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4.6.3 A SURVEY OF CLUES TO THE RELATIONSHIP BETWEEN EROSION RATE AND IMPACT PARAMETERS

by

F. J. Heymann

ABSTRACT

It is well known that in impingement erosion the rate of material loss due to a given amount of impinging water increases strongly with impact velocity, decreases if the angle of impingement deviates from the normal, and tends to decrease if the impinging water is subdivided into smaller droplets. The purpose of this survey is to search for clues to generalized formulations for these relationships, both from the experimental results reported in the literature, and from theoretical considerations some of which are taken from the literature and some of which are here proposed for the first time.

Since the interpretation and correlation of test data is confused by the uncertainty as to which attributes of an erosion-time curve should actually be used to quantify experimental results, it is found necessary first to establish a definition of 'rationalized erosion rate'. In order to permit rational formulations for erosion rate dependencies, it is also necessary to review some of the consequences of a drop impact, especially those concerning which the literature shows disagreement or lack of specific information. A position is taken concerning the impact pressure developed under a round drop, and the maximum value of the lateral outflow velocity after impact. The evidence concerning the effective impact area and time duration still seems inconclusive. A simple formulation is proposed for the value of the shock wave velocity C to be used in impact pressure calculations at high impact velocities, and is shown to have good accuracy in the range of practical interest.

With respect to the velocity dependence of erosion, it is shown that this does not follow exactly any of the simple empirical 'laws' which most authors have used, though all of these can approximate the data over limited ranges. A rationally-based formulation is proposed which does fit the characteristics of experimental curves much more closely.

With respect to drop size effect, it is found that some of the experimental data can be correlated successfully on the assumption that the 'threshold condition' for rationalized erosion rate is represented by $V^2 D$ = constant,

where V and D are impact velocity and drop size. However, a rational model which successfully predicts both the velocity and drop-size dependence is still elusive. Several possible reasons for drop-size effects are discussed qualitatively.

Finally, with respect to the impingement angle effect, there is agreement that the normal component of the impingement velocity is generally the determining factor. Under some conditions, however, the tangential component also seems significant, and a tentative explanation is offered for this observation.

FREQUENTLY-USED SYMBOLS

A	contact area between drop and target
C	velocity of stress wave, pressure wave, or shock wave
C_o	acoustic velocity of undisturbed liquid in drop
C_2	acoustic velocity of target material
C_s	Shock wave velocity in drop
D	drop diameter
E	'rationalized erosion rate' (see section 2)
M_o	impact mach number, V_o/C_o
M_e	edge Mach number, $\frac{1}{C}\left(\frac{dR_p}{dt}\right)$
R	drop radius
P	impact pressure
R_p	radius of area of contact between drop and target
V	impact velocity or particle velocity
V_c	critical or threshold impact velocity
V_{cd}	critical velocity for a given drop diameter
V_o	initial velocity of drop relative to target
V_i	particle velocity given to target
V_s	particle velocity in drop, $V_s = V_o - V_i$
V_R	lateral outflow velocity after impact
Z	acoustic impedance, ρC
Z_o	acoustic impedance of drop, $\rho_o C_o$
Z_2	acoustic impedance of target, $\rho_2 C_2$
ρ	density
ρ_o	density of undisturbed liquid in drop
ρ_1	density of compressed liquid in drop
ρ_2	density of target material
ϕ	the acute angle between target surface and drop surface at the edge of the contact area
θ	the angle of incidence of the impingement velocity, measured from the normal to the target surface

1 INTRODUCTION

In recent years the problem of liquid impingement damage to solid surfaces has received heightened attention, primarily because of the increase in flight speeds of aircraft and missiles and the increase in peripheral tip speeds of steam turbine blades, both of which are subject to this type of attack. An impressive body of experimental data has been generated, not only to evaluate materials but also to investigate the dependence of repeated impact damage on such external or operating variables as the impact velocity and angle, and the size of the impinging liquid drops.

Notable among the comprehensive investigations are those of Pearson, which were reported in the open literature by Baker et al.[1], of Busch, Hoff et al.[2,3], of Hobbs[4], and of Fyall et al.[5]. Among the earlier but still valuable contributions are those of Honegger[6] and of Brandenberger and De Haller[7]. Most of these authors have deduced empirical relationships between erosion rates and impingement variables, which are approximately valid for their data; but no rational theories exist as yet to predict such relationships.

Our purpose in this study is to review these and other results found in the literature, in order to select or to deduce, if possible, generalized formulations for these relationships; and also to present some theoretical hypotheses which may contribute to the understanding of the mechanisms involved and to the eventual formulation of a more complete rational theory.

2 EFFECT OF EXPOSURE TIME, AND CHOICE OF EROSION PARAMETERS

One of the greatest difficulties in the interpretation or correlation of empirical data lies in the identification or definition of the dependent parameter, namely the 'erosion rate'. All would be well if, under given conditions, the loss of material due to erosion proceeded at a constant rate and could be uniquely represented by the slope of the cumulative mass loss versus time curve. As is well known, however, this is not what happens. On the contrary, the rate of erosion generally passes through a number of different stages, and which of those is the most significant stage is still a matter of some dispute and may differ for different conditions of erosion attack.

Almost all recent authors concerned with erosion have stressed this time dependence, have enumerated and named the various stages of the rate-time pattern for their reported results, and have stated which stage they believe to be the most meaningful measure of erosion rate. Broadly speaking, there are two schools of thought: namely that in which the maximum erosion rate - the 'hump' of the rate-time curve - is considered the most significant, and that in which a final steady-state stage is postulated to be the most significant. The former view is maintained by Hobbs[4,8], Baker et al.[1] and Hoff et al.[3]; whereas the latter is espoused by Thiruvengadam[9] and by Smith et al.[10].

Heymann[11] has surveyed in some detail the various observed erosion rate-time patterns and the physical and geometric surface changes which could explain them; he has also shown that when erosion takes place primarily through a fatigue-like mechanism, the erosion rate-time pattern will be affected by the statistical distribution of the life-time to failure of the individual erosion fragments created, and therefore indirectly by the distribution of droplet sizes and impact velocities. With that statistical model, almost all of the observed rate-time patterns can be qualitatively predicted, depending on the assumed distribution curve and dispersion parameters for the erosion fragment lifetimes. It has not been established, however, whether the observed patterns are in reality determined primarily by these statistical effects or by the other physical and geometric effects; Smith et al[10] provided some strong evidence for the significance of surface geometry.

In any event, most authors seem to agree that the decrease of the erosion rate from a maximum toward a lower (possibly steady-state) value is either due to, or at least accompanied by, a significant roughening of the eroded surface. However, while Baker et al.[1] maintain that at this stage the damage is already so serious that the usefulness of actual components in service will have been destroyed, Smith et al[10] assert that it is in this latter stage that a component will operate during most of its life. Obviously, therefore, an important factor in making one's choice must be the intended application of the material and the amount of damage which can be tolerated under service conditions. However, since both Baker et al.[1] and Smith et al.[10] were concerned with steam turbine blades, there should have been some agreement upon this point.

This divergence of opinion seems to underline the need for a rational manner of characterizing erosion damage, so that a direct comparison can be made between the degree of surface damage corresponding to the various stages of the erosion rate-time curves found by different investigators. One rational measure is the volume loss per unit area of the eroded surface, which has been referred to as 'erosion depth' by Hoff et al.[3], and as 'mean depth of penetration' or 'MDP' by Hammitt (e.g. 12) and others.

This, however, does not tell the entire story since a given average erosion depth may occur with differing degrees of roughness, and it is the change in roughness which is presumed by many to be the primary cause of the changes in erosion rate. While Young and Johnston[13] have made some studies of the changes in surface roughness during cavitation erosion tests, the conventional techniques for measuring the surface roughness of machined or prepared surfaces will probably fail on deeply eroded surfaces. It seems very desirable, therefore, that a method be developed for measuring and characterizing the roughness or non-uniformity of a surface which is deeply and irregularly pitted. For quantitative measurement of the very early states of surface damage, Hancox and Brunton[22] have used the optical reflection coefficient.

Heymann[11] concluded that the most meaningful practical measure of erosion rate for a given application would be the total time taken to reach a given level of surface damage or average erosion depth. Such a measure has also been proposed by Hoff et al.[3], who have referred to it as 'erosion resistivity'. It is not a practicable approach, however, when one wants to deduce generalizations from widely-varying test data under widely differing conditions and with widely different damage levels. In the correlation attempts which follow we shall revert, therefore, to the simple approach which has been adopted by most of the authors (e.g. 1,4) whose work we shall survey: we define the erosion rate as the slope of the straight line which approximates or encompasses the steepest portions of the cumulative erosion-time curve. (See Fig.1.)

We have already mentioned the desirability of expressing the erosion rate in rationalized units. This is particularly important when the results at different impingement conditions are to be compared, since the target areas involved and the quantity of water impinging may be changed as a consequence. For instance, if in a wheel-and-jet apparatus the jet diameter is changed, this effectively alters the area of the target subjected to impact and also the quantity of water involved in each impact; and if the impact velocity is changed by changing the speed of rotation, this also alters the weight of water impacting per unit time. In order to permit valid comparisons and correlations, it is therefore essential to express the erosion and the duration in a rationalized form which will compensate for these test variations.

As stated before, at present the most obvious rationalized erosion parameter is volume loss per unit area. The appropriate rationalized 'time' parameter is not quite so obvious. One could make a case for selecting the number of impacts per unit area. At present, however, preference is given to the volume of liquid impinged per unit area. This is attractive because results expressed in this way will show directly the effect of subdividing a given quantity of impinging liquid into particles of different sizes or shapes, and because it makes the 'rationalized erosion rate' (E) a non-dimensional quantity, as follows:

$$E = \frac{\text{Volume of material lost per unit area per unit time}}{\text{Volume of liquid impinged per unit area per unit time}}$$

From a fundamental point of view the mass of impinged liquid may be more significant, since it is a measure of the impinging energy flux (see Ref.3), but for our present purposes the above definition is acceptable.

3 EFFECT OF IMPACT VELOCITY

A Empirical formulations

The literature contains a considerable body of data relating erosion to impact velocity, dating from the classic work of Honegger[6]. These results have been analyzed in detail elsewhere[14], and here we shall restrict ourselves to summarizing the most important findings.

There are various functional forms to which one can attempt to fit such data; the most obvious ones are briefly discussed below. Here E = erosion rate and V = velocity.

$$E = a V^n . \tag{1}$$

This represents a simple power relationship, and implies that some erosion will take place no matter how low the velocity is. Usually, however, it is thought that there is a critical or threshold velocity V_c below which erosion is absent for all practical purposes. An abvious type of relationship to reflect that is

$$E = a(V - V_c)^n . \tag{2}$$

Equations (1) and (2) should plot as straight lines on logarithmic coordinates of E versus V or $(V - V_c)$ respectively.

A third relationship which has been proposed on the basis of an analogy with a fatigue S-N curve is

$$V = a - b \log \frac{1}{E}$$

or, in a form which is equivalent but more consistent with the previous types of equation listed,

$$E = a e^{nV} . \tag{3}$$

This equation of course does not predict a critical velocity and could be combined with the separate condition that there is a transition to $E \to 0$ at some value $V = V_c$. Equation (3) will plot as a straight line on semi-logarithmic coordinates.

Most of the empirical relationships proposed in the literature have been of the aforementioned types.

One of the earliest comprehensive sets of tests data at various velocities was given by Honegger[6]. His conclusion was that while the behaviour of the various materials differs considerably, the rate of erosion 'may be generally expressed as':-

$$E \propto (V - 125)^2 \qquad (4)$$

where V is the impact velocity in m/sec. While this equation fits a 'mean curve' drawn through the band of experimental curves, some individual curves suggest exponents that are much higher.

Vater[15] has regarded erosion as analogous to a multiplicity of fatigue or corrosion-fatigue failures, which suggests a relationship like equation (3). Fig. 2 shows some of his data plotted in that manner.

Pearson[1], as a result of extensive tests, has proposed a relationship for stainless steel which for normal impact reduces to one like equation (2):

$$E \propto (V - 390)^{2 \cdot 6} \qquad (5)$$

where V is the impact velocity in feet per second. (In a later section of this paper we will suggest that the value V_c = 390 ft/sec is valid only for a particular drop size.)

Fyall et al.[5] present the following equation for the erosion rate of 'perspex', also similar to equation (2):

$$\text{Weight loss rate} \propto (V - 208)^{3 \cdot 37}$$

where V is the impact velocity in miles per hour.

This, however, refers to the velocity of a target within a given rainfall: thus the rate of water impingement increases linearly with velocity and the 'rationalized erosion rate' would be given by

$$E \propto (V - 208)^{2 \cdot 37} . \qquad (6)$$

Hoff et al[3] have preferred to express the velocity dependence in terms of the absolute velocity, as in equation (1), and state that for the majority of metals, ceramics and high polymers the erosion rate varies between the 5th and 7th power of velocity, and even reaches the 13th power for glassea. This, again, includes the effect of the increasing rate of water impingement with velocity, so that for the rationalized erosion rate of most materials the power could be between 4 and 6.

Hobbs[4] has also concluded that 'at the higher velocities the rationalized erosion rate was found to increase with the fifth power of impact velocity. Within the limits of experimental accuracy this law was the same for all five materials tested. At low velocities this relation did not hold and the results obtained suggest that for each material there may be a threshold velocity below which no damage occurs'.

As a further illustration, data presented by Hobbs in a discussion to a paper by Leith and Thompson[16] were analyzed and suitably plotted in Ref.14. It was there shown that these results, too, provide no evidence pointing toward any particular simple type of empirical formulation, but can be approximated over portions of the test range by any of the following equations:

$$E \propto V^{4 \cdot 4}$$

$$E \propto (V - 270)^{2 \cdot 4}$$

$$E \propto e^{0 \cdot 0084\, V} .$$

The foregoing cited results, together with the more detailed analyses of the original data found in the literature (see Ref.14), led to the conclusion that even the best available erosion-velocity data do not follow exactly any law such as represented by equations (1) through (3), but can, over limited ranges, be approximated by any of them. When this is done, one does find reasonable consistency in the approximate magnitude of the exponents 'n', for a wide variety of materials. If a direct power law like equation (1) is used to fit the data, the exponent will generally be between 4 and 6. If a 'threshold relationship' like equation (2) is adopted, the exponent is likely to fall between 2·3 and 2·7. (This excludes very brittle materials such as glasses, in which catastrophic failures occur above certain velocities.) In no case does it seem justifiable to use any of these simple curve-fitting equations for the purpose of extrapolating out of the test range.

B <u>Rational approaches to velocity effect</u>

None of the simple equations discussed previously has any true physical foundation, and a simple physical law is not at hand.

A purely stress-based argument could be made if it is assumed that, at moderate velocities, erosion fragments are formed as a result of fatigue failures. The impact stress may be taken to be proportional to V, if it is related to the water-hammer pressure $\rho C V$. (See section 5 for detailed discussion of impact stress.) If one assumes that erosion is taking place as a steady-state process, and that the mean size of erosion fragments is independent of V, then the volume rate of erosion E would indeed be proportional to 1/N, where N is the mean number of impacts required to generate a loose erosion fragment. In turn, N could be assumed to be related to the impact stress and hence to the velocity V in a manner similar to the relation between cycles to failure and stress in conventional fatigue tests. If these assumptions were correct, a V - (1/E) curve should exhibit similar characteristics to a S-N fatigue curve.

Now, according to Manson[17], the relationship between elastic strain range $\Delta\varepsilon_{el}$ (or stress range) and life cycles N is approximately $\Delta\varepsilon_{el} \propto N^{-0.12}$, which would lead to the analogy

$$E \propto V^{8.3} . \qquad (7)$$

This is not in accord with the bulk of experimental data, and the analogy is certainly much oversimplified. For one thing, the lateral outflow of a drop after impact contributes toward damage, as well as the stress field created by the impact presure. Moreover, the yield point for most materials is only about twice the endurance limit, whereas in erosion we are concerned with impact velocities which vary over a much greater range than that. A low velocities, damage seems to occur even though impact pressures are below the endurance limit, and at higher velocities the impact stresses may exceed elastic limits, and erosion is accompanied by considerable plastic deformation. Manson[17] states that in plastic strain-cycling fatigue the relationship between plastic strain range $\Delta\varepsilon_{pl}$ and life cycles N approaches $\Delta\alpha_{pl} \propto N^{-0.6}$, which would imply approximately

$$E \propto V^{1.7} \qquad (8)$$

if $\Delta\varepsilon_{pl}$ could be assumed to be proportional to the impact stress or V, which is not true, of course. However, the exponents of equations (7) and (8) do bracket the experimental range of $n = 4 - 6$, which we have observed earlier.

The observation that erosion is accompanied by heavy plastic deformation in the surface and in the eroded particles has led to energy-based arguments such as Thiruvengadam's theory[18] that the erosion strength, at least for ductile materials, is proportional to their strain energy to failure. It also logically leads to attempts to relate the erosion rate to the kinetic energy flux of the impinging liquid. Busch et al.[2] have pointed out that since the kinetic energy flux (for a given rate of impinging liquid) is proportional to V^2, the proportion of the energy which is absorbed by the material must also increase with V if the observed E-V relationships are to be explained.

The energy balance involved in liquid/solid impact is very complex and has not yet really been analyzed with sufficient rigor for quantitative conclusions. Part of the kinetic energy of the impinging drop will remain as the kinetic energy of the lateral outflow velocities; part will be dissipated in the shock or pressure waves passing through the drop and part in the shearing associated with the change of direction of the liquid flow; part will be dissipated in the target material, and here too, the energy dissipation associated with stress waves should be considered as well as the quasi-static plastic strain hysteresis energy associated with each impact stress cycle. The picture is further complicated by the rather large amount of energy which will temporarily be stored as elastic strain energy in the target and which will reappear in one of the previously-mentioned forms.

The energy dissipated in the target material is that associated with fracture and therefore with erosion. But it does not necessarily follow that the volume of material removed is proportional to that energy. Two reasons account for this: one is that (at least in the case of larger drops at moderate velocities) erosion fragments produced by the random linking-up of fatigue-like cracks are not likely to be deformed to the fracture point throughout their volume, and therefore the accumulated plastic strain energy may be more related to the surface area of the fragment than to its volume, or, at the least, be non-uniformly distributed within the volume. The other is that in fracture due to the repeated stressing, the total energy input increases greatly with the number of cycles to failure. This is evident in McAdams' results for impact fatigue tests[19], and has recently been documented for a large collection of fatigue data by Halford[20]. Even if one postulates that the damaging energy is the same in all cases and the excess hysteresis energy is dissipated through non-damaging processes, the fact remains that <u>all</u> of the dissipated energy is supplied by the impinging droplets and even if the energy absorption by the target material is known, that in itself will not truly establish the erosion rate.

Nevertheless, Hoff et al[3] have attempted an 'energy balance of erosion' in more detail. They define 'erosion strength' $\underline{f}$ as the ratio of impinging energy flux to volumetric erosion rate; $\underline{f}$ therefore has the dimensions of stress but is a property both of the material and of the impact conditions. Their final expression for $\underline{f}$ involves a critical stress and a critical strain of the target material, the integral under the stress-strain curve up to the impact pressure, the acoustic impedances of the target material and the liquid, and a mechanical loss integral which accounts for the non-damaging energy dissipated in the material.

A very significant point to note about the foregoing is that both stress properties and energy properties of the target material are involved, which indicates that any simple approach based purely on stress or purely on energy will not be adequate. This is consistent with the findings of Hammitt et al (e.g. 21,12) that the best correlations between the erosion resistance of materials and their other mechanical properties involve at least one stress-type and one energy-type property.

Hoff et al.[3] draw qualitative conclusions from their analysis, but do not derive any directly useful formula relating erosion almost wholly on the response of the target material, and neglects entirely the question of what proportion of the impinging energy flux is made available to the target. This is not a trivial question, since, as has been pointed out for instance by Hancox and Brunton[22], the initial high impact pressures (of magnitudes near the waterhammer pressure ρCV) decay very rapidly toward the stagnation pressure $\rho V^2/2$, and the kinetic energy represented by the mass of liquid 'arriving' at the surface thereafter probably remains entirely within the liquid. Therefore the potentially damaging energy offered to the target material will be a function not only of the total mass and velocity of the impinging liquid, but also on the size and shape of the liquid particles into which it is subvidided, and on the surface geometry of the target and the possible effect of a retained liquid film on that surface.

An analysis which is less sophisticated than that of Ref.3 with respect to the material response, but does attempt to account for some of the above-described effects, is presented in Appendix A. The general approach of this analysis is due to W. D. Pouchot of the Westinghouse Astronuclear Laboratory. It results in the relationshop

$$E \propto \left(\frac{V}{V_c}\right)^4 \left(1 - \frac{V_c}{V}\right). \qquad (9)$$

Equation (9) is portrayed graphically in Fig.3. Curve (a) represents the relative erosion rate E plotted versus (V/V_c) in order to compare its predictions with empirical data represented in the form of equation (1), and curve (b) represents E plotted versus $[(V/V_c) - 1]$ in order to compare it with empirical data in the form of equation (2). The similarity of these curves to the previously-reviewed experimental data is quite startling: in a typical set of erosion test data the impact velocities may range from $V/V_c = 1{\cdot}5$ to $V/V_c = 3{\cdot}0$. The corresponding range in terms of $(V/V_c - 1)$ is from 0·5 to 2·0. If these points on Fig.3 are joined by straight-line approximations, their slopes are, respectively, 5 for curve (a) and 2·5 for curve (b). These are exactly the average values of the exponents which have been found to represent experimental data in terms of equations (1) or (2). Not only that, but the curvatures of the predicted curves agree with those plotted from the more meticulously-derived experimental data.

