

Examination Cover Sheet

<b>COURSE:</b> ELEC	NUMBER: 273	SECTION(S): D, R	
<b>EXAMINATION:</b> <input checked="" type="checkbox"/> <b>FINAL</b> <input type="checkbox"/> ALTERNATE <input type="checkbox"/> DEFERRED <input type="checkbox"/> MID TERM <b>VERSION:</b> _____	<b>DATE:</b> December 14, 2018	TIME: 2:00-5:00  Exam Length: 3 hours	PAGES: 4  Including cover
<b>INSTRUCTOR(S):</b> Dr. C. W. Trueman, Dr. F. Fayyaz		<b>DIVISION:</b>	
<b>MATERIALS ALLOWED:</b> <input checked="" type="checkbox"/> Booklets <input type="checkbox"/> IBM (Scantron) <input type="checkbox"/> Blue <input type="checkbox"/> Green <input type="checkbox"/> Printed Translation Dictionary Other _____ <input checked="" type="checkbox"/> Calculator <input checked="" type="checkbox"/> ENCS Approved <input type="checkbox"/> Other _____		<b>INSTRUCTIONS:</b> <input checked="" type="checkbox"/> Return all <input type="checkbox"/> Answer on Exam <input type="checkbox"/> Open book <input type="checkbox"/> Crib sheet Details _____	

Please print your name, I.D. number and section in the appropriate spaces below.

**STUDENT NAME:** \_\_\_\_\_

**I.D. NO.** \_\_\_\_\_ **SECTION:** \_\_\_\_\_

**SPECIAL INSTRUCTIONS:**

Closed book exam. No books or notes are permitted.

Do not tear pages out of the exam booklet.

Cell phones or other electronic devices are forbidden in final examinations.

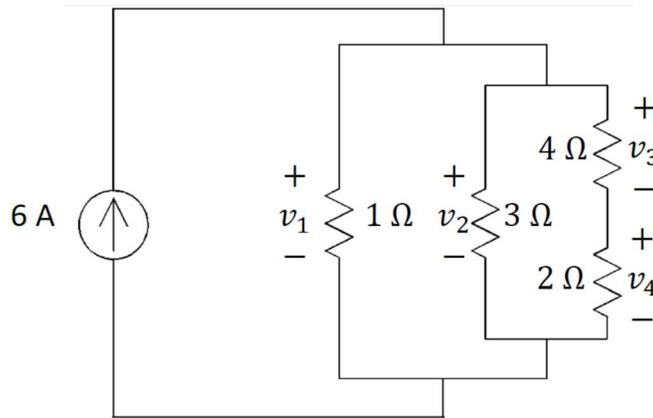


Figure 1a

1a) Find the voltages  $v_1$ ,  $v_2$ ,  $v_3$ , and  $v_4$  in the circuit of Figure 1a. (4 marks)

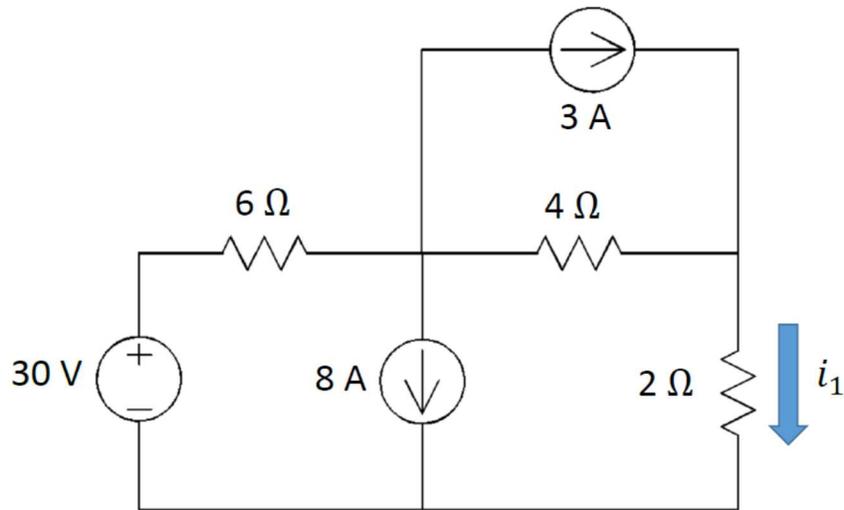


Figure 1b

1b) For the circuit of Figure 1b, calculate current through 2Ω resistor using superposition theorem. (6 marks)

