx (S_A^L - S_A^{\circ \alpha}) = (S_A^L - S_A^{OL}) + (S_A^{OL} - S_A^{O\alpha})$$

 $\approx -R \ln X_A^L + \Delta S_{f(A)}^{\circ}$ [12]

where, as before, the entropy of mixing of the liquid has been assumed to be ideal.

A similar substitution for $(S_B^L - S_B^{\alpha})$ may be made. Substitutions for $(S_A^L - S_A^{\beta})$ and $(S_B^L - S_B^{\beta})$ may be made as before, with the following results:

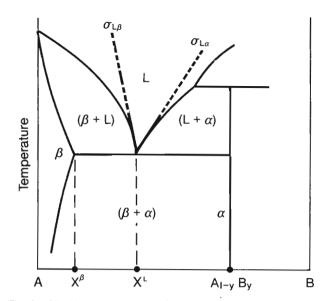


Fig. 6—Liquidus slopes at a eutectic involving the compound $A_{1-y}B_y$.

$$\frac{\sigma_{L\beta}}{\sigma_{L\alpha}} = \frac{\left[(1-y) \left(-R \ln X_A^L + \Delta S_{f(A)}^{\circ} \right) + y \left(-R \ln X_B^L + \Delta S_{f(B)}^{\circ} \right) \right] \left(X_B^{\beta} - X_B^L \right)}{\left[X_A^{\beta} (R \ln (X_A^{\beta} / X_A^L) + \Delta S_{f(A)}^{\circ} \right) + X_B^B (R \ln (X_B^B / X_B^L) + \Delta S_{f(B)}^{\circ}) \right] \left(y - X_B^L \right)}$$
[13]

An example of the application of Eq. [13] is provided by the K-Na^[1,2] phase diagram shown in Figure 1. At 6.92 °C the compound Na₂K melts incongruently. From Figure 1, $\sigma_{L\beta}$ is read as 165. From Eq. [13], with y = 2/3, $X_B^{\beta} \approx 1.0$, $X_B^L = 0.60$ and the entropies of fusion of K and Na taken from Reference 3, we calculate $\sigma_{L\alpha} \approx 24$. As shown in Figure 1, this agrees very well with the measured liquidus slope.

In order to show the sensitivity of the calculations to the stoichiometry of the compound, $\sigma_{L\alpha}$ was recalculated for y=0.75 (i.e., for Na₃K) and plotted on Figure 1. (Clearly, for $y=X_B^L=0.60$, $\sigma_{L\alpha}=0$, and for $y=X_B^{\beta}\approx 1.0$, $\sigma_{L\alpha}=\sigma_{L\beta}$.) Equation [13] can thus be used to check that the slope ratio is consistent with the known stoichiometry of the compound or, conversely, to use the measured slope ratio to estimate the stoichiometry of the compound.

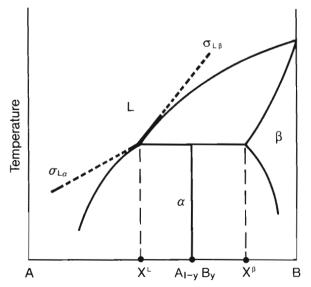


Fig. 5—Liquidus slopes at a peritectic involving the compound $A_{1-y}B_y$.

Case 4—Eutectic with an Intermediate Compound

This case is illustrated in Figure 6. The α phase is the compound $A_{1-y}B_y$ ($X_A^{\alpha}=1-y$, $X_B^{\alpha}=y$). With the same assumptions as in case 3 above, we find that Eq. [13] also applies in this case.

An example is provided by the eutectic in the K-Na system (Figure 1). From the figure we read $\sigma_{L\beta}=190$. From Eq. [13] with y=2/3, $X_B^\beta=0.05$, $X_L^\beta=0.32$, and the entropies of fusion from Reference 3, we calculate $\sigma_{L\alpha}=149$. As shown in Figure 1, this agrees very well with the measured liquidus slope. The slope $\sigma_{L\alpha}$ was recalculated with y=1/2 (i.e., NaK) and with y=1. Results are also plotted on Figure 1 to show the sensitivity of the calculations. (Note that when y=1, Eq. [13] reduces to Eq. [7] with $X_B^\alpha=1$.) Hence, Eq. [13] can be used to estimate the stoichiometry of a compound from the measured slope ratio.