This can be seen by comparing curve (a) of Fig.3 with Fig.4, which is taken directly from Hobbs[4]. Even more remarkable agreement is demonstrated by Fig.5, in which equation (9) is fitted to the data of Pearson. The data points shown are taken from the 660 μm drop curve in Fig.8 (Fig.5 of Ref.1), for which V_c was established as 390 ft/sec. The dotted line represents equation (9) with a suitably chosen constant of proportionality.

In short, regardless of whether all the assumptions which led to the derivation of equation (9) need be accepted in full, it does appear that this equation represents, better than any hitherto proposed, the experimentally observed erosion rate versus velocity relationships.

4. SOME ASPECTS OF THE IMPACT PROCESS

A Introduction

In order to develop any rational models for erosion damage, certain consequences of the impact of a liquid drop onto a solid surface must be known. In a simplified way we may list these as follows, recognizing also that they must be time dependent:

(a) The impact pressure.
(b) The area over which the pressure acts.
(c) The velocity of the lateral outflow of the liquid after impact.
(d) The approximate decay time of high impact force.

Considerable attention has been given to these questions in the literature of the last ten years, but the subject is so complex that most authors have addressed themselves to deriving broad approximations, and disputes exist concerning even some of these. It will be our intent here to summarize and put into focus some of these hypotheses, and to add some additional thoughts. (We do not propose to discuss explicitly the damage mechanisms, which have been thoroughly treated by many authors (e.g. 1,3,7,22,23,24,40), except to point out that damage can be produced by both the direct impact pressure and by the shear or impact stresses due to the lateral outflow interacting with surface irregularities.)

With regard to the impact pressure, the conceptual starting point is usually the model of two flat-ended cylinders of elastic material colliding end-on. If one of the cylinders is liquid, it must be assumed to be laterally restrained so that plane pressure waves can be sustained in it. Let the initial velocity of cylinder '0' (projectile) be V_o and that of cylinder '2' (target) be zero. Let the velocity of the interface after impact be V_1. Then the change in particle velocity in cylinder '2' is V_i and that in cylinder '0' is $(V_o - V_i)$. The mechanisms by which these changes in particle velocity are spread through the length of the cylinders are pressure or stress waves originating at the interface. For equilibrium, their magnitudes must be equal, say P. The magnitude of a stress wave in an elastic medium is related to the associated particle velocity change by the acoustic impedance Z, or $P = (\Delta V) Z$. Also $Z = \rho C$, where C is the velocity of the stress wave relative to the undisturbed material. From the above we may state:

$$P = Z_2 V_i = Z_o(V_o - V_i) \tag{10}$$

from which

$$V_i = \left(\frac{Z_o}{Z_o + Z_2}\right) V_o \tag{11}$$

and

$$\left.\begin{aligned} P &= \left(\frac{Z_o Z_2}{Z_o + Z_2}\right) V_o \\ &= \frac{Z_o V_o}{1 + \frac{Z_o}{Z_2}} \end{aligned}\right\} \tag{12}$$

If cylinder '0' is a liquid and cylinder '2' a solid of much higher acoustic impedance, i.e. $Z_2 >> Z_o$, then equation (12) reduces to

$$P = Z_o V_o = \rho_o C_o V_o \tag{13}$$

which is the simple waterhammper equation.

The real situations with which we are concerned deviate from this simple model in the following respects:

(a) The 'target is not a cylinder but a semi-infinite solid or, sometimes, a plate. The materal under the impact area is therefore partially restrained by the surrounding material and the interface velocity V_i may be less than given by equation (11). This has usually been neglected, particularly when $V_i << V_o$ as is frequently true.

(b) The 'projectile', if liquid, is not laterally restrained and when a pressure wave has reached a free boundary, lateral flow will begin, accompanied by a pressure release wave travelling inward.

(c) The projectile is not normally a flat-ended cylinder but a more or less round-ended drop, so that contact is made initially at one point and the contact area grows as the engagement of the two bodies progresses.

(d) Equations (12) and (13) assume, implicitly, that $Z = \rho C$ is an invariant for a substance. This can be assumed correct when $V << C$ and C can be taken as the acoustic velocity of the substance. At higher impact velocities and pressures this approximation is no longer valid; the wave is a finite compression shock wave whose velocity of propagation is a function of the pressure behind it and hence indirectly on the impact velocity V_o.

Let us consider the last three of the above mentioned points in turn.

B Lateral flow after impact

A graphical description of the assumed behaviour of an ideal flat-ended cylinder (of radius R) upon impact has been given, for instance, by Bowden and Field[23]. As the plane pressure wave passes along the cylinder, pressure release waves initiated from its intersection with the free cylindrical boundary and propagate inward. As a result, the area over which the full waterhammer pressure acts is progressively reduced, and at a time R/C after impact the waterhammer pressure will have been relieved over the whole liquid-solid interface. Also, the pressure gradient between the interior of the drop where waterhammer pressure still exists, and the outer boundary where the pressure must be zero, results in radial or lateral outflow from the original confines of the liquid cylinder.

The magnitude of the lateral outflow velocity V_R is a matter not entirely settled, though many independent investigators have shown experimentally that in real situations it can be several times the impact velocity V_o. According to Bowden and Brunton[24], in the ideal case of a flat ended cylinder the radial velocity should always be equal to the impact velocity, and the higher velocities observed are ascribed to a 'wedging effect' due to the tapered or rounded front surface of the liquid cylinder or drop. Some photographic evidence of this is presented. On the other hand, both Engel[25] and Jenkins and Booker[26] have assumed that, regardless of geometry, the order of magnitude of V_R should be such that its stagnation pressure ($\rho V_R^2/2$) is equal to the pressure generated under the impact. If the latter is taken as $\rho C V_o$ this leads to

$$V_R = \sqrt{2C V_o}; \quad \text{or} \quad \frac{V_R}{V_o} = \sqrt{\frac{2C}{V_o}} . \tag{14}$$

Jenkins and Booker[26] present experimental data supporting this hypothesis.

The following argument can be presented in favour of the latter view; the impact pressure $\rho C V_o$ is a result of the enforced instantaneous or step velocity change of the liquid particles, as the steep-fronted shock wave moves past them. The acceleration of the liquid particles into lateral flow, however, is not the reverse of this process. Nothing requires that acceleration to be instantaneous; the pressure-release wave is not a shock wave, since a rarefaction shock wave cannot exist in a liquid[35]. Because the acoustic velocity increases with pressure, the high-pressure 'face' of the pressure release wave moves faster into the liquid than the low pressure 'face', and the pressure release wave therefore disperses into a pressure gradient. The acceleration of the fluid particles can therefore be expected to follow the Bernouilli law, as implied by equation (14).

Since the total kinetic energy of the lateral outflow, considering the whole drop of mass 'm', can be no greater than $m V_o$, it is clear that high lateral flow velocities greater than V_o, regardless of the mechanism responsible, can apply only to a small portion of the drop mass and can occur only at the expense of an outflow velocity less than V_o for another portion of the drop.

C Impact of curved surfaces

In most practical cases with which we are concerned the impact takes place between a solid surface and a roughly spherical drop, or the cylindrical surface of a jet. The state of knowledge concerning that type of impact is still somewhat unsatisfactory.

A fairly rigorous analysis of the impact and spreading of a liquid drop on a rigid surface was presented by Savic and Boult[27], but it neglects compressibility of the fluid and therefore predicts infinite pressure at the first instance of impact. Its practical usefulness is therefore restricted to the latter stages of the collision process.

Engel[25] presented a detailed but approximate analysis for the same case, involving plane wave theory, which led to the conclusions that the maximum impact pressure was developed at some time after initiation of impact, and has the value

$$P = \frac{\alpha}{2} (\rho C V) . \tag{15}$$

The value of α was deduced to be approximately 0·4, from photographic observations of a spreading drop after impact at moderate speed.

Some of the assumptions of Ref.25 have been questioned by Bowden and Field[23], who present a simple argument to suggest that in the first stage of impact the impact pressure must still be that corresponding to elastic impact, given by equations (12) or (13). The essence of the argument can be shown with reference to Fig.6, which shows a flat rigid surface at velocity V in the process of impacting a stationary curved liquid body. In the apparently overlapping area, the liquid has actually been compressed by means of a compression wave and its particle velocity equals that of the rigid surface. If the angle ϕ between the two surfaces at the end of the contact area is small enough, then the rate at which the radius of contact grows is greater than the speed of the compression wave: as a result the liquid on the free surface has received no information of the impact and cannot flow. The condition for this is clearly that the 'edge Mach number' is greater than unity, or

$$M_e \equiv V/C \sin \phi > 1 \tag{16}$$

and if the radius of the liquid surface is R, then the limit of this condition is reached when the impact area radius R_p, is given by

$$R_{p_{max}} = R\, V/C\ . \tag{17}$$

This condition is shown on Fig.6. It may then be assumed that up to that point the impact pressure is the full value given by equation (12) or (13).

A somewhat more accurate picture can be inferred from a contribution by Skalak and Feit[28], which deals with the impact of a blunt solid body onto the surface of a semi-infinite compressible fluid. It seems reasonable to suppose that if the radius of curvature of the solid body surface is large compared to the dimensions of the contact area, the results obtained from this analysis would not be significantly altered if the curvatures were exchanged, and hence would apply to the drop impact problem in the early stages when the contact area radius is small compared to the drop radius. The geometry considered by Ref.28 is more amenable to analysis, however, and Skalak and Feit present an essentially rigorous time-dependent, three-dimensional wave equation approach, restricted to low values of the impact Mach number V/C. (The only questionable simplification is that the penetration of the body into the liquid is ignored in order to linearize the boundary conditions. The boundary condition then reduces to specifying the particle velocity imposed by the impacting body, over the time-dependent contact area but at the level of the original free surface.)

One major conclusion reached in Ref.28 is that although the contact pressure distribution is non-uniform, yet so long as the 'edge Mach number' M_e remains greater than unity the total contact force is given by

$$F = \rho C \bar{V} A \tag{18}$$

where $\bar{V}$ is the mean velocity of the solid body over the area of contact A. (The possible compressibility of the solid body is implied by the definition of $\bar{V}$.)

Thus, as far as load is concerned, this conclusion is identical with that reached from Bowden and Field's simple argument: the average pressure over the contact area is equal to that given by the one-dimensional analysis, i.e. equations (12) or (13). The important difference is that this pressure is not uniform as Bowden and Field had assumed.

The nature of the pressure distribution which can be inferred is interesting, because it is opposite to that of Hertzian contact stresses, which have sometimes been supposed to apply also to drop impact. Numerical calculations in Ref. 28 for the impact of blunt wedges show that the pressures are maximum near the edges of the contact area and minimum at the centre. The lower the 'edge Mach number', the greater is the non-uniformity. When M_e is less than unity, the calculations for the wedge case predict a pressure singularity (i.e. theoretically, infinite pressure) at the contact area edge. This suggests that the impact of a spherical drop may result in a 'ring' of high and increasing contact pressure spreading out over the target surface as the contact area grows and M_e diminishes. (Eventually, of course, the high pressures will be relieved as the result of tangential outflow.)

The physical explanation for these results may correspond to Engel's[25] assumption that the pressure will be increased when 'wavelets' propagating from the earlier contact points catch up with the growth of contact area and reinforce the newly-generated pressure 'wavelets'. The important difference from Engel's conclusion is that the average pressure during the early stages is not less than that for the one-dimensional case.

In the later stages, i.e. when $M_e < 1$, the detailed results of Ref. 28 are probably not transferable to the drop impact case. Intuitively it would appear that the actual opportunity for lateral outflow grows rather slowly at first. Thus it is not entirely surprising that Hancox and Brunton[22] found the characteristic surface tearing damage, attributed to high-speed lateral flow, to begin only at much greater distances from the impact centre than is predicted by equation (17). What is surprising, however, is that these distances were independent of impact velocity and subtended an angle $\phi = 17$ degrees in drops (actually jets) of different sizes, whereas equation (19) predicts ϕ and $(R_p)_{max}$ to be proportional to velocity. Measurements of maximum impact load, reported in Ref. 22, also implied that the effective impact area was independent of impact velocity. No full explanation was offered. The question of the maximum effective impact area for a curved surface must therefore be left unresolved at present. The same applies to the time duration of the maximum impact pressure, which is closely related to the previous considerations.

Further confirmation of the ability of equations (12) or (13) to predict the the impact pressures for liquid drops has been offered by Jenkins and Booker[26]. They determined photographically the radial flow velocities after impact, over a wide range of impact velocities up to some 3700 ft/sec, and found that the corresponding pressure calculated from equations (12), using the appropriate shock wave velocities.

Additional evidence for high impact pressures, of magnitudes at least equal to $\rho C.V$, was given by Jolliffe[29], who deduced the loads and impact areas from the size of dislocation rosettes produced on lithium crystals by the impact of liquid drops.

Some final comments may be appropriate, concerning the dilemma posed by the factor $\alpha/2$ in equation (15), proposed by Engel[25]. The α factor has also been discussed by Engel in Ref.30, and in a discussion to Ref.18. Ref.25 suggested that α would tend toward unity as V increased, but Ref.30 amended this and proposed the relation

$$\alpha = \frac{0{\cdot}41}{1 + 0{\cdot}59(Z_o/Z_2)} \qquad (19)$$

where Z_o and Z_2 are the acoustic impedances of drop and target respectively. Therefore, it was stated in the discussion to Ref.18, the maximum impact pressure on a rigid surface cannot exceed about $0{\cdot}2\ \rho V C$. This, of course, contrasts strongly with the conclusions of Refs.23, 26, 28 and 29. Two points can be made to help resolve the discrepancy:

(a) The experimental observations which led to the assigning of numerical values to α both involved the complete collision process: in Ref.25 this was the time taken for a drop to spread completely, and in Ref.30 the depth of pits created by drop impacts (usually of mercury) on soft metal targets. Thus any deductions made therefrom would seem to apply better to the time-average impact pressure over all stages of the collision, than to the maximum pressure which will decay as a result of radial outflow.

(b) The experimental observations which led to equation (19) are susceptible to an alternate explanation. Equation (19) was deduced by comparing the empirical constants in prediction equations for the pit depths formed in soft metals by impacts of mercury drops and steel spheres[31], which involved the time during which the impact pressure exists. That time was taken to be $2D/C$, where D is the drop or sphere diameter and C the acoustic velocity in the projectile material. However, the assumption that this applies to both cases is open to question. In the rigid sphere case, this represents the time for a compression wave to travel through the sphere, reflect, and return as a tension wave to relieve the stress at the contact face. In a liquid projectile, however, Bowden and Field[23] suggests that the presure release waves come from 'the nearest free boundary', so that the impact time will be shorter. In the case of a heavy drop forming a deep pit, it may be reasonable to suppose that pressure waves would have to travel almost to the 'equator' of the drop and reflect from there; this could be consistent with an impact duration of about 0·8 D/C, which would fully explain the difference between the two empirical constants without requring modifications to the expression for impact pressure.

In summary, the following principal observations apply when the impacting liquid has a curved surface:

The contact area begins with a point (or line) and increases as the engagement between the two bodies continues. Thus the total impact force begins at a zero value and, at least in the first stages of the process, increases proportionately with the area, until in the later stages it is decreased because of the relief of the interface pressure through lateral flow. This is in contrast to the case of the flat-ended cylinder impact, where the force must rise almost instantaneously to its maximum value and then immediately begin to decay.

The average impact pressure over the contact area should be the same as that for a flat-ended liquid column at the same velocity, until substantial lateral flow takes place. This can occur only after the 'edge Mach number' has dropped to below unity. Theory suggests, however, that a ring of pressure very much higher than the average may be developed near the boundary of the contact area, before lateral flow develops.

D <u>Shock-wave velocity and impact pressure at high impact velocities</u>

Many authors have pointed out that at high impact velocities the value of C implicit in equations (10) through (13) can no longer be approximated as the acoustic velocity of the undisturbed liquid, but must be taken as the shock wave velocity corresponding to the pressure rise created by it.

To clarify this conceptually, one may imagine a long cylinder of liquid impinging end-on upon a rigid surface: a compression shock front, which forms the boundary between the liquid brought to rest and that still moving, then leaves the plane of impact and propagates upstream through the liquid cylinder at a velocity C relative to the approaching liquid. From simple momentum considerations one can then deduce the relationship $P = \rho C V$, but it is also obvious that C must always exceed V no matter how great the latter, since otherwise the shock front could not move away from the impact surface and no 'storage space' would be created for the liquid brought to rest.

The exact relationship between the pressure rise across a shock front (P) and its velocity of propagation (C) is complicated and involves the thermodynamic properties of the medium at high pressures. Most of the theoretical and experimental work on this subject has been concerned with underwater detonations. Among the most basic theoretical work appears to be that of Kirkwood and his associates[32,33,34], which has been drawn upon (at least in part) by many subsequent contributors such as Cole[35], Richardson et al.[36], Rice and Walsh[37], and Cook et al.[38]. All of these have tabulated shock front velocities C (and sometimes particle velocities V) versus pressures, but no closed-form analytical formulations are given.

It would be desirable, however, to have even an approximate simple explicit relationship for C as a function of the particle velocity V, which could be used to calculate P directly and be incorporated into the other analyses which involve C or P. Such a relationship is derived in Appendix B, on the basis of a simple physical argument. It can be expressed in the form

$$\frac{C}{C_o} = 1 + k\left(\frac{V}{C_o}\right) \tag{20}$$

where C_o is the acoustic velocity of the liquid at atmospheric conditions, and k an empirical constant which can most easily be determined by plotting the exact tubular data in the form $(C/C_o) - 1$ versus V/C_o. Data for water from several of the previously cited sources has been so plotted in Fig. 7, and it may be seen that the points do follow very nearly a straight-line relationship at first, although the slope begins to diminish at high impact Mach numbers (V/C_o). A line of slope $k = 2$ remains within the scatter band up to about $V/C_o = 1{\cdot}2$, which is well beyond the range of practical liquid impact conditions now being considered.

It must be emphasized that in equation (20), V represents the particle velocity change in the liquid drop. For the case of liquid impact on a <u>rigid</u> surface this corresponds to the impact velocity V_o and equation (20) can be simply combined with equation (13), to yield

$$P = \rho_o C_o V_o \left(1 + k\frac{V_o}{C_o}\right) \tag{21}$$

where V_o is the impact velocity, and ρ_o the density of the undisturbed liquid.

When the compressibility of the target material must also be considered, then the situation becomes more complicated. The first assumptions we shall make is that the target material responds elastically and that the stress wave velocity in it, C_2, can be approximated by its acoustic velocity. Although these will not always be true, the response of a metal to impact in which plastic deformation occurs is too complicated to be considered here; it involves both elastic and plastic stress waves, propagating at different velocities.

Let us introduce the following additional notation:

$M_o \equiv V_o/C_o$ = impact Mach number

$V_s \equiv V_o - V_i$ = effective change of particle velocity in liquid drop

C_s = velocity of shock wave in liquid drop.

We may then re-state equation (20) in the following form

$$\frac{C_s}{C_o} \cong 1 + k\, M_o \left(\frac{V_s}{V_o}\right) \qquad (22)$$

and, since $p = \rho_o\, C_s\, V_s$, we can state with complete generality that

$$\frac{p}{Z_o\, V_o} = \frac{C_s}{C_o}\,\frac{V_s}{V_o} = \left(1 + k\, M_o \frac{V_s}{V_o}\right) . \qquad (23)$$

If certain conditions are met, the interaction between the Mach Number correction and the target compressibility correction can be neglected, and V_s/V_o can be taken as identical to that implied by equation (12), which is

$$\frac{V_s}{V_o} \simeq \frac{Z_2}{Z_o + Z_2} = \frac{1}{1 + (Z_o/Z_2)} . \qquad (24)$$

It is shown in Appendix B that the condition which justifies this approximation is

$$\frac{k\, M_o}{2 + \dfrac{Z_2}{Z_o} + \dfrac{Z_o}{Z_2}} << 1 . \qquad (25)$$

When this condition is not met, the exact value of V_s/V_o (which is given in Appendix B) may be used, or a better approximation also derived in Appendix B is

$$\frac{V_s}{V_o} \simeq \frac{(Z_2/Z_o) + k\, M_o}{1 + (Z_2/Z_o) + 2k\, M_o} . \qquad (26)$$

The above approximation should be valid for most practical cases when $M_o < 1$ and $Z_2 > Z_o$. For instance, with $M_o = 0{\cdot}5$ and $Z_2/Z_o = 4$, the exact value of V_s/V_o

is 0·702, that from equation (26) is 0·714 and that from equation (24) is 0·8. Note that the error is magnified in the calculation of pressure since V_s/V_o occurs to the second power in equation (23).

In summary, a close approximation to the relationship between shock wave velocity and particle velocity change is given by equation (20), with k = 2 for water. From this the impact pressure on a rigid surface follows directly as equation (21). When target compressibility must be considered, the non-dimensional particle velocity change in the drop can be obtained from equations (26) or (24), and substituted in equations (22) and (23) to compute the non-dimensional shock wave velocity and impact pressure.