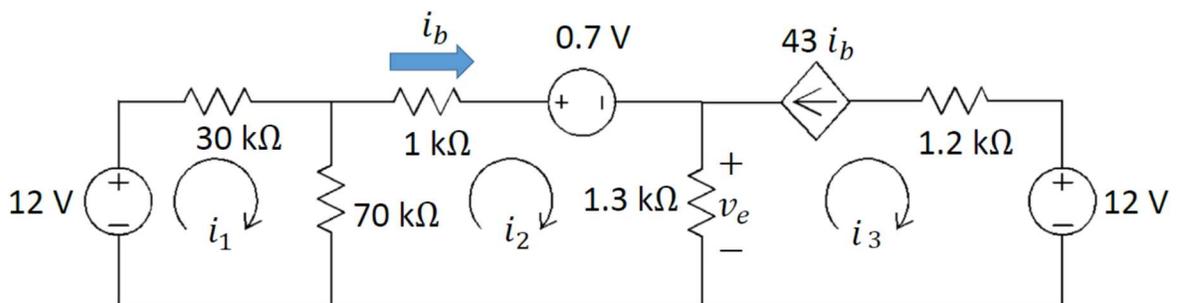


Figure 2

2a) For the circuit of Figure 2, write a set of mesh equations using mesh currents  $i_1$ ,  $i_2$  and  $i_3$ . (6 marks)

2b) Solve the mesh equations to find  $i_b$ . (2 marks)

2c) Find the value of  $v_e$ . (2 marks)

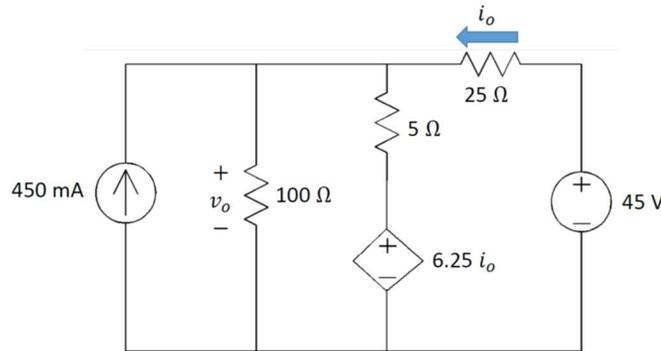


Figure 3

3a) For the circuit of Figure 3, find the value of  $v_o$  using nodal analysis. (6 marks)

3b) Find the value of  $i_o$ . (4 marks)

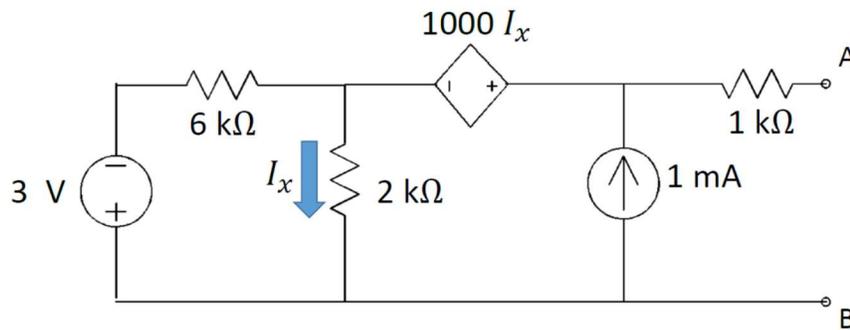


Figure 4

For the circuit of Figure 4:

4a) Find the open circuit voltage across terminals AB. (3 marks)

4b) If terminals AB are connected together by a short circuit, find the current flowing in the short circuit. (3 marks)

4c) Find the Thevenin Equivalent Circuit at terminals AB. (2 marks)

4d) If a load resistor of 2 kΩ is connected across terminals AB, what is the voltage across the load resistor? (2 marks)

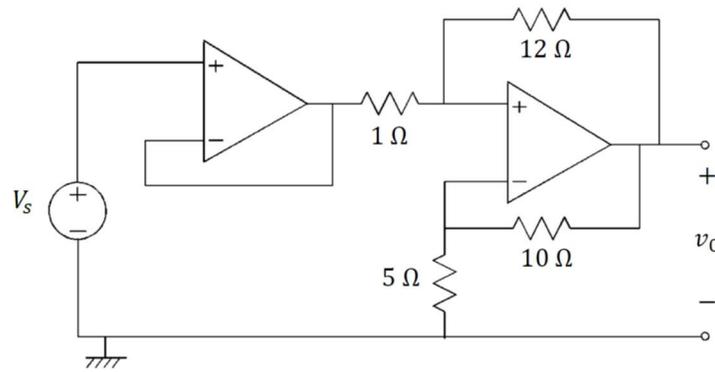


Figure 5

5. Find the output voltage  $v_o$  of the operational amplifier circuit of Figure 5, with  $V_s = 5$  volts. Assume that the op-amps are ideal with infinite gain. (10 marks)

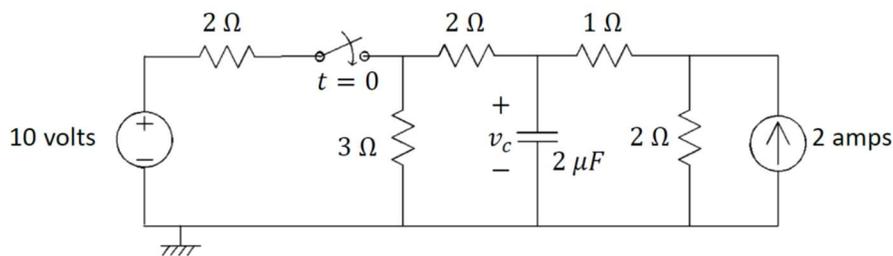


Figure 6

In the circuit of Figure 6, the switch has been open for a long time. At  $t = 0$  the switch closes and remains closed for  $t > 0$ .

- 6a) Find the initial value of the capacitor voltage  $v_c$  just after the switch closes. (2 marks)
- 6b) Find the final value of the capacitor voltage  $v_c$  as  $t \rightarrow \infty$ . (2 marks)
- 6c) Find the time constant  $\tau$ . (2 marks)
- 6d) Write the equation giving the capacitor voltage  $v_c(t)$  as a function of time for  $t > 0$ . (2 marks)
- 6e) What is the value of the capacitor voltage at  $t = 2.2\tau$ ? (2 marks)