III. CALCULATING SOLIDUS COMPOSITION FROM THE LIQUIDUS SLOPE

For many binary systems, the liquidus has been measured, but data on the solidus are imprecise or lacking. If the mutual solubility of the solid components is extensive, then a good approximation can often be made of the solidus composition by means of a thermodynamic relationship presented earlier. ^[8] In general, to calculate the solidus composition at a given temperature it is necessary to know the composition and slope of the liquidus at the same temperature as well as the excess Gibbs energy of the liquid and the excess entropy of the solid. However, in systems with extensive solid solubility, it is usually a good assumption that the excess entropy of the solid is small. Furthermore, when the components are very miscible in the solid phase, the liquid solution is generally close to ideal. Under the

assumptions that the excess Gibbs energy of the liquid and the excess entropy of the solid are both zero, the general relationship^[8] reduces to:

$$X_{A}^{S}[\ln X_{A}^{L} - (T/X_{A}^{L})(dT/dX_{B}^{L})^{-1} - \Delta S_{f(A)}^{\circ}/R]$$

$$+ X_{B}^{S}[\ln X_{B}^{L} + (T/X_{B}^{L})(dT/dX_{B}^{L})^{-1} - \Delta S_{f(B)}^{\circ}/R]$$

$$= X_{A}^{S} \ln X_{A}^{S} + X_{B}^{S} \ln X_{B}^{S}$$
 [14]

where X_A^L , X_B^L , X_A^S , X_B^S are the liquidus and solidus compositions in equilibrium at temperature T, dT/dX_B^L is the liquidus slope, and $\Delta S_{f(A)}^{\circ}$ and $\Delta S_{f(B)}^{\circ}$ are the entropies of fusion of A and B.

An example of the application of Eq. [14] is provided by the Cs-K (A-B) system^[9] in Figure 7. At -15 °C (258 K), $X_B^L = 0.27$ and (dT/dX_B^L) as read from the figure is -150. Entropies of fusion were taken from Reference 3. With these substitutions, Eq. [14] may then be solved for $X_B^S = (1 - X_A^S)$ by trial and error. The result is $X_B^S = 0.175$ which agrees within 0.005 with the measured solidus composition.

IV. CONCLUSIONS

Equations have been derived which can be used to test binary phase diagrams for thermodynamic consistency. The general relationship, Eq. [3], for the ratio of phase boundary slopes at an invariant can be simplified through various approximations so that in many cases the entropies of fusion of the components are the only thermodynamic data required. It is better to formulate the general equation in terms of entropies rather than enthalpies, since entropies are more amenable to approximations.

Several examples of the application of these equations have been given. Although these examples involved liquid-solid equilibria and eutectic or peritectic invariants, the same equations apply to solid-solid equilibria, and eutectoid or peritectoid invariants.

APPENDIX

Equation [3] for the ratio of the slopes $\sigma_{\gamma\beta}$ and $\sigma_{\gamma\alpha}$ as defined in Eq. [2] and as illustrated in Figure 3 will be derived. The derivation applies to all four cases of Figures 3(a) through (d). For the slopes of the other boundaries at an invariant we use the notational scheme illustrated in Figure 3(a). That is, for example, $(dT/dX)_{\alpha\gamma}$ is the slope of the phase boundary of the α -phase in equilibrium with the γ -phase.