For some exploratory purposes it may be desirable to use an even more approximate form of the relationship between P and V_o for impact on a rigid surface, in the form $P \propto V_o^n$. When equation (21) is plotted on log-log coordinates in non-dimensional form, one finds that it can reasonably be approximated over the range $0{\cdot}06 < M_o < 0{\cdot}6$ by

$$\frac{P}{\rho_o C_o^2} \simeq 2{\cdot}4\, M_o^{1{\cdot}3} .$$

This expression also implies the approximation

$$\frac{C}{C_o} \simeq 2{\cdot}4 \left(\frac{V}{C_o}\right)^{0{\cdot}3}$$

which, however, deviates proportionately even more from the 'exact' relationship of equation (20). Both of these have been chosen so as to agree with equations (20) and (21) at the points $V/C_o = 0{\cdot}1$ and 0·5. The exponent 1·3 agrees with an approximation deduced in an unpublished report by D.Pearson. For very low values of M_o such a formulation must fail, however, and the above expression yields values of $C < C_o$ and $P < \rho_o C_o V_o$ when $M_o < 0{\cdot}055$.

5 EFFECT OF DROP SIZE

A Empirical findings

Despite the fact that the maximum impact stress is generally a function of the material properties and the impact velocity, and should be independent of the size of the impacting drops, there is ample evidence that both the size and shape of the impacting liquid masses do affect the erosion measured. However, the quantitative data in the literature from which generalized relationships could be deduced is very scant.

A frequently cited test is that of Honegger[6], in which he compared the erosion produced in a wheel-and-jet type apparatus by impact with one 1·5 mm water jet, with that produced by nine 0·5 mm jets, arranged in a closely-stacked triple-echelon. The results are described as follows: "The splitting up of the jet is accompanied by a considerable reduction of the erosion, the numerical value of the reduction largely depends upon the speed, and for tests under consideration it varies from 1 to 5 for high speeds and 1 to 10 for low speeds". The test was so contrived as to fulfill the requirements of a rationalized erosion measurement: both the target area subjected to erosion and the volume of impinged water were the same for both configurations. Yet, upon reflection, one must conclude that this was not an entirely valid test of the drop size on a dry surface; a liquid layer from these would almost certainly still be present to cushion the effect of the next three impacts, and similarly so for the last three. Thus no quantitative conclusions should be drawn from these results, though the qualitative findings are of interest.

Systematic tests, also in a wheel-and-jet apparatus with jet diameters varying from 4 mm to 12 mm, were reported by Brandenberger and DeHaller[7] and showed apparently strongly increasing erosion rates with jet diameter. However, when the results are put into rationalized form (see Ref.14) only a fairly weak dependence on jet size is found.

Hobbs[4] also has reported tests in a wheel-and-jet apparatus, with six different jet diameters ranging from 1 mm to 2·5 mm, and he found no difference in the rationalized erosion rate. Hobbs did find, however, that with jet diameters below 1·6 mm, the incubation period lengthened appreciably. A corresponding observation was made by Hancox and Brunton[22], who found that the number of impacts to initiate cracking in polymethylmethacrylate targets varied almost inversely as the jet diameter, in the diameter range of 0·4 mm to 2 mm.

The 'threshold velocity' below which no damage occurs has also been found to depend on drop or jet size. Vater[15] presented a curve (his Fig.22) in which the threshold velocity for repeated impacts appears to vary approximately in the relation

$$V_c^2 D = \text{constant} \qquad (27)$$

where V_c is the critcal velocity and D is the jet diameter.

DeCorso[39] found that the 'visible damage threshold' velocity in single impact tests depended on jet diameter. For example, in copper the threshold velocity with a 1·5 mm jet was 500 ft/sec (150 m/sec), and that with a 0·25 mm jet was 2000 ft/sec (600 m/sec). In stainless steel, they were approximately 850 ft/sec, 950 ft/sec and 1150 ft/sec respectively with jets of 1·5 mm, 1·0 mm and 0·5 mm diameters. Measurements of pit depths produced above the threshold velocity led DeCorso to conclude that "given an equal amount of fluid impacting, for the higher velocities well above the damage threshold, drop size of the fluid does not affect the damage, but that, at velocities near the threshold, damage increases with drop size."

While the observations cited above are not in clear agreement concerning the influence of drop size once erosion has begun, the very comprehensive test data of Pearson reported in Ref.1 exhibit an undisputable drop size effect. These data, obtained ina wheel-and-spray apparatus, are reproduced in Fig.8. Although this shows an anomaly in the crossing of the 920 μm and 1050 μm lines, it seems to confirm that the relative effect of drop size diminishes at high drop sizes and high velocities - i.e. as one gets away from what may be considered the 'threshold' conditions.

We have found two ways in which Pearson's data can be effectively correlated, and both are based on the assumption that the threshold velocity is related to drop size as suggested by equation (27). The threshold velocity of 390 ft/sec cited earlier (equation (5)) was actually obtained with drops of 660 μm diameter, from which the threshold condition $V^2 D = 10^8$ can be deduced for these tests, where V is the feet per second and D in microns.

One successful correlation approach is to plot the erosion rates versus a 'corrected velocity' defined as $K_C V$, where K_C is a 'critical factor' given by $K_V = 1 - 10^8/V^2 D$. This has been done in Fig.9, and as can be seen the different curves of Fig.8 are neatly collapsed into one.

Another correlation approach, more consistent with the velocity formulations discussed earlier, is to plot the erosion rates versus the 'reduced velocity' $(V - V_{cd})$, where the critical velocity V_{cd} is now a function of drop diameter D and is given in this instance by $V_{cd} = \sqrt{10^8} D$. This has been done in Fig.10 and also proves quite successful, though the scatter is somewhat greater than in Fig.9.

However, these data do <u>not</u> coalesce into one curve when plotted versus V/V_{cd}, which they should if both equation (9) and equation (27) were correct. We must therefore admit that a successful model incorporating both velocity and drop size dependence still eludes us.

B Physical reasons for drop size effect

One might ask why there should by a drop size effect at all. According to the discussions in section 4, the maximum pressure should be independent of drop size, and the area over which it acts ought to be related in a straight-forward manner to the drop dimensions if similarity is preserved, so long as the impact velocity remains constant. That is, the load should vary as D^2 with spherical drops, and with D in the sideways impact against cylindrical jets as is common in test apparatus.

Even this conclusion, however, has been contradicted by load measurements obtained by Brunton[40] and by Hancox and Brunton[22]. In these tests cylindrical jets were struck by a barium titanate pressure transducer acting as the target. While the loads were fairly linear with velocity, the variation with jet diameter appears to be to the 1·15 power in Ref.22 and to the 1·4 power in Ref.40. The reason for this is by no means obvious, and no explanation is offered in Refs.22 and 40.

To begin a search for the causes of a drop-size effect, one might ask the following question: what properties of the impacts, or of their effect on the target surface, vary when one reduces the size of droplets into which a given amount of water, impinging on a given target area in unit time, is subdivided?

The total area subjected to the impact pressure actually increases, since the number of drops increases as D^{-3} while the impact area per drop decreases as D^2, assuming similarity. This implies that each target area element will actually be subjected to a greater number of stress pulses. Thus, if merely the impact stress level and number of stress cycles were the governing criterion, then erosion would be expected to <u>in</u>crease with <u>d</u>ecreasing drop size, which contradicts all experience.

It is not really known, however, what the true criterion is, either for the onset of erosion damage or for the loss rate once erosion begins. While some general correlation attempts have been made between the endurance limit and the ρ C V values corresponding to threshold velocities found through experience[18], it has also been shown[22] that surface deformation can occur at ρ C V values far below the yield point. It cannot be assumed, therefore, that impact stress is the sole criterion. It may be significant, for instance, that another consequence of the increased total impact area (when drop size is reduced) is that the total kinetic energy 'arriving' per unit impact area is reduced in proportion to drop size. Also the time duration of impact stresses is lesser with smaller drops.

A number of mechanisms or phenomena may be considered which might result in the damage potential of a drop decreasing more rapidly than its volume. Broadly speaking, these may be divided into considerations relating to the attenuation of the impact itself, considerations relating to the time duration of impact and to the energy transfer between drop and target, and considerations relating to size effects on the target properties. Let us briefly discuss these in turn.

Impact attenuation

Hoff et al[3] found that drop size had little influence on rationalized erosion rates when drop diameters exceed 1 mm, but became increasingly significant with smaller drops. They ascribed this to deceleration of small drops by the pressure and velocity gradients in the air ahead of the moving specimen, and therefore concluded that 'the essential part of the drop-size dependence in rain erosion can be attributed to a velocity dependence'. Ref.3 does not offer a quantitative estimate of this effect, but calculation methods are available by which this effect can be evaluated, at least for cylindrical targets and subsonic velocities[41]. Such an effect may be particularly marked at supersonic target velocities when the raindrops must penetrate the shock-wave ahead of the target. Jenkins[42] has considered the possible disintegration of larger drops under those conditions, but has not explicitly derived the size and velocity of the drop at impact.

The deceleration and/or disintegration mechanism will be of significance for targets moving in air, as in actual or simulated rain erosion. It is less likely to play a role in actual or simulated low-pressure steam turbine blade erosion, where the ambient condition is steam at very low density. Here another attenuation mechanism may be of considerable importance, namely the presence of a residual liquid film on the blade or target. If the film thickness depends only on the total rate of liquid impinging, its attenuation effect obviously increases for smaller drops. This effect has been analytically explored by Pouchot[43], with particular reference to the test conditions and results reported in Ref.1. The conclusion was reached that the effect on the primary impact pressure and area is probably small, but the effect on the damaging potential of the high-speed lateral flow after impact could be significant.

Either of these attenuation effects could conceivably account for the non-linearity of load with jet diameter, cited earlier from Refs.22 and 40.

Still another attenuation effect may be that due to the interaction between drop size and the surface roughness, particularly when created by the erosion itself. In Ref.11 it was suggested that when the surface irregularities are small compared to drop diameter, they provide a greater opportunity for attack by the lateral flow after impact. However, when the height of irregularities becomes comparable with drop dimensions, then the lateral flow will be disrupted and the initial impact itself will often be attenuated when it occurs on a sloping surface. This was proposed as one explanation for an increase and then decrease of erosion rate as the surface becomes more roughened, but it can also be used as an argument that, for a given roughness, smaller drops will have a lesser damage potential than larger ones.

Time and energy transfer effects

Time and energy transfer effects are generally interrelated. The basis for all of these is the generally accepted fact that the duration of the pressure pulse exerted on impact will depend on drop size, since it is determined by the time required by a pressure release wave to initiate or reflect from a free surface and propagate into the contact area. In addition, theoretical considerations discussed earlier suggest that, in a spherical drop, both the maximum contact area of high impact pressure, and the duration of that pressure, are functions not merely of the drop size but also of the impact velocity.

Hypotheses concerning energy transfer may be adapted from the argument used by Engel[31] to predict pit depths due to impacts on soft metals. This states that the total energy imparted to the target by a single drop impact is proportional to the impact pressure, to the area over which the pressure acts, and to the displacement of the interface which is the product of the particle velocity (V_i) given to the interface and the time duration of the impact pressure. If this is combined with some assumed criterion for establishing the damaging portion of that energy, then a threshold relationship will result which may involve drop size well as velocity, depending on the assumptions adopted both for the critical criterion and for the impact area and time duration. The threshold criterion may be, for instance, a critical stress (or particle velocity), or a critical total displacement, or perhaps a critical energy input per unit area.

The critical criterion may also be a time interval itself. Rinehart and Pearson have stated (Ref.44, page 18) that "certain materials, notably those which exhibit a definite yield point in their static stress-strain curve, have a measurable time delay associated with the initiation of plastic deformation". This delay time appears to be a function of stress and temperature, and, if comparable to the duration of the pressure pulse, it obviously could reduce quite drastically the ability of the impact pressure to do work upon the target material.

A hypothesis based on a 'threshold energy density per impact' was described in Ref.14 and led to the prediction that erosion rate is a function of $V^2(1 - Q/V^2 D)$, where Q is a critical constant. While this relation does not fit the observed velocity dependence of erosion, it did lead to the successful correlations of the erosion rates with different drop sizes, as described in the previous section. Our real knowledge of what occurs and what criteria are important is still so uncertain, however, that all such hypotheses are at present very much speculative, and at best can suggest directions for further inquiry.

Effects related to size of impact area

A number of possible drop size effects are related primarily to the size of the impact pressure area and of the quasi-static stress field created by the impact.

One of such could be the strain rate effect which may exist for plastic wave propagation in certain metals, although this still appears to be a point of controversy for researchers in that field (e.g. Ref.45). The strain rate for the usual type of longitudinal impact tests is defined as the particle velocity divided by specimen length. For impact on an essentially semi-infinite body, the 'specimen length' could perhaps be related to the depth of the quasi-static stress field; with smaller drops this will be smaller and will therefore result in a higher effective strain rate. If there is a strain rate effect, such that for a given strain the stress increases with strain rate, then the total strain and relative energy transfer for a given impact velocity would be less with smaller drops.

Another possible cause for a drop size effect, based entirely on quasi-static considerations, is that the impact areas may well be small enough where a size effect of the material itself becomes important. We know that the impact area at the moment of peak pressure will be a small fraction of the projected area of the drop. Size effects have been found in the values of endurance limits of notched specimens, and this has been explained by Peterson[46] by the argument that for fatigue failure to occur, the endurance limit must be exceeded not merely at a 'point' or 'line' but across a dimension which is on the order of 0·002 to 0·003 inch. Since erosion damage can be akin to a fatigue process, a similar size effect could be expected. It is noteworthy that size effects have been found in other material removal processes: Backer et al[47] discovered a large increase in the shear energy required to remove a unit volume of material, as the chip size (or depth of cut) decreases in turning, micro-milling and grinding operations; the depth of cut in these tests ranged from about 0·010 inch down to 2×10^{-5} inch. These findings have been considered by Finnie[48] to be of relevance to erosion by solid particle impingement.

A final argument relates to the probability that larger drops will create larger erosion fragments. This has not been definitely established, to our knowledge, and would be a suitable subject for an experimental study. (Hoff et al.[3] did show that the surface roughness pattern is much coarser when caused by larger drops.) The argument supposes that while the volume of removed material is probably a function of the damaging energy input to the target, it is not necessarily proportional to it. If it is true that (at least at intermediate impact velocities) erosion fragments are formed by the intersection of fatigue cracks propagating from existing erosion pits or stress concentration points (1, 3, 11, 22), then the plastically-deformed portion of the total volume of an erosion fragment is likely to decrease with increasing fragment size, and the energy input required per unit volume of eroded material will decrease correspondingly. This argument may apply primarily to larger erosion fragments.

Even more generally speaking, regardless of how the fragments are formed, the total free surface created per unit eroded volume increases with decreasing particle size. Since the formation of new surface requires energy, the energy input to create smaller fragments must be greater than that to create larger fragments of the same total volume. Rabinowicz[49] has shown that, while the surface energy becomes comparable with volume energy only in very small particles; it does play an important role in the formation of wear particles. Ref.49 indicates that wear particle dimensions are on the order or 100 μm; erosion fragments may well be in the same range. (Walsh and Hammitt[50] have measured cavitation erosion particle sizes and found those in the 12 μm to 75 μm range to predominate. Impingement erosion particles, however, may be larger.)

If either of the two foregoing arguments holds, and if it is true that fragment size is related to drop size, then a drop-size effect on erosion rate would follow.

In conclusion, we can only say that a large number of possible reasons for a drop size effect suggest themselves, and their closer analysis would be required to determine which will be the most significant under any particular set of conditions; the discussions do suggest that some will be more important with respect to tangential flow damage, others with fatigue damage, and still others when each impact produces substantial plastic deformation. It should be remembered that any mechanism which results in diminished damage from smaller drops, must more than compensate for the theoretically increased impact area associated with smaller drops, when the total volume of impinging water remains the same.

6 EFFECT OF IMPINGEMENT ANGLE

Only recently have investigators made quantitative estimates of the influence of impingement angle. The consensus appears to be that the normal component of the impingement velocity is primarily responsible for the damage, with the tangential component playing a secondary role, if any.

Thus, according to Fyall et al.[5], for initially smooth surfaces the normal impact velocity can be used successfully for correlations valid during the initiation and earlier stages of erosion, but when the surface has been roughened by erosion, the tangential component also becomes significant because the true local impact angles can become more normal to the absolute velocity. No quantitative estimate is made for the latter effect.

Busch, Hoff et al.[2,3] indicate that the normal component of velocity governs the erosion rate even up to substantial damage levels. They show loci of equal average erosion rates plotted on a field of absolute velocity versus angle of incidence, which closely correspond to loci of constant normal velocity component ($V_n = V \cos \theta$). Absolute velocities ranged from 600 to 120 km/hr, and angles of incidence from 0 to 60 degrees.

Hobbs[4] has reached an equivalent conclusion, documented in a different way. He has plotted the erosion rate (E) versus the cosine of the angle of incidence, with the absolute velocity (V) held constant: these showed that E varied as the fifth power of cos θ, which agrees with his previously-established relationship between E and V under normal impact conditions. The angle of incidence in these tests was varied from 0 to about 40 degrees.

Pearson, in Ref.1 has proposed a somewhat more complex empirical relationship, implicit in his correlation equation which can be expressed as follows:

$$E \propto (V \cos \theta - V_c)^n / \cos \theta \, . \qquad (28)$$

The physical meaning of this equation is that erosion is in the first instance a function of the normal component of the impact velocity, but that the additional erosion due to a tangential component is accounted for by the $1/\cos \theta$ multiplier.

To show some of the consequences of equation (28), Fig.11 plots the ratio of erosion rate at angle θ to that at normal incidence, (E_θ/E_0), versus θ for several absolute velocities. The values for V_c and n are taken as 400 and 2·6 respectively, as given in Ref.1 for 12% chromium stainless steel.

Some independent support for this formulation may be provided by data points also shown in Fig.11, which have been deduced from erosion-time curves given by Busch and Hoff[52]; these were obtained in a supersonic rain erosion facility, with target cones of different angles, but same base diameter. The material was pure aluminium; the absolute impact velocity was Mach 1·2, or approximately 1320 ft/sec. The critical velocity V_c for aluminium would certainly be far lower than that for stainless steel - perhaps on the order of 100 ft/sec. If one computes E_θ/E_0 from equation (28) with $V = 1300$ and $V_c = 100$, n remaining 2·6, one obtains curve D, which fits the data points reasonably well.

One interesting result in the literature is at variance with the conclusions of all of the previously-cited references: Brandenberger and DeHaller[7] tested one material in a relatively low-speed wheel-and-jet apparatus at various combinations of specimen velocity (V) and jet velocity (U). The 'jet velocity' in a wheel-and-jet apparatus is in a direction perpendicualr to the specimen velocity, and the absolute impact velocity is given by $W^2 = V^2 + U^2$. Thus the velocity W is inclined at an angle, $\theta = \tan^{-1}(U/V)$, from the normal to the specimen surface. The measured erosion rates varied significantly with tangential velocity U, even when the normal velocity V was held constant.

The original data of Ref.7 were analysed in Ref.14, and it was there shown that:

(1) When plotted against normal velocity V, there is a different curve for each value of U. A correction based on Pearson's assumption ($E_{V,\theta} = E_{V,o}/\cos\theta$) did not suffice to bring them into line.

(2) When the data are plotted against the absolute velocity W, they fall quite well into one curve.

The search for the cause of this apparently anomalous behaviour may lead to a more general understanding of the behaviour of liquid particles in oblique impact. The significant difference between the test conditions of Ref.7 and those of the other cited investigations is that the body of impacting water (the jet) was essentially infinite in the direction of the tangential component of absolute velocity.

In summary, it appears that under the usual impact conditions the normal velocity component can be considered to govern the erosion rate, at least as a first approximation. The data and correlation equation presented by Ref.1 suggest, however, that the tangential component of impact velocity does result in some increase of the erosion rate, while in the particular test configuration of Ref.7 the erosion rate is found to depend on the absolute impact velocity. Once a surface is strongly roughened, however, local impact angles may well be normal and lead to an increasing significance of the absolute value of impact velocity.

7 SUMMARY

In order to deduce rational or empirical relationships between erosion rate and other parameters, it is first necessary to define what quantity one chooses to represent the erosion rate, since the latter generally varies during the course of an erosion test. The parameter adopted here, consistent with the approach used by many other authors, is the approximate maximum erosion rate 'rationalized' by being expressed as mass or volume of impinged water, both per unit exposed area. This is termed the 'rationalized erosion rate, E'.

Examination of the experimental data relating erosion rate to impact velocity V, leads to the conclusion that the velocity dependence can be represented approximately, over the greater part of the test velocity ranges, by any of several simple expressions, such as:

$$E \propto V^n \qquad \text{(usually with } 4 < n < 6\text{)}$$

or

$$E \propto (V - V_c)^n \quad \text{(usually with } 2{\cdot}3 < n < 2{\cdot}7\text{)} .$$

None of these, however, fit well enough to represent a 'law', nor can they be reliably used to extrapolate out of the test range. A somewhat more complicated expression appears to exhibit more closely the character of the observed erosion-rate versus velocity curves. It is:

$$E \propto V^4 \, [1 - (V_c/V)] .$$

This expression derives from a hypothesis relating to the energy available in the compressed high-pressures region of the drop during impact.

Before going on to consider the dependence on drop and angle of incidence, a closer look at the impact process itself seemed desirable. The idealized case of the elastic impact between two rods provides a convenient model for deriving the maximum impact pressure given by

$$P = \frac{Z_o Z_2 V}{Z_o + Z_2}$$

where Z_o and Z_2 are the acoustic impedances of the two rod materials. The impact of a liquid drop onto a larger solid target differs, however, in several major respects from this model. Three of these considerations are discussed in more detail. They are the lateral outflow after impact, the effect of the founded impact surface of the drop, and the dependence of the shock wave velocity C in the drop on impact velocity.

It is concluded that the maximum velocity reached by the lateral outflow during impact should be that corresponding to isentropic expansion from the maximum impact pressure. The outflow itself results in a progressive diminution of the average impact pressure until this reduces to the stagnation pressure $\rho V^2/2$.

The magnitude of the impact pressure generated under a rounded drop has been subject to disagreement, but the most nearly applicable rigorous theory as well as most of the available experimental data suggest that the average contact pressure, during at least the 'first stage' of impact, is still given by P as defined previously. This 'first stage' is that during which the boundaries of the contact area move outward at a speed greater than C, so that the liquid beyond the contact area has no warning and cannot flow outwards to relieve the impact pressure. Experimental evidence suggests that the time duration and area of maximum pressure grow substantially even beyond this point. Theoretical evidence suggests that pressures are maximum at the periphery of the contact area, minimum at the centre.

At high impact velocities the pressure wave propagating from the impact interface through the liquid becomes a shock front and its magnitude is a function of the impact velocity. A simplified rational argument leads to the approximate expression

$$C = C_o + kV$$

where C_o is the acoustic velocity of the undisturbed liquid. Suitable plotting of tabular data in the literature shows that this expression fits the data quite well up to $V/C_o = 1 \cdot 2$, with the constant $k = 2$ for water. Corresponding expressions are derived for the impact pressures, both on rigid and on compressible targets. A much rougher approximation which may be useful for some purposes is $C/C_o = 2 \cdot 4 \, (V/C_o)^{0 \cdot 3}$. This however, is not physically sound and should not be used for V/C_o less than 0·06 or greater than 0·6.

The experimental evidence concerning drop size effect is not entirely consistent, when the erosion rates are rationalized - i.e. based on equal total rates of water impinging. The preponderance of the evidence suggests that more erosion does result from larger drops, this difference diminishing at higher impact velocities. This suggests that the threshold velocity V_c is a function of drop size, and some experimental data in the literature can be explained with the assumption that the threshold or critical condition is given by $V^2 D = \text{constant}$, where D is the drop diameter. Many possible physical reasons for a drop size effect suggest themselves, and are discussed in three categories:

(a) Considerations relating to the cushioning of the impact velocity, as by the pressure gradient in the ambient gas ahead of the target if it is moving, or the retained liquid film on the target.

(b) Considerations relating to the time duration of high impact pressures and the mechanism of energy transfer between the drop and the target.

(c) Considerations relating to the size of the impact area and sixe effects in the strength properties of the target material.

Lastly the dependence on impingement angle is reviewed. The literature is almost unanimous in stating that the rate of erosion is determined principally by the normal component of the impact velocity, though some authors have found that the tangential component contributes to a somewhat increased erosion rate. In one test where the impacting liquid was a jet of 'infinite' length in the tangential velocity direction, the erosion rate was a function strictly of the absolute impact velocity.

In brief, we have tried to generalize, where possible, from the experimental and theoretical observations found in the literature, and have proposed some additional hypotheses to justify these observations. It seems that many of the experimentally-observed dependencies are explainable in terms of at least tentative physical models; these are still fragmentary, however, and a complete and consistent rational formulation for the erosion rate in terms of all impact parameters is still elusive. It is hoped that the generalizations and hypotheses presented here may assist in the further search for a more complete model.

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Appendix A

A HYPOTHETICAL MODEL FOR VELOCITY DEPENDENCE OF EROSION RATE

It has been hypothesized by a number of authors that the rate of material removal in impact erosion should be proportional to, or at least related to, the impact energy transferred to the target material above some threshold criterion. One approach is to consider the work done on the target, by the impact force, moving at the target particle velocity, during the time duration of the high impact pressures. This assumes that the dominant damaging mechanism is the direct impact pressure. It has also been suggested, however, that the dominant damaging mechanism can be the violent radial or lateral outflow from the high-pressure region in the drop, which in turn impinges upon surface irregularities which either existed before or were the result of deformation caused by the direct pressure.

It may be reasonable to propose, therefore, that the damage potential of the drop could be represented by the work done relative to the drop itself, by the impact pressures in excess of those corresponding to a threshold velocity. This will be proportional to the energy available, in the highly compressed drop liquid, to do damage to the target by whatever mechanism or combination of mechanism this may be accomplished.

In general terms, the work done on the drop is equal to the external impact force acting on the drop multiplied by the distance (relative to the drop) through which this force moves. More precisely - since the impact pressure and impact area are time dependent - it will be given by the double integral of impact pressure over impact area and over time, multiplied by the velocity of the external force relative to the drop, which is the impact velocity V if the compressibility of the target can be neglected. For our purposes we may suppose it to be proportional to the product of a characteristic pressure P, a characteristic area A, a characteristic time T, and the impact velocity V. Thus,

$$W \propto P.A.T.V \, . \qquad \text{(A-1)}$$

To introduce a threshold condition, the characteristic pressure will be taken as the impact pressure in excess of that corresponding to a critical velocity V_c, or

$$P = \rho \, C(V - V_c) \, . \qquad \text{(A-2)}$$

The characteristic area may be taken as the maximum area over which the maximum impact pressure must act. If we adopt the hypothesis cited in section 4C of this paper, which is represented by equations (16) and (17), then

$$A \propto \left(\frac{DV}{C}\right)^2 \tag{A-3}$$

where D is the drop diameter.

The characteristic time may be taken as that time after which the impact area A has been attained. This can be calculated by considering the corresponding 'penetration', X, which can be deduced from the geometric relationship $X\,D = R_A^2$, which is approximately true when V/C and hence X/D is small. R_A is the radius of the contact area, $\propto D\,V/C$. Since $T = X/V$, we then obtain

$$T \propto DV/C^2 \, . \tag{A-4}$$

Substituting all of these into equation (A-1), we can obtain

$$W \propto D^3 \rho \frac{V^5}{C^3}\left(1 - \frac{V_c}{V}\right) . \tag{A-5}$$

The above has assumed that the shock-wave velocity C is an invariant. This is not true, however, for high impact velocities. It has been shown that a good approximation is $C = C_o + 2V$, where C_o is the acoustic velocity at atmospheric pressure; and that a rougher but similar approximation is $C/C_o \propto (V/C_o)^{0\cdot3}$, where $0\cdot06 < V/C_o < 0\cdot6$. Making use of the latter, $C^3 = C_o^3\,(V/C_o)^{0\cdot9} = C_o^2\,V$. With this substitution, we can say

$$W \propto D^3\,V^4(1 - V_c/V) \tag{A-6}$$

and, since the number of droplets of diameter D varies inversely with D^3, when we consider a fixed quantity of water its damaging energy potential could be represented by W/D^3.

By our initial assumption, this same quantity should be representative of the erosion rate associated with a fixed mass rate of impinging water. Thus, finally,

$$E \propto V^4(1 - V_c/V). \tag{A-7}$$

It will be noted that this model does not inherently predict either a threshold condition or a drop size effect. The threshold velocity represented by V_c is artificially inserted and must be based on experience or deduced from experimental data. It could, therefore, be considered a function of drop size. It should also be pointed out that extreme care must be taken in applying energy-based arguments to impact processes. It can readily be shown that in an inelastic impact the maximum energy available for deformation of the projectile and target is $M_D V^2/2$, where M_D is the mass of the projectile. Yet the work done 'on' the drop, by the argument we have used, represents V^2 per unit mass of compressed liquid. This is because this mass has not only been compressed but has also been accelerated to a velocity V relative to the coordinates against which the motion of the impact force is measured.

Appendix B

DERIVATION OF EQUATIONS RELATING TO SHOCK WAVE PRESSURE AT HIGH IMPACT VELOCITIES

Consider the one-dimensional case of flat-ended liquid cylinder impacting on a rigid surface, as shown in Fig. 12. The undisturbed liquid is assumed to flow from right to left at velocity V. It is brought to rest by a shock-wave travelling from left to right at a velocity C relative to the undisturbed liquid, and therefore, at an absolute velocity (C-V). Let 'E' be the location of the shock-wave at time T, and 'G' be the location of a liquid plane at time T, such that at time T + dT both the shock wave and the liquid plane arrive at location 'F'. Clearly the length $\overline{EF} = (C-V)\ dT$, $\overline{FG} = V\ dT$, and $\overline{EG} = C\ dt$. As liquid passes through the shock-wave and is brought to rest, its pressure changes from P_o to P_1 and its density changes from ρ_o to ρ_1. Thus, during the time dT, all the liquid which previously extended from E to G, at density ρ_o, is now compressed into the region E to F, at density ρ_1. In the one-dimensional case, conservation of mass then requires that

$$\rho_o\ C\ dt = \rho_1 (C - V)\ dt \tag{B-1}$$

whence

$$C = \left(\frac{\rho_1}{\rho_1 - \rho_o}\right) V . \tag{B-2}$$

Now, strictly speaking, ρ_1 is a function of P_1, the pressure developed to the left of the shock wave. However, at very high pressures the compressibility of a liquid must decrease markedly, as the molecules are pressed closer together. We may therefore argue that in the limit ρ_1 can be regarded as approximately constant in comparison with ρ_o, so that, at very high impingement velocities V, equation (B-2) approaches

$$C = k\ V \tag{B-3}$$

where k is some constant. On the other hand, at very low values of V, we know that C approaches the ordinary acoustic velocity C_o. Combining these two limiting results in the simplest manner possible, we propose the relationship

$$\left. \begin{aligned} C &= C_o + k\ V \\ \text{or} \quad \frac{C}{C_o} - 1 &= \frac{k\ V}{C_o} \end{aligned} \right\} . \tag{B-4}$$

Equation (B-4) states, in effect, that the fraction by which C exceeds C_o should be directly proportional to the Mach number (V/C_o) of the impingement velocity.

The pressure rise $P_1 - P_o = P$ can be deduced from the fact that the quantity of liquid originally between E and G, moving with velocity V, is brought to rest in the time dT by the action of the pressure difference $P_1 - P_o$. Thus, by Newton's second law

$$P = P_1 - P_o = (\rho_o\ C\ dt)\ \frac{V}{dT}$$

or

$$P = \rho_o\ C\ V \qquad \text{(B-5)}$$

which, of course, is well-known but is derived here to emphasize that the undisturbed liquid density, ρ_o, is the proper density to use. Combining equations (B-4) and (B-5) leads to the result

$$\begin{aligned} P &= \rho_o(C_o\ V + k\ V^2) \\ &= \rho_o\ C_o\ V\ 1 + k\ \frac{V}{C_o}\Big) \end{aligned} \qquad . \qquad \text{(B-6)}$$

Let us now consider the case where the target surface is not rigid, so that the particle velocity change in the liquid is no longer given by the impact velocity V. For the reasons discussed in the body of this paper, we shall assume the stress wave velocity in the target to be an invariant, so that one equation for the interface pressure is

$$P = \rho_2\ C_2\ V_i \qquad \text{(B-7)}$$

where ρ_2 and C_2 are target properties and V_i is the interface velocity during impact, or the target particle velocity.

If we denote the original impact velocity as V_o, then the particle velocity change in the liquid is $V_o - V_i$. Substituting this velocity in equation (B-6), and equating the result with (B-7) gives

$$\rho_2\ C_2\ V_i = \rho_o\ C_o(V_o - V_i)\left[1 + k\left(\frac{V_o - V_i}{C_o}\right)\right] . \qquad \text{(B-8)}$$

To simplify the notation we can substitute $Z_o \equiv \rho_o\ C_o$ and $Z_2 \equiv \rho_2\ C_2$. Furthermore, since we are interested primarily in the particle velocity change in the liquid, we shall denote this by $V_s \equiv V_o - V_i$, whence $V_i = V_o - V_s$. With these substitutions, equation (B-8) can be rewritten as a quadratic equation in V_s/V_o, as follows:

$$Z_2 = (Z_o + Z_2)\frac{V_s}{V_o} + Z_o\, k\,\frac{V_o}{C_o}\left(\frac{V_s}{V_o}\right)^2 . \qquad \text{(B-9)}$$

The formal solution of equation (B-9) can be written as:

$$\frac{V_s}{V_o} = \left\{\left[\frac{1 + (Z_2/Z_o)}{2k(V_o/C_o)}\right]^2 + \frac{Z_2/Z_o}{k(V_o/C_o)}\right\}^{\frac{1}{2}} - \left[\frac{1 + (Z_2/Z_o)}{2k(V_o/C_o)}\right] . \qquad \text{(B-10)}$$

Equations (B-4) and (B-6) can be rewritten in generalized forms as

$$C = C_o + k\,V_o\left(\frac{V_s}{V_o}\right) \qquad \text{(B-11)}$$

and

$$P = Z_o\,V_o\,\frac{V_s}{V_o}\left(1 + k\,\frac{V_o}{C_o}\,\frac{V_s}{V_o}\right) \qquad \text{(B-12)}$$

which permits the calculation of shock-wave velocity and pressure in terms of V_s/V_o. Since the exact expression for V_s/V_o (equation (B-10) is awkward, we seek some useful approximations.

The simplest approximation is obtained by assuming that in equation (B-9) the term in $(V_s/V_o)^2$ is negligibly small compared to that in (V_s/V_o). In that case we can directly state that

$$\frac{V_s}{V_o} \simeq \frac{Z_2}{Z_o + Z_2} \qquad \text{(B-13)}$$

which is identical to V_s/V_o in the case where $C = C_o$. In other words, it ignores the interaction between the effects of a non-constant C and a non-infinite Z_2.

The <u>a posteriori</u> justification for this simplification is, from equation (B-9), that

$$\frac{Z_o\,k(V_o/C_o)}{Z_o + Z_2}\left(\frac{V_s}{V_o}\right) << 1$$

which, with equation (B-13) substituted, leads to the <u>a priori</u> criterion

$$\frac{k(V_o/C_o}{2 + (Z_o/Z_2) + (Z_2/Z_o)} << 1 \; . \qquad (B\text{-}14)$$

A better approximation may be required when the conditions of equation (B-14) are not met. Since in most cases $V_i < V_s$, we may approach this by solving for V_i from equation (B-8). This leads to a quadratic in V_i/V_o, as follows:

$$1 + k\frac{V_o}{C_o} = \left(1 + \frac{Z_2}{Z_o} + 2k\frac{V_o}{C_o}\right)\frac{V_i}{V_o} - k\frac{V_o}{C}\left(\frac{V_i}{V_o}\right)^2 \; . \qquad (B\text{-}15)$$

If we now assume that the term in $(V_i/V_o)^2$ is negligibly small, we obtain

$$\frac{V_i}{V_o} \simeq \frac{1 + k(V_o/C_o)}{1 + (Z_2/Z_o) + 2k(V_o/C_o)} \qquad (B\text{-}16)$$

and, since

$$V_s/V_o = 1 - (V_i/V_o)$$

$$\frac{V_s}{V_o} \simeq \frac{(Z_2/Z_o) + k(V_o/C_o)}{1 + (Z_2/Z_o) + 2k(V_o/C_o)} \; . \qquad (B\text{-}17)$$

The justification of this approach depends on the condition that

$$\frac{k(V_o/C_o)(V_i/V_o)}{1 + (Z_2/Z_o) + 2k(V_o/C_o)} << 1$$

which, with equation (B-16) substituted, leads to the a priori criterion that

$$\frac{k(V_o/C_o)[1 + k(V_o/C_o)]}{[1 + (Z_2/Z_o) + 2k(V_o/C_o)]^2} << 1 \; . \qquad (B\text{-}18)$$

It will be found that, when $Z_2 > Z_o$, the requirement of (B-18) is less stringent than that of (B-14). Thus equation (B-17) is more accurate approximation than (B-13).

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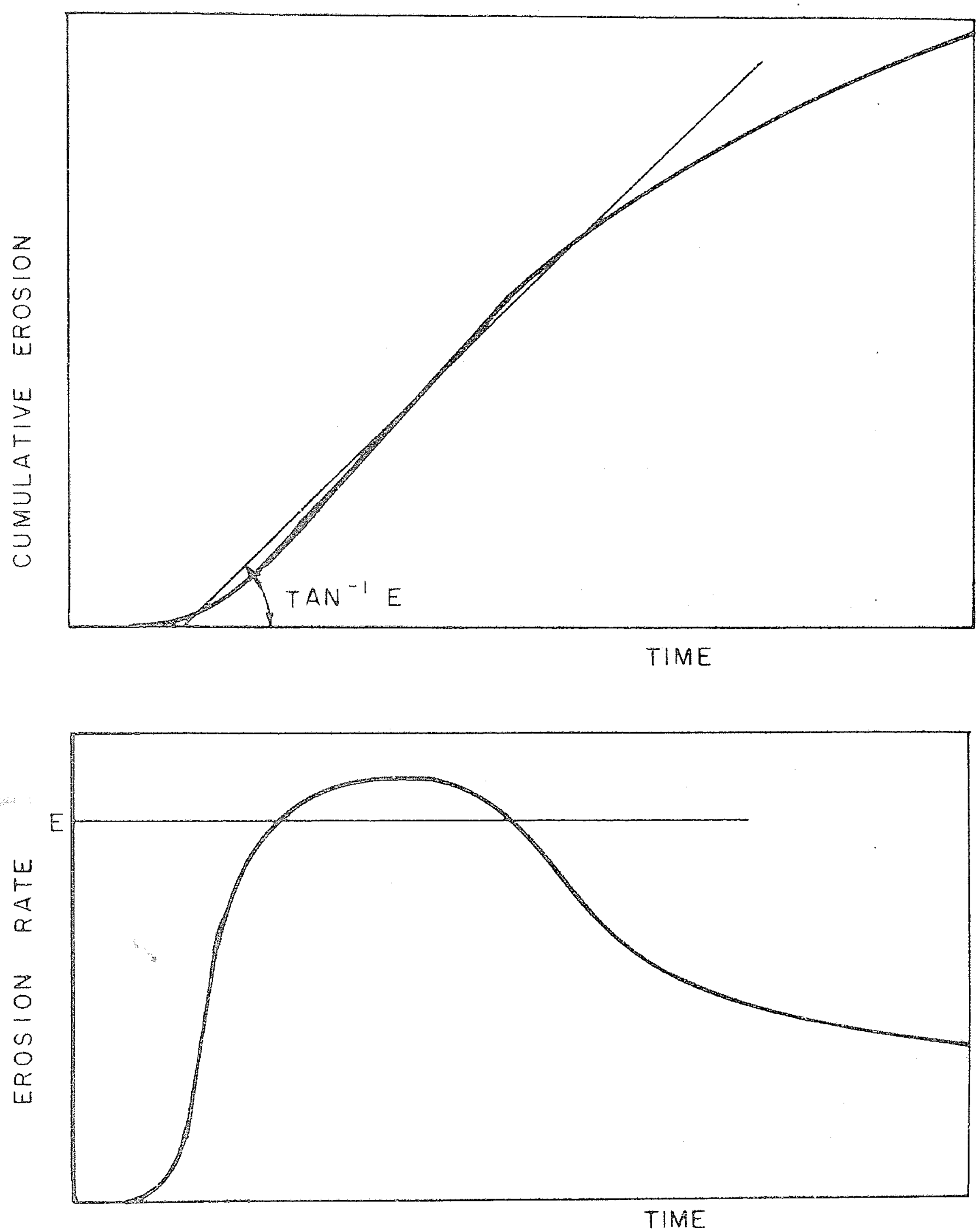
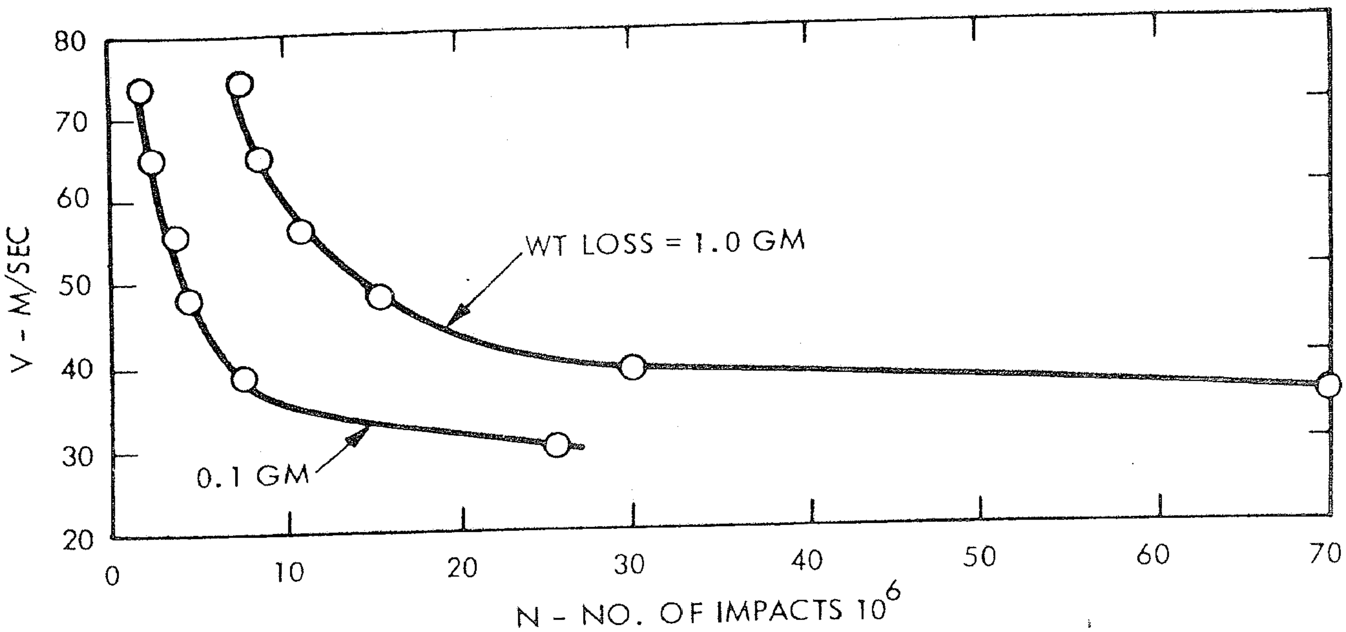
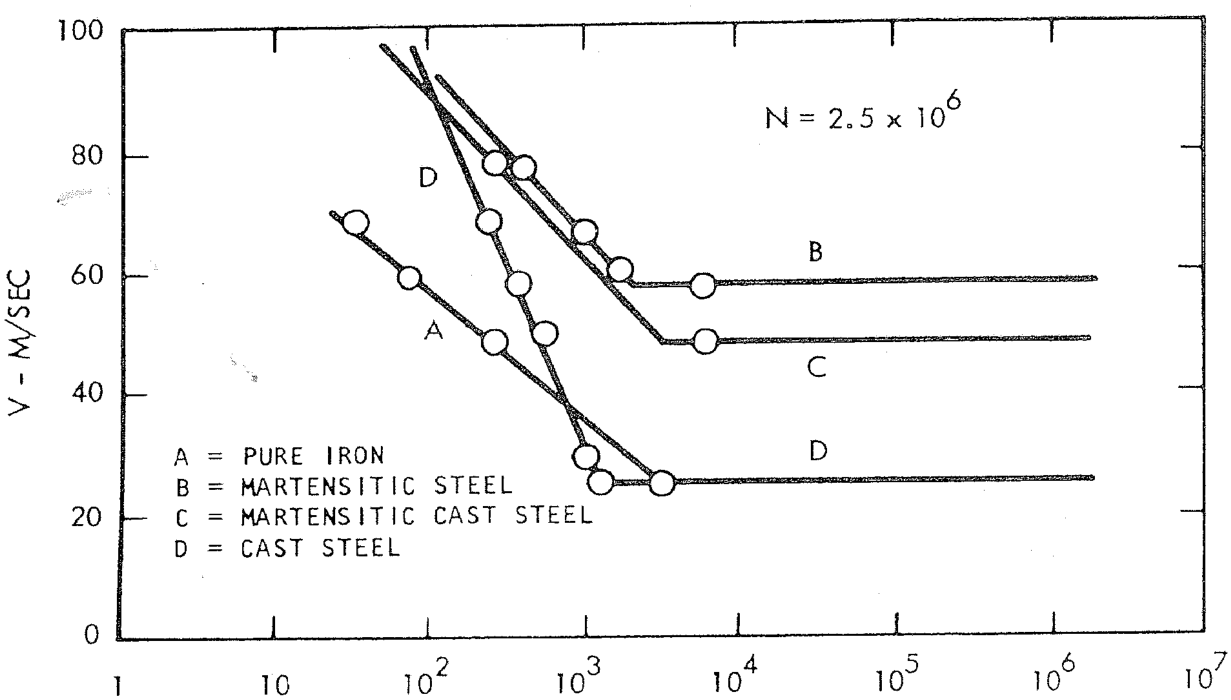