For the three phases in equilibrium at the invariant, the partial Gibbs energies of the components are equal:

$$g_A^{\gamma} = g_A^{\alpha} = g_A^{\beta} \qquad g_B^{\gamma} = g_B^{\alpha} = g_B^{\beta}$$
 [15]

(for a total of 4 independent equations). Taking the first equation, $g_A^{\gamma} = g_A^{\alpha}$, and differentiating, we obtain:

$$\frac{\partial g_A^{\gamma}}{\partial T} + (\partial g_A^{\gamma}/\partial X_B)\sigma_{\gamma\alpha}^{-1} = \frac{\partial g_A^{\alpha}}{\partial T} + (\partial g_A^{\alpha}/\partial X_B) \cdot (dT/dX_B)_{\alpha\gamma}^{-1}$$
[16]

That is:

$$(\partial g_A^{\gamma}/\partial X_B)\sigma_{\gamma\alpha}^{-1} - (\partial g_A^{\alpha}/\partial X_B)(dT/dX_B)_{\alpha\gamma}^{-1} = S_A^{\gamma} - S_A^{\alpha}$$
[17]

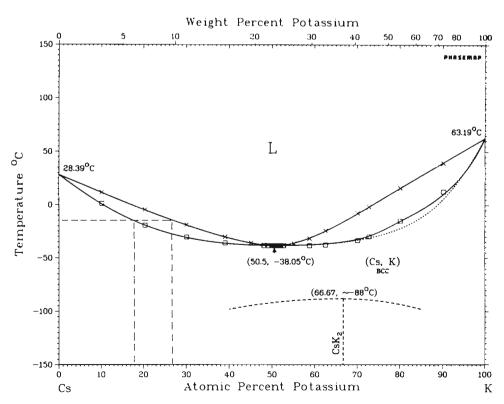


Fig. 7—Cs-K phase diagram reproduced from Ref. 9. Experimental points are from Ref. 10.

where S_A^{γ} and S_A^{α} are partial molar entropies. By differentiating the other three equations $(g_A^{\gamma} = g_A^{\beta}, g_B^{\gamma} = g_B^{\alpha})$, and $g_B^{\gamma} = g_B^{\beta}$ we obtain three more equations similar to Eq. [17].

In each of the three phases, the Gibbs-Duhem equation may be written. For the α phase, for example:

$$X_A^{\alpha} dg_A^{\alpha} + X_B^{\alpha} dg_B^{\alpha} = 0$$
 [18]

The four equations of the form of Eq. [17] and the three equations of the form of Eq. [18] can then be rearranged and substituted into one another to yield Eq. [3].

ACKNOWLEDGMENTS

This work evolved from conversations held many years ago with W. T. Thompson. Financial support was provided by the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

- 1. C. W. Bale: Bull. Alloy Phase Diagrams, 1982, vol. 3, pp. 313-18.
- J. B. Ott, J. R. Goates, D. R. Anderson, and H. T. Hall, Jr.: Trans. Farad. Soc., 1969, vol. 65, p. 2870.
- 3. M. W. Chase: Bull. Alloy Phase Diagrams, 1983, vol. 4, p. 124.
- W. O. Roberts: U.S. At. Energy Comm. TID-20639, 1964; W. O. Roberts: Ph. D. Thesis, Syracuse (NY) University, 1964; reported in F. A. Shunk: Constitution of Binary Alloys, 2nd supplement, McGraw-Hill, 1969, p. 540.
- 5. A. D. Pelton: Bull. Alloy Phase Diagrams, 1985, vol. 6, pp. 43-45.
- O. Kubaschewski and C. B. Alcock: Metallurgical Thermochemistry, 5th ed., Pergamon, 1979, pp. 186-87.
- P. R. Bussey, P. Hubberstey, and R. J. Pulham: J. Chem. Soc., Dalton Trans., 1976, vol. 22, pp. 2327-29.
- A. D. Pelton: Ber. Bunsenges. Phys. Chem., 1980, vol. 84, pp. 212-17;
 A. D. Pelton and H. Kohler: Calphad J., 1982, vol. 6, pp. 39-48;
 A. D. Pelton, H. Kohler, and A. Dubreuil: Proc. Wagner Sympos. "Chem. Metallurgy—A Tribute to Carl Wagner", N. Gokcen, ed., AIME, Warrendale, PA, 1981, pp. 273-82.
- C. W. Bale and A. D. Pelton: Bull. Alloy Phase Diagrams, 1983, vol. 4, pp. 379-82.
- J. R. Goates, J. B. Ott, and H. T. Hall Jr.: J. Chem. Eng. Data, 1971, vol. 71, p. 83.