FIG. 1 — DEFINITION OF "EROSION RATE, E" USED FOR CORRELATIONS



EXAMPLE OF V-N CURVE (ADAPTED FROM FIGURE 15 OF REFERENCE 15).



EXAMPLE OF V-(1/G) CURVE (FIGURE 5 IN REFERENCE 15).

FIG. 2 - EROSION - VELOCITY RELATIONSHIPS PLOTTED IN THE MANNER OF FATIGUE DATA.

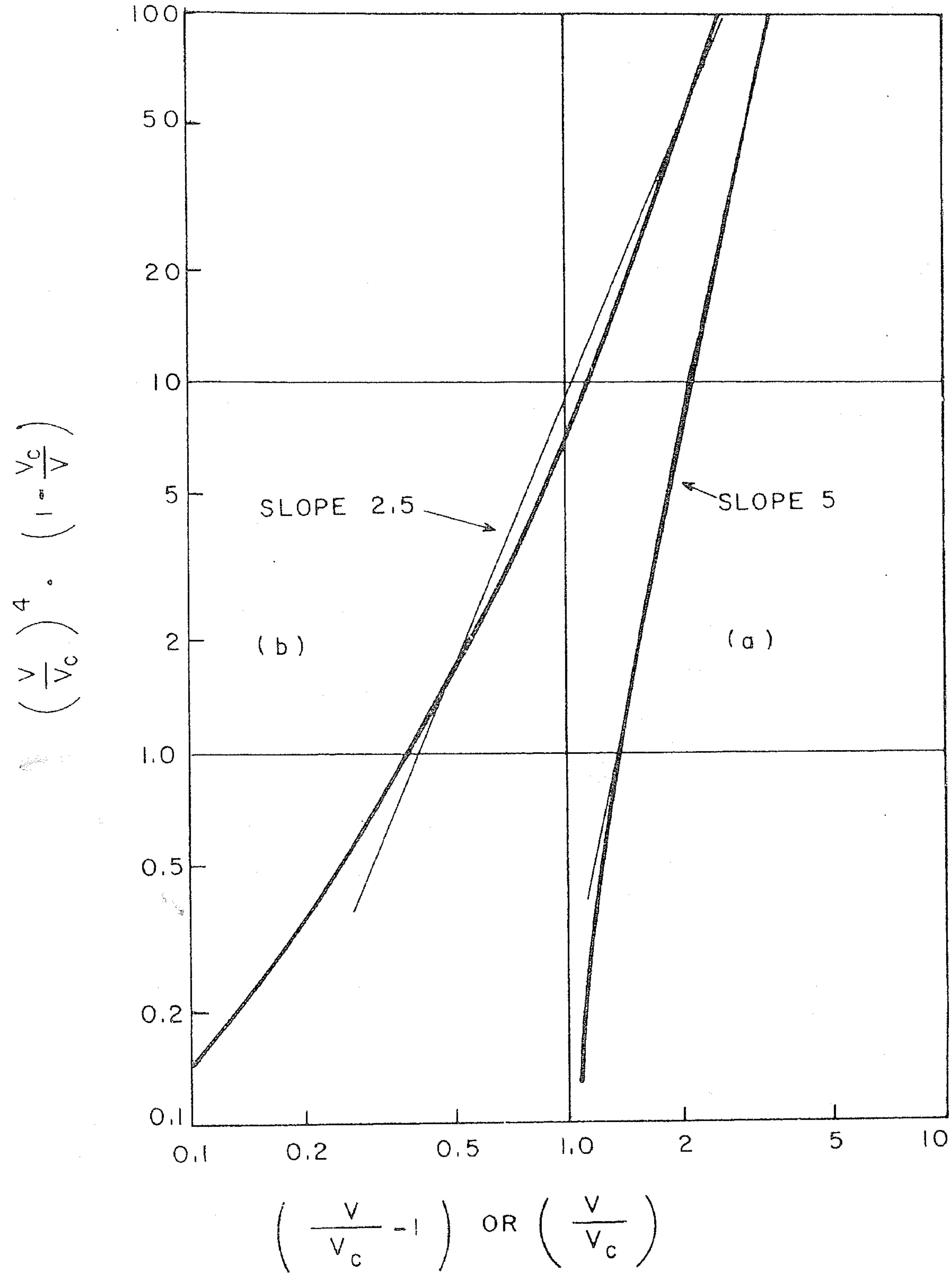


FIG. 3 — EROSION RATE VS VELOCITY ACCORDING TO EQUATION 9

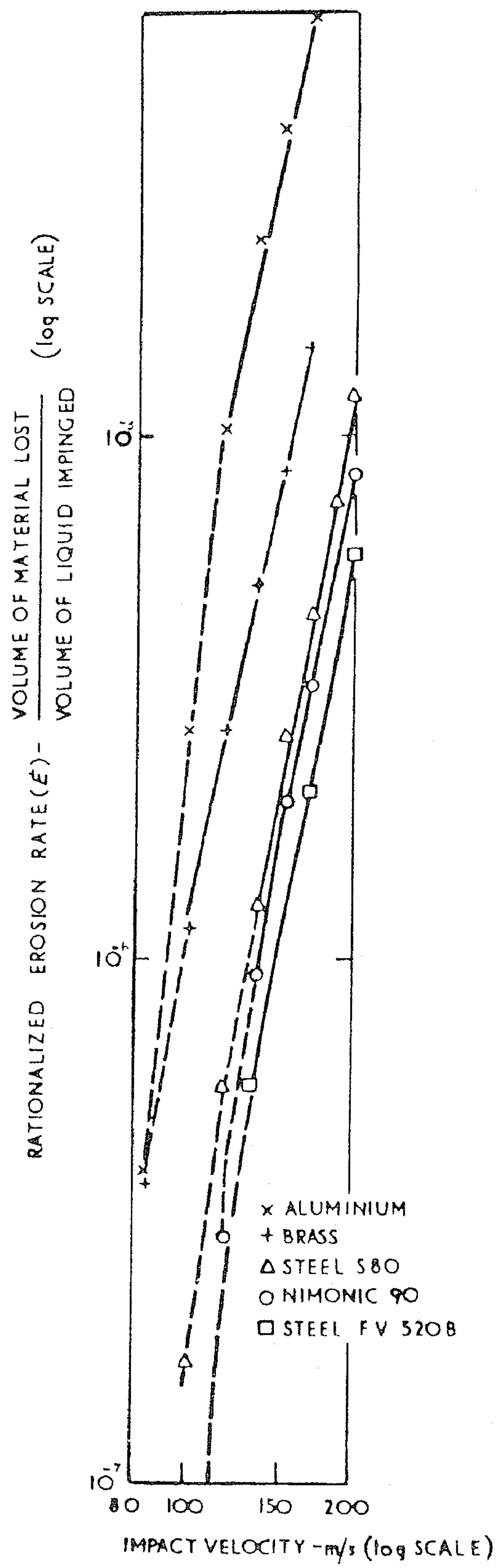
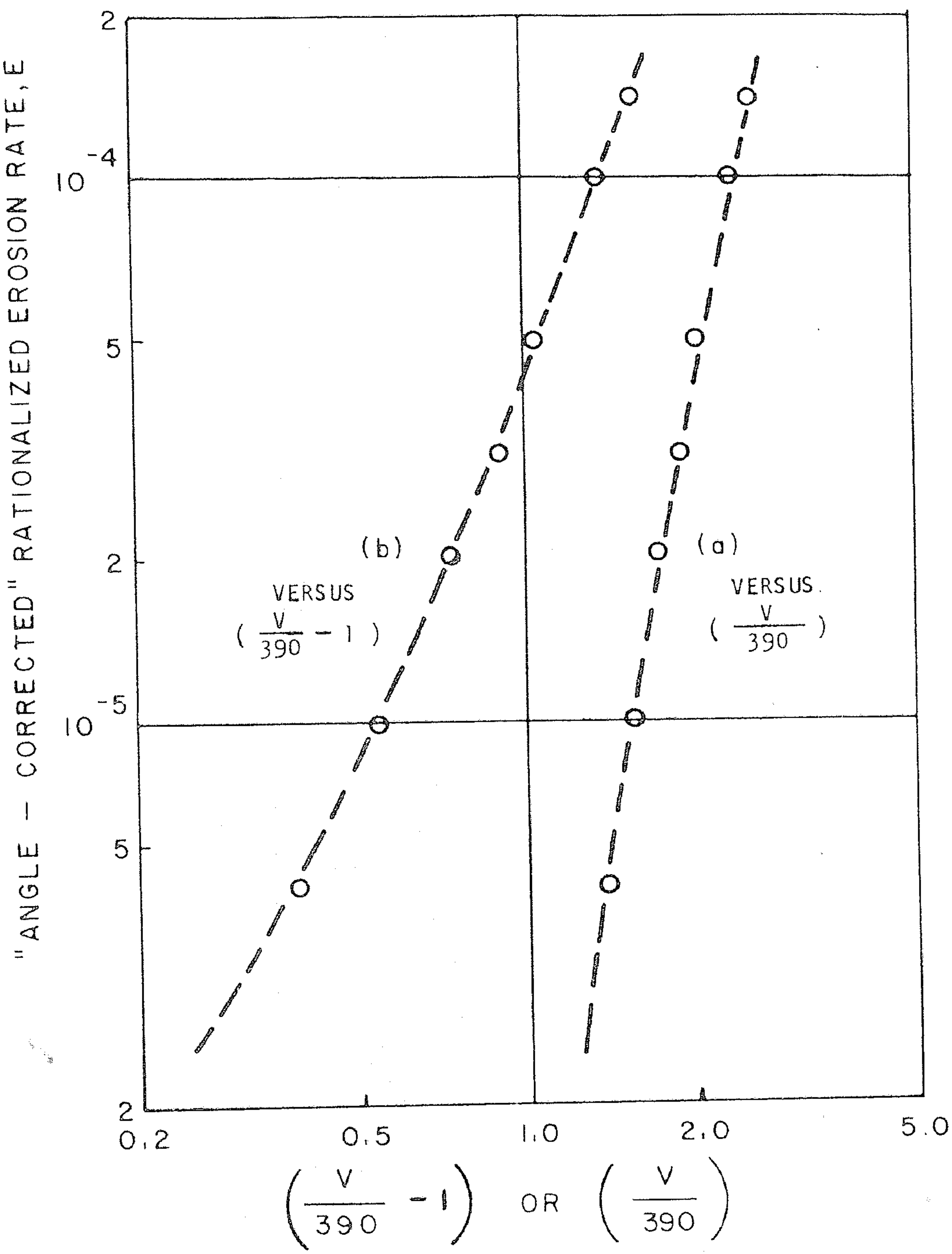


FIG. 4 - VARIATION OF EROSION RATE WITH VELOCITY
[REPRODUCED FROM HOBBS (4)]



[DATA POINTS ARE TAKEN FROM CURVE FOR 660μ DROPS ON FIG. 5 OF REF. 1, FOR WHICH V_c = 390 FT/SEC. DOTTED LINES REPRESENT EQUATION 9, CONSTRAINED TO PASS THROUGH POINT $E = 4.2 \times 10^{-5}$ AT $V/V_c = 2.0$]

FIG. 5 — CORRELATION OF PEARSON'S DATA BY MEANS OF EQUATION 9.

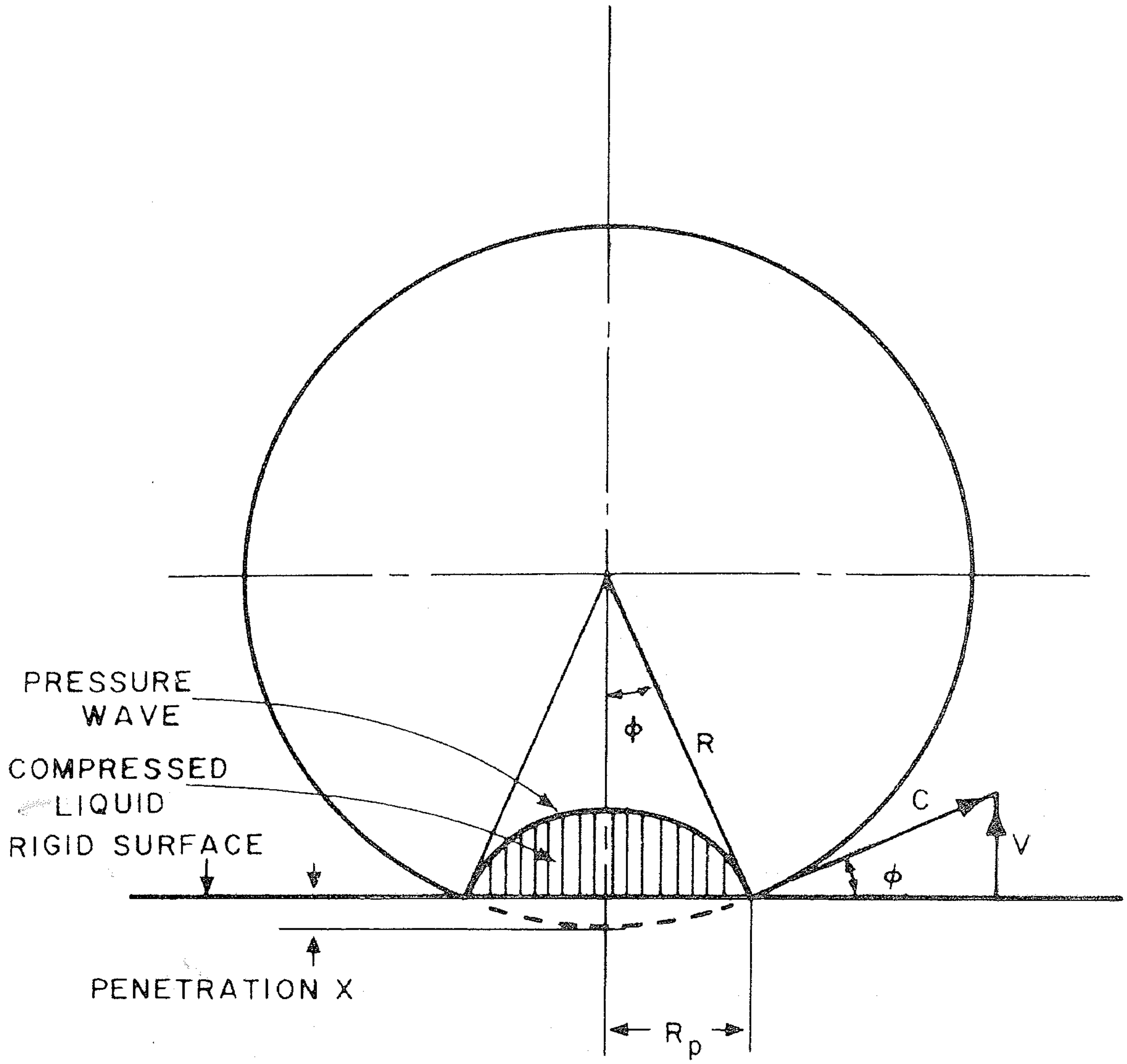


FIG. 6 — INITIAL STAGE OF IMPACT OF LIQUID CYLINDER OR SPHERICAL DROP

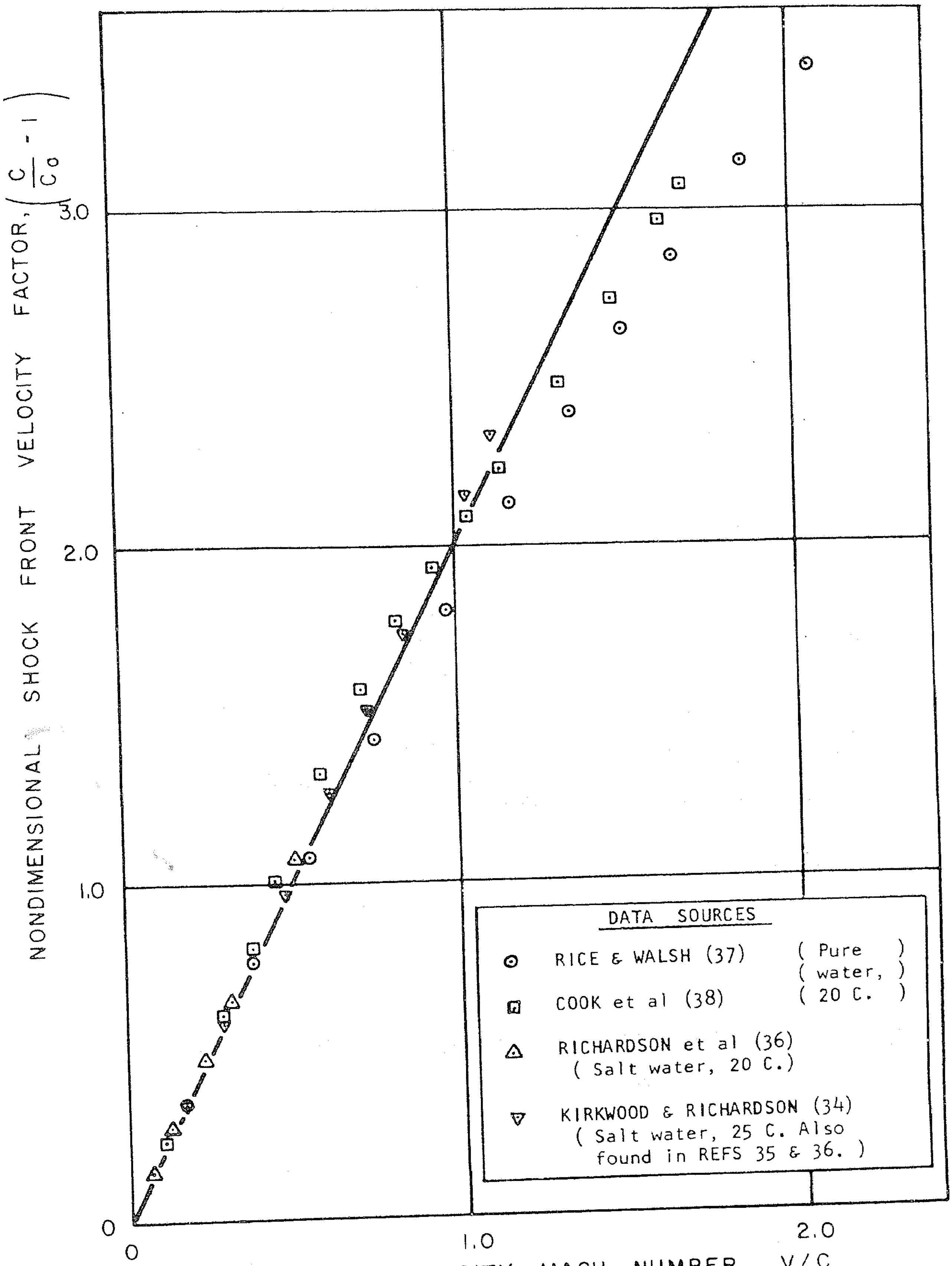


FIG. 7 – SHOCK WAVE VELOCITY VS PARTICLE VELOCITY IN WATER

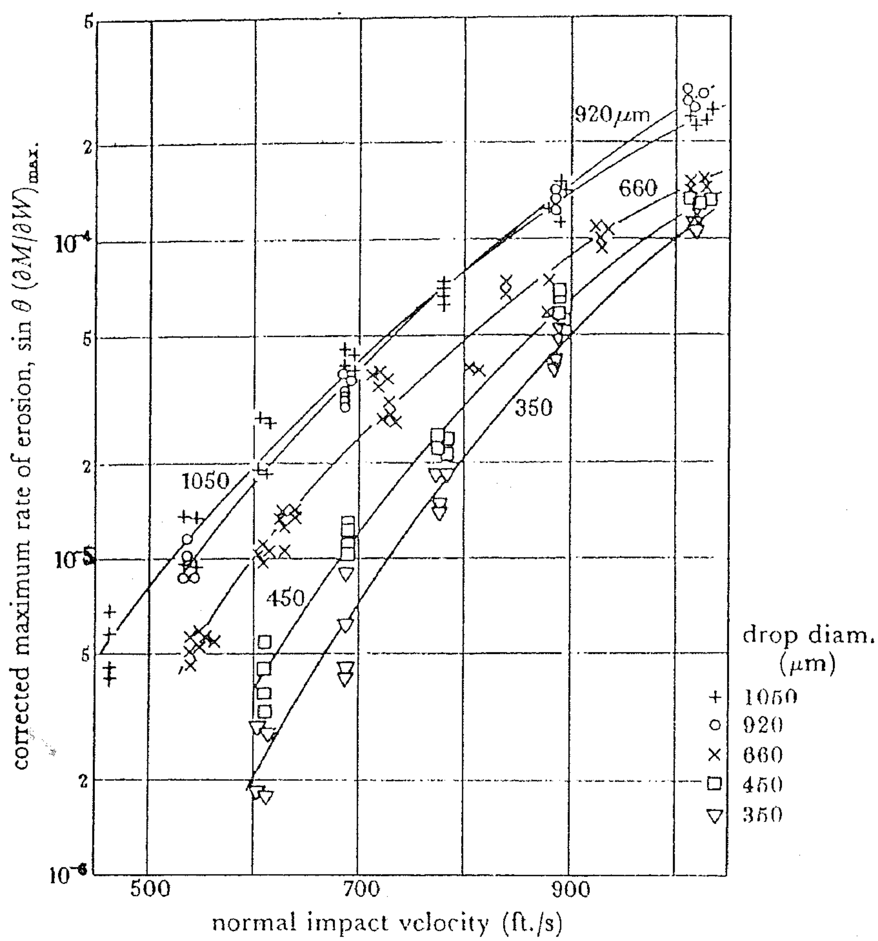


FIG. 8 – EFFECT OF DROP SIZE ON EROSION RATE (REPRODUCED FROM FIG. 5 OF REFERENCE 1)

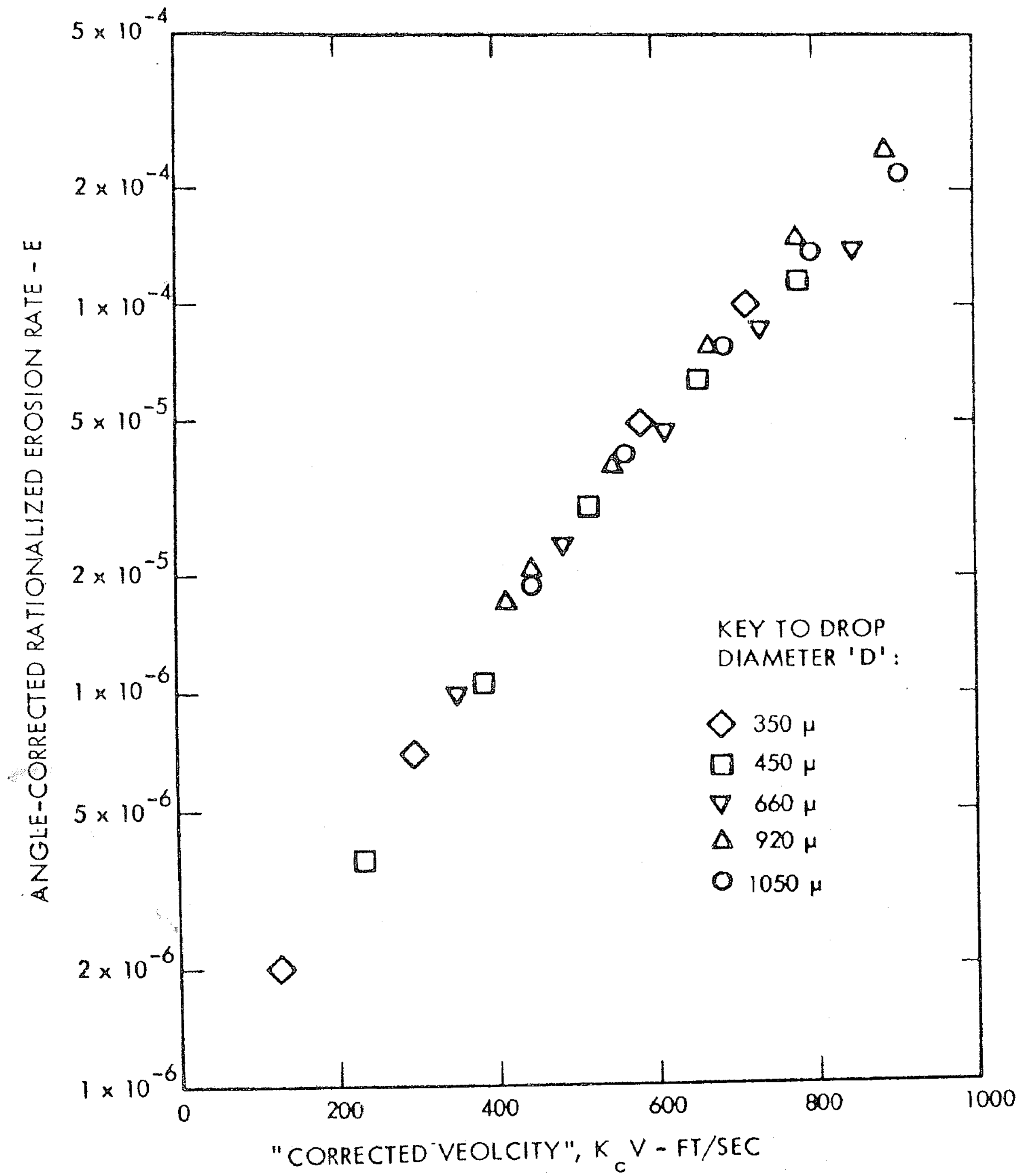


FIG. 9 – CORRELATION OF DATA OF FIG. 8
BY USE OF "CRITICAL FACTOR"

$$K_C \equiv [1 - 10^8/V^2 D]$$

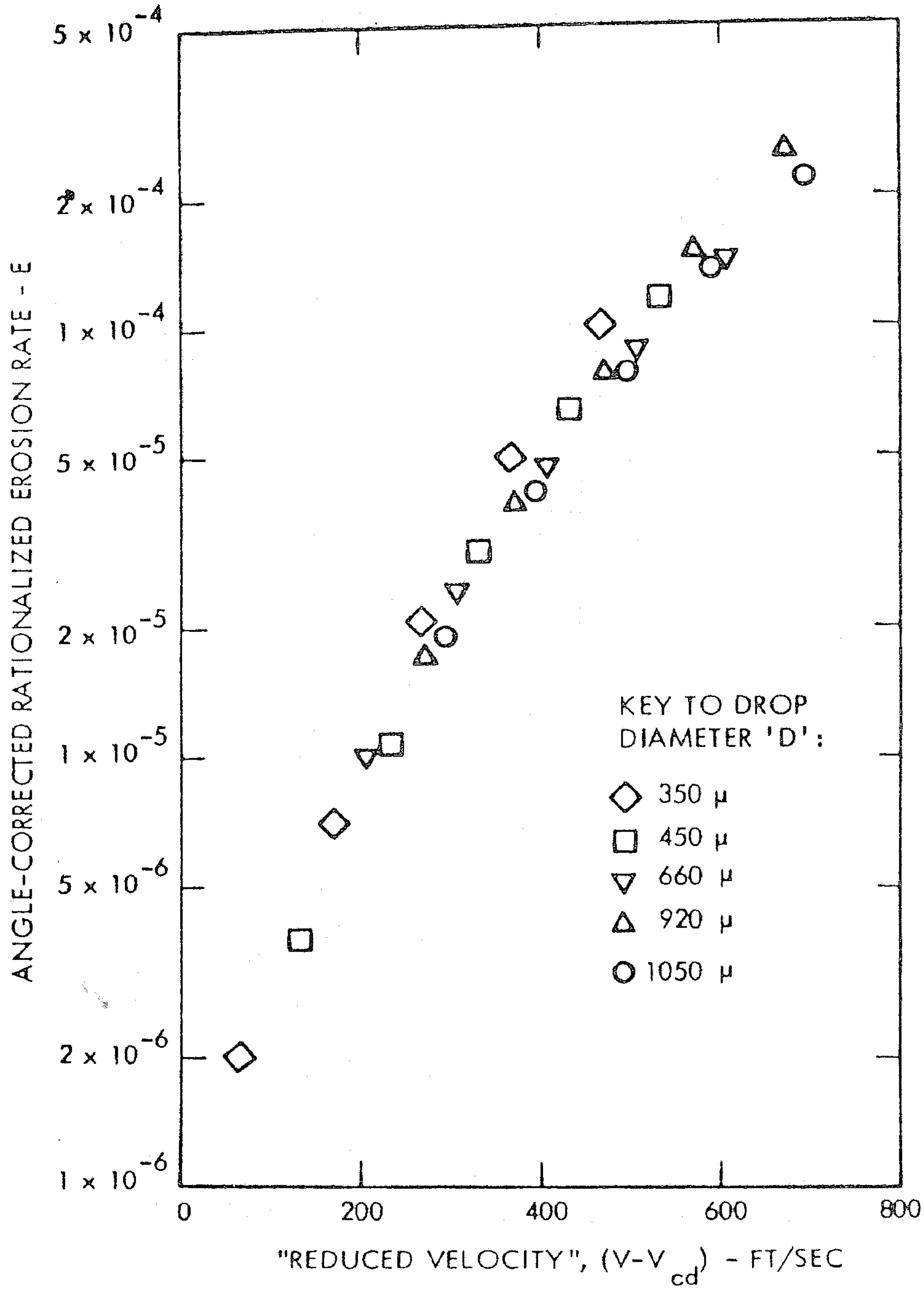
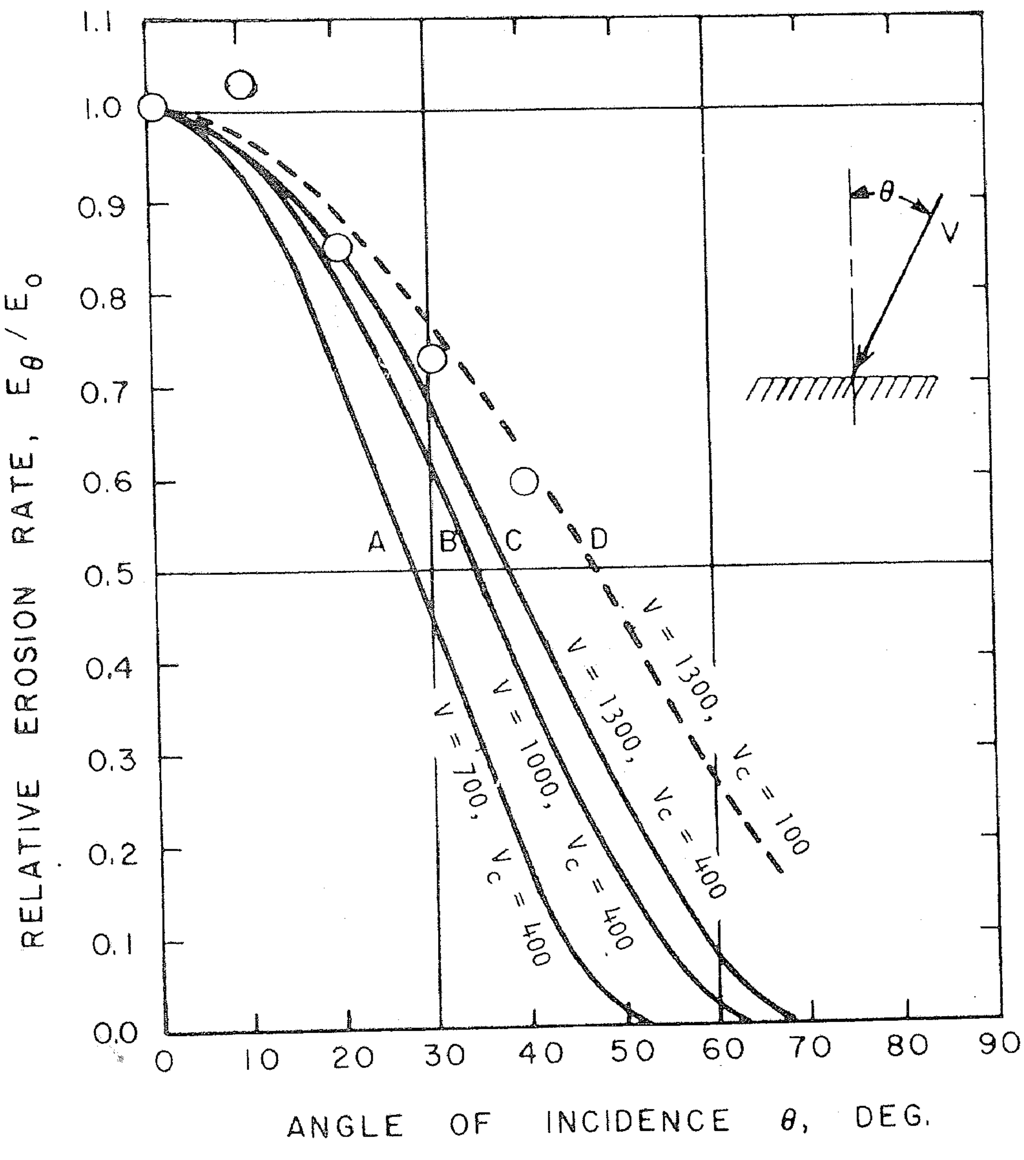


FIG. 10 – CORRELATION OF DATA OF FIG. 8 BY MEANS OF "CRITICAL VELOCITY"

$$V_{cd} \equiv \sqrt{10^8/D}$$



(CURVES ARE COMPUTED FROM EQUATION 28, PROPOSED BY REFERENCE (1). DATA POINTS ARE DERIVED FROM EXPERIMENTAL CURVES FOR ALUMINUM, FROM REFERENCE (52).)

FIG. 11 – VARIATION OF EROSION RATE WITH IMPINGEMENT ANGLE

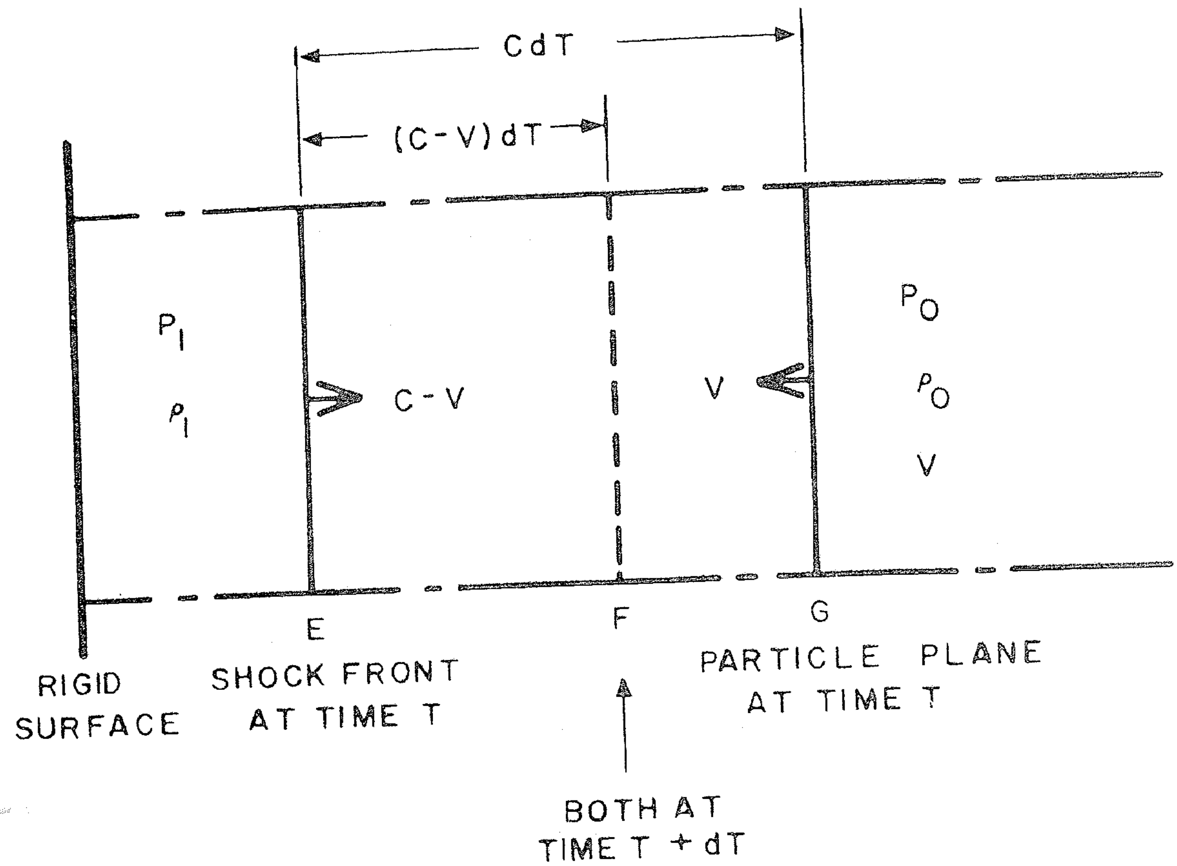


FIG. 12 — RELATIONSHIPS AT A SHOCK FRONT

Dr. O. Engel (General Electric Co. Evendale, Ohio, written contribution)

At the very start I want to thank Mr. Heymann for the courtesy of sending me a preprint of his paper prior to the conference at Meersburg to enable me to comment on the views and conclusions that he has expressed. I have restricted my comment to section 4C. There are a few points in this section that I would like to discuss.

1. The equation

$$P = (\alpha/2)(\rho C V) \, , \tag{15}$$

which Mr. Heymann gives as his equation (15), is based on an implicit assumption that the maximum impact pressure exists in a ring around the central point of impact and is developed at a time Δt after the initiation of impact. However, the statement made by Mr. Heymann, that this equation gives the value of the maximum pressure (which exists in the ring) is incorrect. Quoting from the journal publication containing the derivation of this equation, which Mr. Heymann gives as his Ref.25. "Furthermore, the maximum pressure as it is evaluated at the time Δt, and consequently all the force that has acted to produce water momentum over the time interval Δt, is considered. The instantaneous force that is acting at the last instant of this time interval is not considered". The pressure P given by this equation is the average pressure over the circle of contact between the drop and the planar solid at the time that maximum pressure exists in the ring of maximum pressure.

The time interval Δt, which elapses before maximum pressure exists in the ring of maximum pressure, was found (please see Mr. Heymann's Ref.25) to be given by

$$\Delta t = 4r(1-\alpha)V/C^2$$

where r is the radius of that part of the water sphere which has been traversed by the compressional wave; r is not the original radius of the drop as Mr. Heymann states after equation (15). (Editors Note: This was deleted from published paper.) The relation between r and the original radius of the water sphere before impact, which I am designating here as R to coincide with Mr. Heymann's notation, is given by

$$r = R/(1-\alpha) \; .$$

2. Just below equation (15), Mr. Heymann states that some of the assumptions made by me in deriving this equation have been questioned by Bowden and Field, (please see Mr. Heymann's Ref.23) who maintain that the full water-hammer pressure exists from the first instant of impact until the radius of the circle of contact between the planar solid and the drop reaches the size RV/C. Questions of this kind can only be settled by recourse to experimental observations and by considering the physical significance of the conclusions that result from the logic used in each case.

In the derivation of equation (15) above it was implicitly assumed that the maximum pressure that develops as a result of collision between a liquid sphere and the planar surface of a solid exists in a ring around the central point of impact. If maximum pressure exists in a ring, then the pressure at points on the circle of contact between the drop and the solid that are outside this ring is lower than the maximum pressure. On the other hand, the logic of Bowden and Field leads to a constant pressure, equal to the full water-hammer pressure, across the circle of contact. It remains to be seen which of these pressure profiles (if either) conforms with that which really exists.

The physical reality of the assumption made by me (that maximum pressure exists in a ring around the central point of impact) is borne out by the results of Eugene Cooper (a) who has found that a ring of maximum pressure is produced during the water entry of hard spheres. The validity of the assumption is further borne out by the results of a treatment of the impact of a fluid sphere against a planar solid carried out by Savic and Boult (b), Savic and Boult found that maximum pressure exists in a ring.

3. The logic of Bowden and Field produces the result that the largest value of the radius of the circle of contact between the drop and the planar solid, while the full water-hammer pressure is maintained, is $R(V/C)$. On the other hand, the assumptions used in deriving equation (15) above give the radius of the circle of contact at the time of maximum pressure as $R(V/C)\ [4\sqrt{1 - 4\,(1 - \alpha)^2\,(V/C)^2}\,]$. The loci of these expressions are shown in the accompanying Fig.1 where the radius of the circle of contact divided by the drop radius is plotted against the quotient V/C.

In evaluating the loci of these equations, one may ask: What is the physical consequence of the fact that as V is progressively increased a point is reached at which V = C? In terms of the assumption regarding maximum pressure, which underlies equation (15) above, the condition V = 0.8333 C means that maximum pressure occurs at the first instant of impact when the circle of contact between the drop and the planar surface of the solid is a point circle. For higher values of V, such as V = 2 C, V = 3 C, etc., the radius of the circle of contact is imaginary.

On the other hand, the logic of Bowden and Field yields the result that when V = C the radius of the circle of contact is the drop radius and for higher values of V, such as V = 2 C, V = 3 C, etc., the radius of the circle of contact becomes larger than the drop diameter. What this means in terms of the original argument, which is presented in Mr. Heymann's Fig.6, can probably be clarified best by Bowden and Field.

4. Immediately above equation (19), Mr. Heymann makes some final comments on what he refers to as "the dilemma posed by the factor $\alpha/2$" in equation (15) above. With regard to the necessity for the coefficient α, I have pointed out (c) that "to assume that α is unity where flow occurs would result in the absurdity that after the flow had occurred there would be a gap between the impacting surface and the remainder of the drop and that the impacting surface would never catch up with the remainder of the drop, let alone move through it".

In my first treatment of the impact pressure (c), I implicitly assumed the coefficient α to be a function of impact velocity, and, in estimating the pressure produced at an impact velocity of 880 ft/sec, I arbitrarily took the coefficient α to be close to unity. At a later date (please see Mr. Heymann's Ref.30), I found that the coefficient α is not a function of velocity but is instead a function of the acoustic impedances of the drop liquid and of the solid. Specifically, I found that

$$\alpha = 0.41/[1 + (0.59\ z/z')] \tag{19}$$

where z is the acoustic impedance of the drop liquid and z' is the acoustic impedance of the solid. The value of α for waterdrop collisions with solids, calculated with use of equation (19) is very close to 0.4. This value is in almost exact agreement with an independent experimental determination of α for waterdrop collisions made earlier (please see Mr. Heymann's Ref.25 and 30).

DISCUSSION

5. Using the value $\alpha = 0.4$, the pressure given by equation (15) for collision of a planar solid with a waterdrop is 0.2 CρV. Following equation (19) of his paper, Mr. Heymann says "Therefore, it was stated in the discussion to Ref.18, the maximum impact pressure on a rigid surface cannot exceed about 0.2 ρVC". This, however, was not what I said. The statement made by me was that the pressure P given by equation (15) above can be at most ρCV/5; one must bear in mind that the pressure P given by equation (15) above is not the maximum impact pressure in the ring of maximum pressure but is the average pressure at the time that maximum pressure exists in the ring of maximum pressure.

6. In section (a) (deleted in published paper) following equation (19), Mr. Heymann concludes, "the α of Ref.30 is not the same as the α of Ref.25, and equation (19) implies that P = 0.4 ρCV rather than 0.2 ρCV". This conclusion is not correct. Equation (5) of Mr. Heymann's Ref.30 was derived for "collision between a solid rod A having flat ends ...... and a similar liquid rod B that is at rest ...". For the impact between a solid rod with flat ends and a liquid rod with flat ends, the factor of 2, which appears in equation (15), is absent. The α of Ref.25 is precisely the same as the α of Ref.30.

In section (b) ((a) in published paper) following equation (19), Mr. Heymann states: "the experimental observations which led to the assigning of numerical values to α both involved the complete collision process: In Ref.25 this was the time taken for a drop to spread completely, and in Ref.30 the depth of pits created by drop impacts (usually of mercury) on soft metal targets". In the derivation of equation (15) (please see Mr. Heymann's Ref.25), the average theoretical displacement velocity at which a solid moves through a drop is given as $(1 - \alpha)V$. To assess the coefficient α, the theoretical displacement velocity was set equal to the experimentally determined displacement velocity. The experimental displacement velocity was found from the drop diameter and the time required for the liquid contents of the protruding spherical body of the drop to enter into radial flow. Although this time is not the time required for "a drop to spread completely", it is a time interval of about a millisecond whereas Δt is a time interval of about a microsecond. If the displacement velocity at which a solid surface cuts through a drop after impact is a function of time, then the objection is well taken. No time dependence is expressed in the average displacement velocity $(1 - \alpha)V$, however.

In the derivation of equation (19) (please see Mr. Heymann's Ref.30), the value of α was assessed from the average negative particle velocities produced in liquid-against-solid and in solid-against-solid collisions in conjunction with the empirical constants of the liquid-against-solid and solid-against-solid crater-depth equations. The cratering model used (d) was that "the depth of pit produced at any relative impingement velocity is proportional to the negative velocity produced in the cylindrical core in the target plate as a result of the collision and to the time that this negative velocity exists". The average negative velocity was taken to be the elastic plane-wave particle velocity, v', in the solid. For liquid-against-solid impacts, $v' = \alpha zV/(z' + \alpha z)$ and for solid-against-solid impacts, $v' = zV/(z' + z)$. In Mr. Heymann's notation, the time interval during which this velocity exists was taken to be 2 D/C where D is the drop diameter and C is the speed of sound in the drop liquid. For a 2-mm-diameter waterdrop, the time involved is about 2.5 microseconds. For a 2-mm-diameter waterdrop colliding with the planar surface of a solid at 1000 ft/sec, Δt is about 1 microsecond. The sizes of these time intervals are much more nearly similar than those involved in the assessment of the coefficient α with use of the displacement velocity. To determine whether the objection raised is serious, it would be necessary to consider the attenuation of the particle velocity that occurs in these time intervals.

In section (c) ((b) in published paper) following equation (19), Mr. Heymann states that the experimental observations which led to equation (19) are susceptible to an alternate explanation. The alternate explanation advanced by him is that the impact duration is shorter for a liquid drop than for a rigid sphere. Mr. Heymann estimates that for a "heavy drop forming a deep pit" the impact duration may be 0.8 D/C whereas (presumably) he accepts the value of 2 D/C for a rigid sphere. To be exact the impact duration is only 2 D/C for impacting objects, the front (or impact) face of which parallels the rear (or trailing) face. In the case of a rigid steel sphere, reflections occur from the hemispherical rear face sooner than if this face were flat; consequently, the impact duration for a steel sphere is also less than 2 D/C.

7. Following equation (19), Mr. Heymann seems to imply that the pressure 0.2 $C\rho V$ is too low. A comparison can be made between this pressure and the values of pressure obtained by Savic and Boult (b). Values both of the maximum pressure in the ring and of the pressure at the center of the circle of contact, which were read from a plot of the radial distribution of impact pressure given by Savic and Boult are listed in Table 1. In order to make comparison between the average pressure predicted by equation (15) and the pressures produced by the treatment of Savic and Boult, it is necessary to convert the $\rho V^2/2$ pressures found by Savic and Boult into $C\rho V$ equivalents.

DISCUSSION

The $C\rho V$ equivalents for three different values of V are given in Table 1. For a waterdrop impact at 1000 ft/sec, which is a velocity of interest both to turbine engineers and to aeronautical engineers, $V = C/5$ if the speed of sound in water is taken to be 5000 ft/sec. The $C\rho V$ equivalents for this velocity, taking $\rho = 1$, are plotted against the ratio a/A in Fig. 2; a is the radius of the circle of contact and A is the radius of the drop. For waterdrop impacts, the coefficient α is about 0.4. The pressure given by equation (15), taking α to be 0.4, is also shown in Fig. 2. From Fig. 2 it can be seen that the average pressure at the time of maximum pressure, given by equation (15), is in substantial agreement with the pressures found by Savic and Boult for an impact velocity of 1000 ft/sec.

It can be seen from inspection of the $C\rho V$ equivalents given in Table 1 that the average pressure given by equation (15) does not apply at an impact velocity as high as 5000 ft/sec ($V = C$) or at an impact velocity as low as 50 ft/sec ($V = C/100$). At a velocity of 5000 ft/sec the assumptions on which equation (15) is based break down; the impacting surface is travelling as fast as the compressional wave and, consequently, maximum pressure occurs at the first instant of impact. The velocity of 50 ft/sec may be too low for a treatment of the pressure with use of elastic wave theory.

Extrapolations are precarious at best, but, if I were to extrapolate the curve of the $C\rho V$ equivalents of Savic and Boult's peak pressures given in Fig. 2, I would find the value at the point $a/A = 0$ to be at least $C\rho V$ and possibly higher.

Continued on next page.

Table 1

Impact pressures from the treatment of Savic and Boult

a/A	Maximum pressure existing in ring				Pressure at center circle of contact			
	ρV^2	CρV equivalent			ρV^2	CρV equivalent		
	—	V = C/100	V = C/5	V = C	—	V = C/100	V = C/5	V = C
0.1	3	0.03	0.6	3	0.64	0.0064	0.128	0.64
0.2	1.5	0.015	0.3	1.5	0.62	0.0062	0.124	0.62
0.3	1	0.01	0.2	1	0.60	0.0060	0.120	0.60
–	–	–	–	–	–	–	–	–
0.6	0.5	0.005	0.1	0.5	0.52	0.0052	0.104	0.52

The density ρ in the plot given by Savic and Boult (b) appears to have been taken equal to one.

References

(a) Eugene Cooper, communicated by letter

(b) P. Savic and G.T. Boult, The fluid flow associated with the impact of liquid drops with solid surfaces. National Research Council of Canada Report MT-26, Ottawa, 2 May 1955

(c) Olive G. Engel, Impact pressure in solid-liquid sphere collisions. WADC Technical Report 53-192 Part 1, Wright Air Development Center, 1953 Wright-Patterson Air Force Base, Ohio, 1953

(d) Olive G. Engel, Pits in metals caused by collision with liquid drops and soft metal spheres. National Bureau of Standards Journal of Research 62, 229 1959

Mr. Heymann

Dr. Engel has raised a number of points concerning my discussion of current knowledge and hypotheses relating to the impact pressure development under a round drop.

I am happy to accept some of the relatively minor corrections offered, and will address myself primarily to those points which affect the substance of the conclusions which I had reached. To facilitate reference to Dr. Engel's specific remarks, I have requested the editors to add reference numbers at the beginning of certain paragraphs. It should be understood that these numbers were not an original feature of Dr. Engel's discussion.

DISCUSSION

Certain points refer to material which had been deleted from my paper in the process of abbreviating and revising the originally-circulated advance copy into the final publication form. However, what they refer to will be obvious from the discussion.

In para.1 and again in para.5, Dr. Engel clarifies the interpretation which should be put upon the pressure given by equation (15). I must concede that I had misinterpreted it, but it has also been misinterpreted, or vaguely referred to simply as "the impact pressure", by most other authors who have cited it. Indeed, Ref.18 described it as "the maximum pressure developed by the impact", which Dr. Engel did not contravene in her cited discussion to that reference. The clarification is welcome, and also valuable because it underscores the difficulty of defining any single meaningful "impact pressure", when the instantaneous pressures in fact vary both over the contact area and with time. This is a point which I had tried to bring out implicitly, but it deserves additional emphasis and may be at the root of much of the disagreement.

If I now interpret it correctly, equation (15) is intended to represent the pressure, averaged both over time and area, from the initial moment of contact up to the time Δt at which the maximum instantaneous pressure is assumed to exist. This, therefore, at least partially justifies the use of observations which involve a goodly portion of the total impact process, to determine the value of α. This I had questioned and Dr. Engel has discussed my objections in para.6.

However, as Dr. Engel concedes, both of the observations used involve time spans which exceed Δt, or go beyond the point at which the maximum pressure is said to be developed. This, together with Dr. Engel's quotation in her para.4, points to what may be the fundamental difference between her view and that of Bowden and Field[23] which I share. Dr. Engel apparently assumes that radial flow takes place from the beginning of contact, as evidenced by the "thin cylinder of liquid" referred to in section 3(c) of Ref.25 and shown in Fig.14 therein. The opposing contention is that until after the "edge Mach number" drops below unity (which is conceptually similar but not identical to Dr. Engel's time Δt) the compressed liquid is entirely bounded by the target surface and by surrounding liquid which has been reached by no compression wave and knows nothing of the impact: the compressed liquid therefore cannot flow. This is what I call the "initial stage" of the impact; when its limit is reached, the compression wave overtakes the contact perimeter and radial flow can begin. It would seem that the nature of the liquid behavior changes radically thereafter, and so does the magnitude and distribution of the contact pressure. If this contention is correct, it not only eliminates the justification quoted in para. 4 for introducing the factor α during the "initial stage", but also invalidates the drawing of inferences for the "initial stage" from events whose duration transcends that stage. For confirmation of this contention I should like once more to refer to Ref. 28, which appears to be the most rigorous applicable analysis currently available, and which concludes that the area-averaged pressure, at any instant during the "initial stage" of the impact, is given simply by ρCV. Some experimental confirmations are cited in my paper.

It should be pointed out that Dr. Engel's Δt does not exactly correspond to the so-called "initial stage". According to Ref. 25. "Δt is defined by the condition that the radiating compressional wavelet initiated at a point in the first instant of impact should just reinforce the compressional wavelet that is started at time Δt later ...". (The underlining is mine.) It can be shown (see Ref. 23) that Δt, as defined above, is more than twice the duration of the "initial stage" defined earlier. I must concede, therefore, that radial flow will have taken place, and will have effected a reduction in the average impact pressure, during that portion of Δt following the end of the "initial stage". However, reinforcement of pressure wavelets at the periphery will begin as soon as the "initial stage" has terminated and the edge Mach number has dropped below unity, and I suspect that maximum edge pressure will be developed before Δt. This raises a question as to the real significance of any average pressure calculated either at the time Δt or over that time span, particularly if, as I contend, a higher average pressure has existed throughout the well-defined "initial stage".

In para. 3 and her Fig. 1, Dr. Engel compares the radius of contact predicted by her at time Δt with that corresponding to the end of the "initial stage", given by Bowden and Field's model or equation (17). The difference derives not only from the inclusion of α but also from the difference between the definitions of Δt and of the "initial stage". Dr. Engel questions the physical significance of the contact radius by Bowden and Field's model exceeding the drop radius when $V/C > 1$, but does not explain the physical significance of that same condition which is predicted by her model over the range of approximately $0.25 < V/C < 0.8$.

Moreover, I can not agree that "as V is progressively increased a point is reached at which $V = C$". I contend that this can never physically occur since the compression shock wave velocity C is not the acoustic velocity C_o, but is itself dependent on V and always exceeds it, as was shown in section 4D and Appendix B of my paper. If we accept the approximate relationship given by equation (20), with k = 2, then the condition $V/C_o = 1$ results in $V/C = 0.333$, and as V/C_o approaches infinity, V/C approaches the limit value 0.5. Admittedly this approximation loses accuracy above $V/C_o = 1.2$ and also is based on a one-dimensional model not directly applicable here. In the time-dependent two-dimensional case, the value of C will vary with the locally-developed pressures, both in space and in time. But the qualitative conclusion must still hold: namely, that regardless of the impact velocity the compression wave will overtake the contact perimeter, permitting lateral flow to develop, before the contact radius equals the drop radius.

The first statement in para.6 refers to a point now deleted from my paper because it was relatively trivial. However, while the relationships in Ref.30 may have been derived for a flat-ended model, they were applied to the reconciling of empirical results obtained from impacts of liquid and solid spheres, through the introduction of the factor α. The relationships used in Ref.30 do imply that the effective impact pressure under a liquid sphere would be α times the effective impact pressure under a solid sphere, if the target were rigid. The validity of Dr. Engel's statement therefore hinges on whether it is also valid to assume that the effective impact pressure on a rigid target under a solid sphere is $(1/2)\ \rho CV$.

In para.7, Dr. Engel claims substantial agreement between her results and those of Savic and Boult[27b], for the case where V/C = 0.2. To obtain a true comparison, in view of the interpretation to be given to equation (15), one would have to make some appropriate integration to obtain a time-and-area averaged value over the whole period Δt, which, for V/C = 0.2, can be found from Fig.1 to extent to a/A = 0.8 in Fig.2. This would be difficult to do on the basis of the information given. Moreover, the analysis of Savic and Boult is based on potential flow theory of ideal fluids and would seem to have little validity in the early stages of the impact when compressibility phenomena and not flow phenomena predominate. Their peak pressure at time zero is stated in Ref.27 to be infinite, not merely "at least ρCV" as is surmised by Dr. Engel's final statement.

I am glad that (in paras.1 and 2) Dr. Engel has emphasised, and has cited confirmation for, the view that the maximum pressures developed under a drop are found in a ring, i.e. at or near the periphery of the contact area. In this respect, I am sure, her model is more realistic than the simple model of Ref.23. This view is consistent with the results of Ref.28 which I have discussed in my paper, and it deserves wider recognition.

From Ref.28 one can infer that the pressure distribution is almost uniform in the very early stages of impact when the edge angle ϕ is still very small, and becomes more and more non-uniform as the contact area grows bigger. This seems intuitively reasonable, in that during the first instants of impact the contact radius is so small compared to the radius of curvature of the drop, that the one-dimensional model, with uniform pressure of ρCV, should be a good approximation.

My view would be that some time after the "initial stage" of impact is over, and a substantial radial flow has been initiated, an approach to the average pressure such as presented by Dr. Engel will become valid, but I suspect that a more rigorous analysis would show that the maximum "ring" pressure is developed rather soon after the end of the "initial stage" and considerably earlier than the time Δt as defined by Dr. Engel. Therefore, I believe that the best index which we have at present for the severity of the impact is still the average impact pressure developed during the "initial stage", which is the full value of ρCV if the target surface is rigid.

I hope that Dr. Engel's discussion and this reply will serve to throw light onto some of the difficulties and differences of opinion which still impede the realistic definition and calculation of the impact pressures under a round drop. Perhaps it will stimulate someone to undertake a more rigorous analysis, possibly using a time-incremented numerical approach suitable for digital computer application.

Dr. J.E. Field

In Miss Engel's comments on Dr. Heymann's paper she refers to an earlier paper by us (Bowden and Field 1964) in which it was argued that when a drop of radius "r" strikes a rigid surface at a velocity V then a uniform pressure ρCV will exist over an area of radius $X = \frac{rV}{C}$. These predictions are somewhat different to those developed from Engel's (1955) treatment of the same problem. The essential difference between the two approaches is that Engel assumes outward flow from the initial stage of impact onwards, whereas we used a simple argument to show that flow was not possible until the radius of impact exceeded the value $\frac{rV}{C}$. The idea of no flow has been substantiated by more sophisticated mathematical treatments of impact in which compressible conditions exist (see, for example, Walsh, Shreffler and Willig (1953), Harlow and Pracht (1966)). The literature on explosive welding also gives evidence for the importance of jetting on weld formation, and what is relevant here, the lack of welding when the angles involved are small and jetting does not occur (Bahrani, Black, Crossland, 1967). The complexity and likely error of Engel's treatment, we feel, largely stems from this wrong assumption of flow in the first stages of impact.

It is regrettable that Miss Engel's comments on the predictions from our treatment should be so misleading. The line on the graph (Engel, Fig. 1) showing $\frac{X}{r} = 1$ when V equals C (the sound wave velocity, 1450 metres/sec) is wrong. Our paper clearly states (p. 334) that when high pressures are involved then C has to be replaced by C', the appropriate shock-wave velocity. In other words the formulae ρCV and $X = \frac{rV}{C}$ have both to be modified. In fact $\frac{V}{C}$ is always less than 1 and so X never exceeds "r", the radius of the drop. There is therefore no failure of our ideas when V = 2C, = 3C, etc. as Miss Engel suggests.

Cook, Keyes and Ursenbach (1962) give data of shock pressure against shock velocity (C'). It is clear that to use a value of C' = 1500 metres/sec is only valid for impact pressures of up to about 1 kilobar (10^9 dynes/sq cm). For higher pressures the value of C' quickly rises reaching 2400 metres/sec for a pressure of 10 kilobars, and 4200 metres/sec for a pressure of 50 kilobars. Since the pressure involved in high speed impact is given by $\rho C'V$ the question arises which value to take for C'. This may be overcome in two ways, either by the method of successive approximations or as follows (Field 1962).

The data of Cook et al (1962) gives a curve that can be expressed by the equation,

$$\rho C'V = A(C' - C)^n \quad . \tag{1}$$

The value of "n" can be shown to be about 1.6 at the start of the curve; approximately 2 in the range 10 to 50 kilobars, and increasing to 2.03 over the final portion. Further, for n = 2, A equals 0.68 (when velocities are in km/sec, and pressures in kilobars). Thus in the range of interest for high velocity liquid impact,

$$\rho C'V = 0.68(C' - C)^2 \tag{2}$$

which can be solved to give, assuming $\rho = 1$

$$C' = C + \frac{V}{2A} + \left\{\frac{V^2}{4A^2} + \frac{V}{A}C\right\}^{1/2} \quad . \tag{3}$$

In his paper Heymann gives a relatively simple expression for calculating C' from C and V, but it is only applicable to about V = C. Equation (3), above, is valid over a much wider range. This equation has been used to calculate C for values of V up to 3C (4350 metres/sec). Fig. 3 gives the plot of pressure $\rho C'V$ against V and Fig. 4 shows how the ratio $\frac{X}{r} = \frac{V}{C'}$, increases for values of V equal to 0 to 3C. It is quite clear from these figures that it is essential to replace the sound velocity C by the shock velocity C' as soon as V exceeds a few metres/sec. Failure to do so gives incorrect predictions for both the impact pressures and the area of contact for compressible behaviour.

In Miss Engel's discussion on the Heymann paper, and in much of her published work, she does not replace C by the shock velocity (C'). The figures she quotes in her Table 1 are for impact velocities of C/100, C/5 and C; where there are calculated values they would need modifying, some quite considerably. Similar corrections would also have to be applied to Miss Engel's other data.

1. Bahrani, A.S., Black, T.J. and Crossland, B., 1967, Proc. Roy. Soc. A, <u>296</u>, 123
2. Bowden, F.P. and Field, J.E., 1964, Proc. Roy. Soc., A, <u>282</u>, 331
3. Cook, M.A., Keyes, R.T., and Ursenbach, W.D., 1962, J. Appl. Phys. <u>33</u>, 3413
4. Engel, O.G., 1955, J. Res. Nat. Bur. Stand., Wash., 54, 281
5. Field, J.E., 1962, Ph.D. Thesis, Cambridge
6. Harlow, F.H., and Pracht, W.E., 1966, Phys. Fluids, <u>9</u>, 1951
7. Walsh, J.M., Shreffler, R.G., and Willig, F.J., 1953, J. Appl. Phys. <u>24</u>, 349

<u>Captions</u>

Fig. 3 shows the variation of ρCV and $\rho C'V$ with V.

Fig. 4 shows the dependence of the ratio $\frac{X}{r}$ on V.

<u>Dr. Brunton</u>

The point about the origin of the high speed radial flow is an interesting one. It must be true that the high velocities are associated with a large pressure gradient near the edge of the drop. The dependence of the radial velocity on the drop shape has lead us to suppose that the pressure gradient arises from the constricted flow around the air wedge between the liquid and solid. The action here is the same as that which produces the jet in the shaped charge and the re-entrant jetting in explosive welding. In these processes the solids are behaving as fluids. In liquid impact we are dealing directly with fluids so that it would be surprising if such jetting actions were not to be found.

An alternative approach to the radial flow problem is to relate the velocity to a stagnation pressure which is taken to be the water-hammer pressure. Supporting experimental evidence is available which favours this explanation. However, a criticism of this approach is that while the water-hammer pressure is calculated assuming non-steady stress wave effects, the radial flow velocity calculation neglects wave effects and treats the flow of the same compressed liquid as a steady state process. The stagnation pressure is assumed to be the non-steady water-hammer value.

The point that the release process is not the reverse of the impact compression is true to the extent that release waves are necessarily broader than compression shocks. However, it is still a discontinuous process with the wave fronts moving at acoustic velocities appropriate to conditions in the liquid. It is arguable whether one can treat the pressure gradient across such a wave by the simple Bernoulli equation.

The shaped charge jetting action is concerned with the steady flow of liquid around the wedge angle. As seen in a moving coordinate system this is a steady process so that pressure and velocity can be related in the usual way by the Bernoulli equation. Flow of this nature might be expected to give rise to the high edge pressures described in Ref.28 in the paper.

In our studies of single impact damage at high velocities we have found evidence of deformation caused by outward flow which is in many respects similar to that observed in explosive welding. The surface around the edge of the impact area is rippled and waved in a regular pattern and in a manner reminiscent of surface waves in explosive welding or in solid projectile impact. Again, we have found that in hard metals where the overall deformation is slight that a deeper indentation is sometimes observed around the edge of the contact area than is found elsewhere. We are of the opinion that high edge pressures are possible but that they are of very short duration over any given area and that normally their effects are masked by general deformation.

Mr. Heymann

Dr. Brunton's discussion is a welcome contribution to a topic which seems to me rather unsettled at this time, and is worth being pursued more thoroughly. I did not mean to suggest that a "wedge" or "shaped charge" effect could not occur, and any language that carries this implication is regretted and should be ignored.

With regard to the acceleration of the fluid particles across the pressure release wave, I am not knowledgeable enough in fluid mechanics to take a strong position. While I still doubt that it can be treated as the inverse of the compression shock process, further thought causes me, too, to doubt the applicability of the simple Bernouilli equation which, after all, is valid only for incompressible (and loss-less) flow, which we certainly do not have here.

I cannot agree, however, that the shaped charge jetting action is more analogous to a steady-state process than is the action of a pressure release wave, at least in the one-dimensional case which would be the initial model. Both are transformable to steady-state flow processes by using suitably moving coordinates. The high velocities and implicit pressures in the supposed jetting action must, likewise, entail compressible fluid behavior and therefore make the Bernouilli equation strictly inapplicable here also.

Nevertheless, let us see what results would be predicted for a round drop impact, by combining the wedge jetting action assumptions of Ref. 24 with the value of the wedge angle which occurs at the moment that lateral flow becomes possible, by the model of Ref. 23 or by my equation (16). Making the usual small-angle approximations, the wedge angle ϕ is V_o/C, and the radial flow velocity V_R, given in Ref. 24 as $V_R = V_o \cot (\phi/2)$, becomes simply 2C. V_R is therefore predicted to be independent of the impact velocity V_o except insofar as C is dependent on it. Thus the "velocity magnification", V_R/V_o, is $2C/V_o$, compared to $\sqrt{2C/V_o}$ given by equation (14) on the basis of the Bernouilli relationship from the waterhammer pressure. The maximum stagnation pressure by this model can be shown to be $(1/2)\ \rho\ V_o^2 \operatorname{cosec}^2 \phi$, which becomes simply $(1/2)\ \rho\ C^2$, also not directly dependent on V_o. This represents the pressure predicted by this model at the perimeter of the impact area, at the moment considered. Note that nothing comparable to the waterhammer or shock impact pressure enters into this model at all: the dependence on C is brought about through defining the wedge angle ϕ as that contact angle at which the "edge Mach number" becomes subsonic and flow can initiate.

Rigorous analysis of the impact pressure distribution under a round or wedge-shaped body[28] also predicts high pressures at the perimeter, greater than the water-hammer pressure ρCV. Thus, even the direct application of the Bernouilli relationship, to the pressure in the region from which the lateral flow initiates, would yield velocities higher than given by equation (14) if that local pressure were known. Ref.28 is of no direct help in this matter, however, since the only results given are those for wedges, for which the edge pressure at subsonic edge Mach numbers is calculated to be infinite.

The many simplifications and uncertainties inherent in these two competing models will become obvious upon reflection; nevertheless this discussion may point the way to further analytical or experimental studies which could help to resolve this interesting question.

Dr. G. Langbein

The main physical reason for a drop size effect must be, that the smaller drops produce lower surface roughness connected with smaller breakouts. Smaller breakouts have a higher surface energy per volume. Therefore the erosion strength for small drops should be higher, especially for brittle materials (as reported in paper 4.6.2).

If the surface energy is small compared with the inner energy, in the case of ductile materials, there should be no drop size effect if one would normalize the erosion depth e by the average particle size a and the length w of the impinging water volume by the drop diameter d. In other words, there must be a general function g defined by the relation

$$\frac{e}{a} = g\left(\frac{w}{d}\right) .$$

This function one can calculate by means of statistics. The reason for a drop size independant erosion function is the similarity of the pressure field at large and little drops

$$p = \frac{v}{\frac{1}{\rho_1 c_1} + \frac{1}{\rho_2 c_2}} \quad f\left(\frac{x}{r}, \frac{x}{r}, \frac{z}{r}, \frac{tc_1}{r}\right)$$

if a drop hits a continuum.

r = drop radius

c = sound velocity

ρ = density

1 = water

2 = specimen material

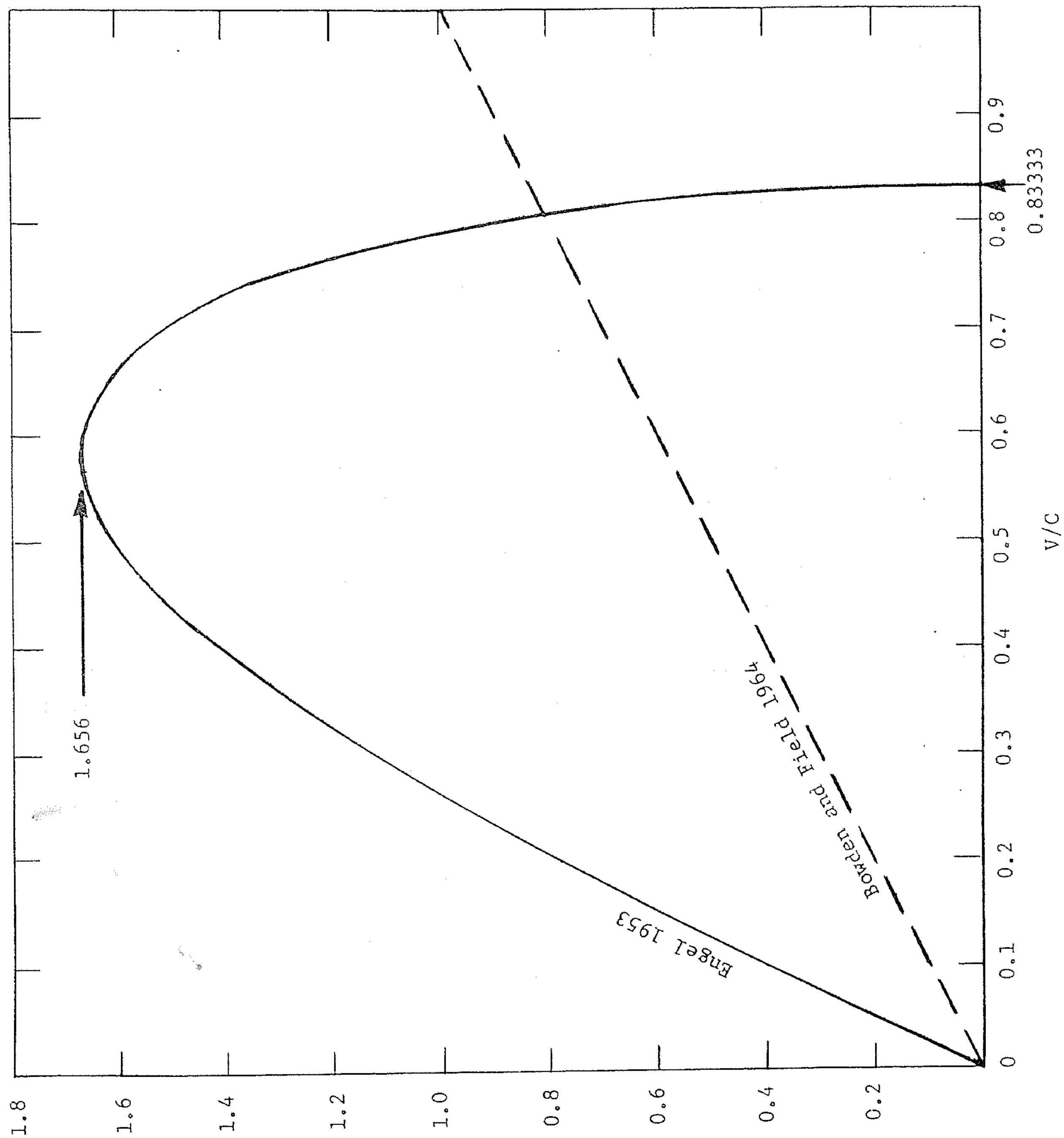
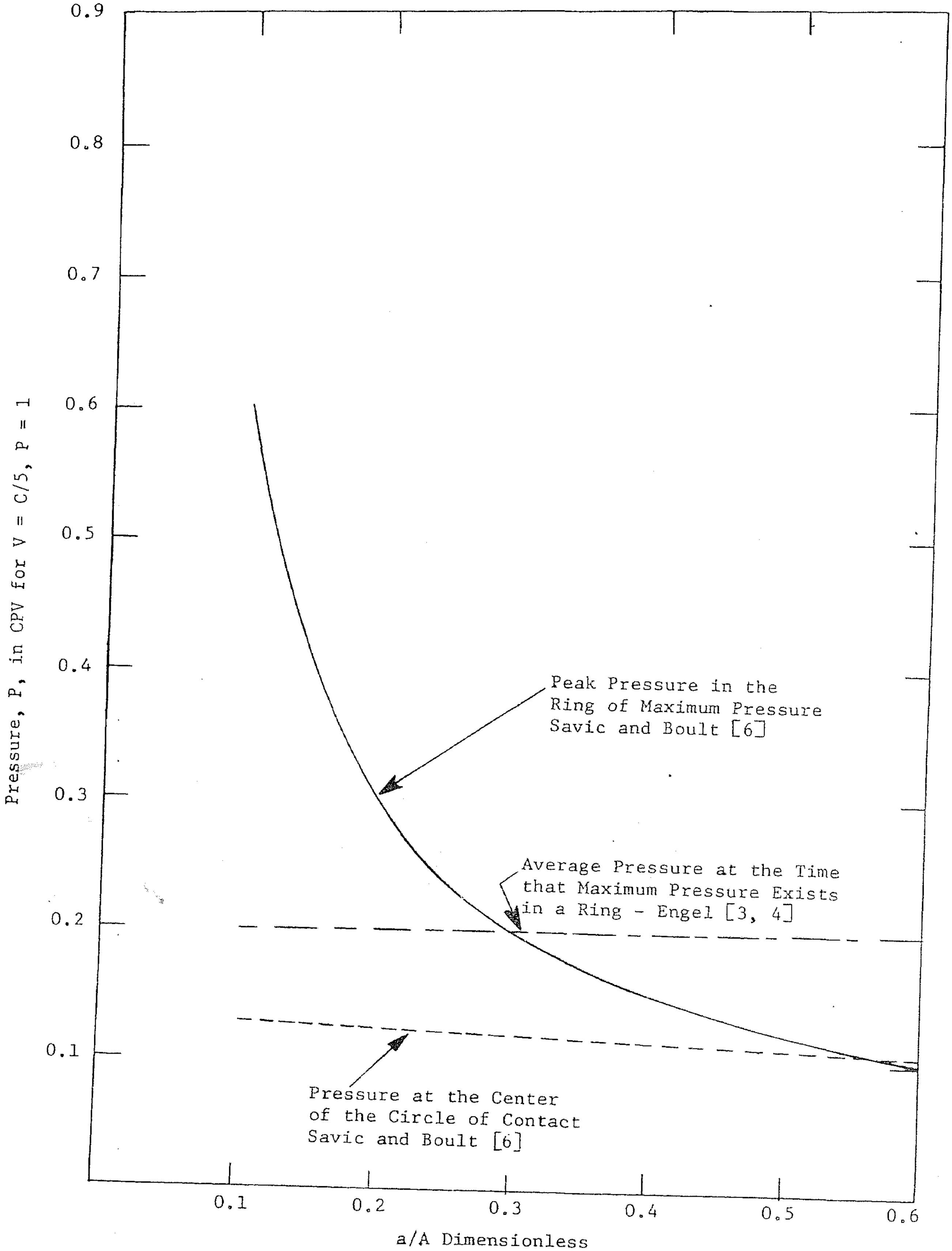

1.656
0.83333
Engel 1953
Bowden and Field 1964
V/C
$X_{2\delta}/R$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

Fig.I



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Pressure, P, in CPV for V = C/5, P = 1
Peak Pressure in the
Ring of Maximum Pressure
Savic and Boult [6]
Average Pressure at the Time
that Maximum Pressure Exists
in a Ring - Engel [3, 4]
Pressure at the Center
of the Circle of Contact
Savic and Boult [6]
0.1
0.2
0.3
0.4
0.5
0.6
a/A Dimensionless

Fig. 2

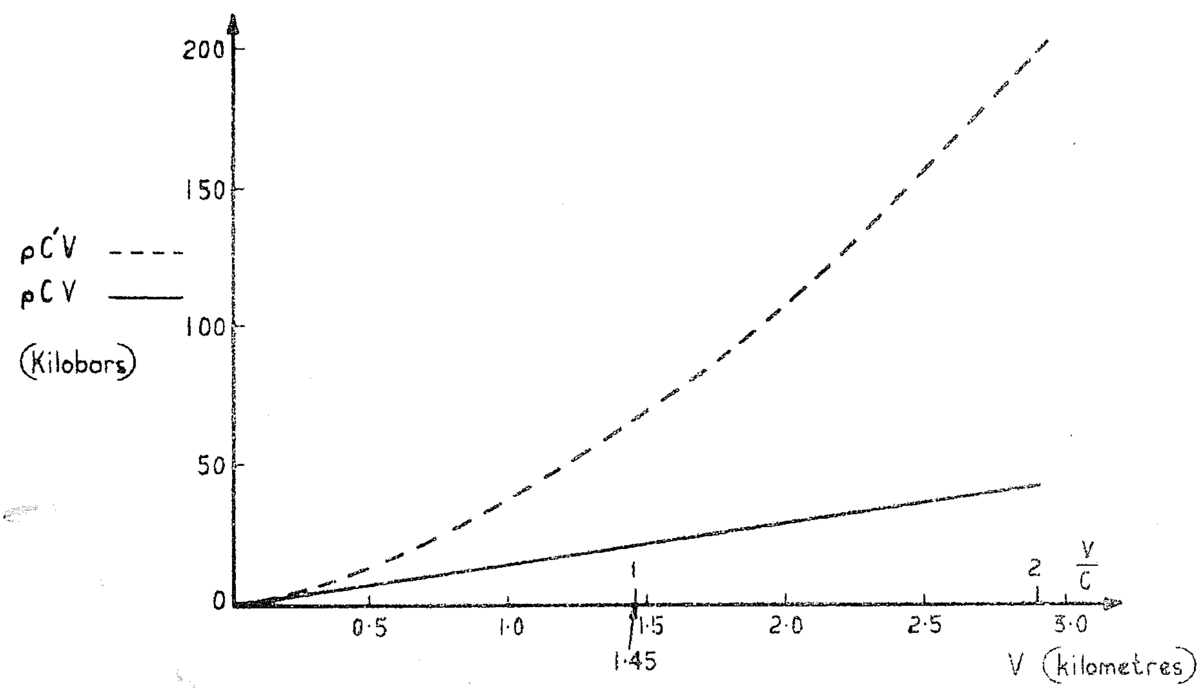


Fig.3 Variation of ρCV and ρC′V with V

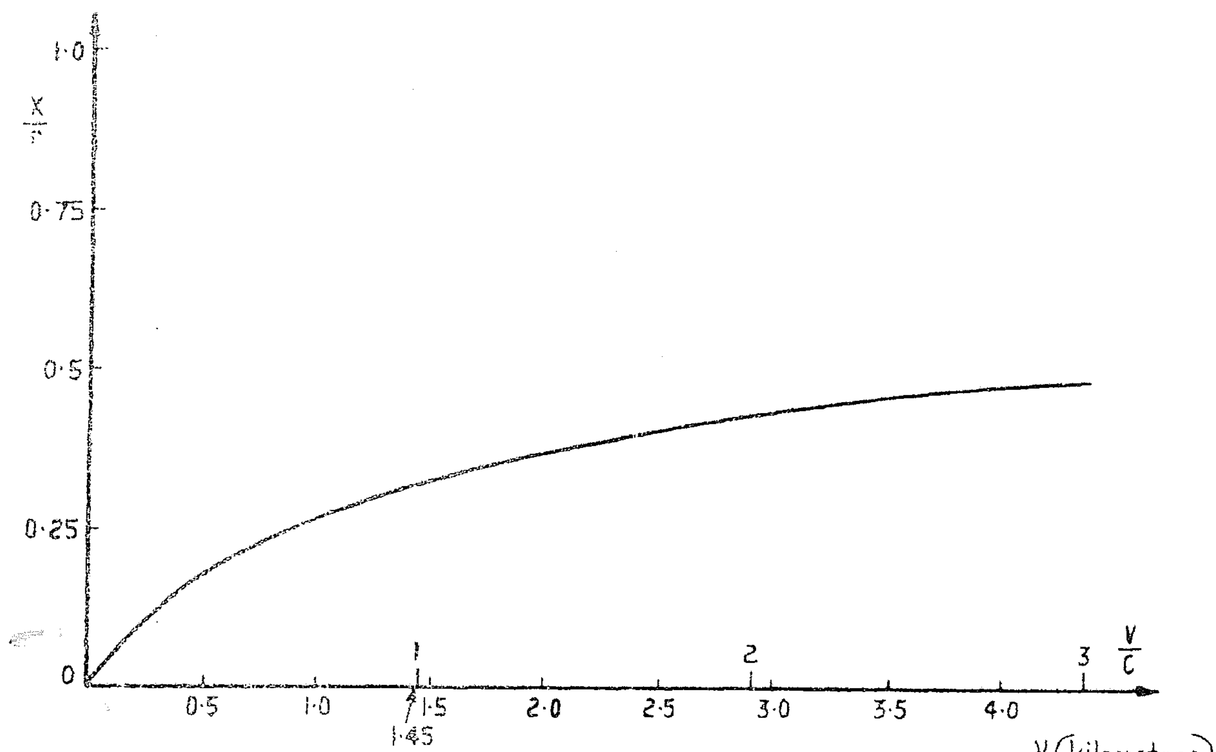


Fig. 4 Dependence of the ratio $\frac{X}{r}$ on V