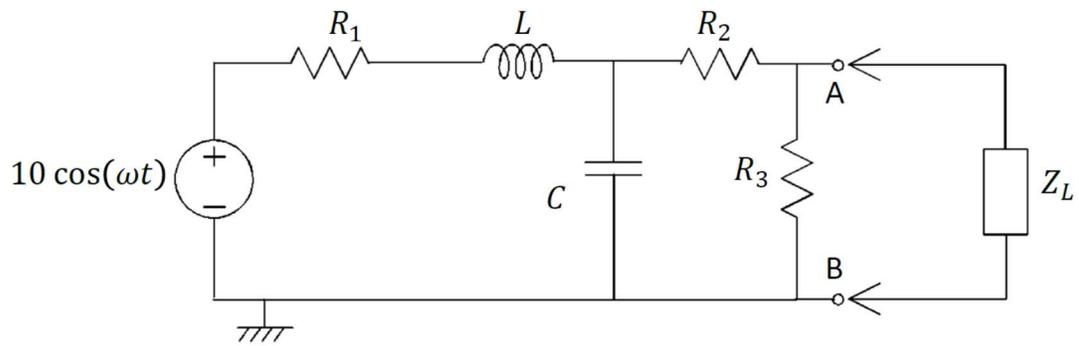


Figure 7

In the circuit of Figure 7, the operating frequency is 60 Hz. The component values are  $R_1 = 1 \Omega$ ,  $R_2 = 2 \Omega$ ,  $R_3 = 2 \Omega$ ,  $L = 2.653$  milliHenries, and  $C = 1,326$  microFarads.

- 7a) With the load impedance  $Z_L$  removed, find the Thevenin equivalent circuit at terminals AB. (6 marks)
- 7b) With a load impedance  $Z_L$  connected to terminals AB, what value of  $Z_L$  dissipates the maximum amount of average power? (2 marks)
- 7c) With  $Z_L$  chosen as in question 7b, how much power does  $Z_L$  dissipate? (2 marks)

# Solution to the ELEC 273 Final Exam 2018

December 18, 2018

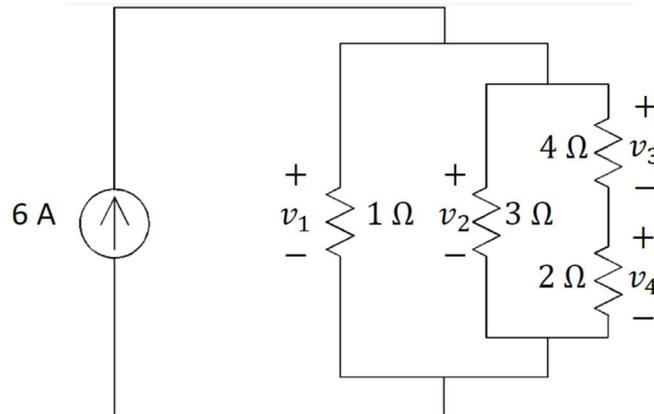
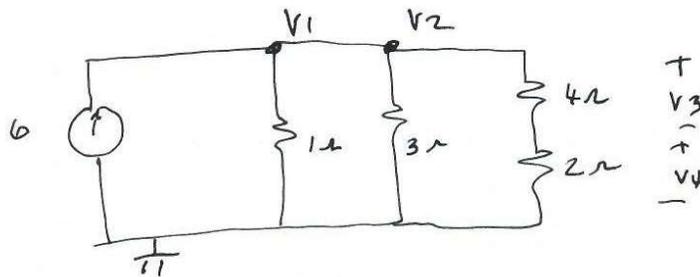


Figure 1a

1a) Find the voltages  $v_1$ ,  $v_2$ ,  $v_3$ , and  $v_4$  in the circuit of Figure 1a. (4 marks)



$$R_{1v} = 1 \parallel 3 \parallel (4+2)$$

$$1 \parallel 3 = \frac{1 \times 3}{1+3} = \frac{3}{4}$$

$$\frac{3}{4} \parallel 6 = \frac{\frac{3}{4} \times 6}{\frac{3}{4} + 6} = \frac{18}{27} = \frac{6}{9} = \frac{2}{3}$$

$$V_1 = V_2 = 6 \text{ A} \times \frac{2}{3} \Omega = \frac{12}{3} \text{ V} = 4 \text{ V.}$$

$$V_3 = V_2 \times \frac{4}{6} = \frac{4 \times 4}{6} = \frac{16}{6} = \frac{8}{3} \text{ V.}$$

$$V_4 = V_2 \times \frac{2}{6} = \frac{4 \times 2}{6} = \frac{8}{6} = \frac{4}{3} \text{ V.}$$

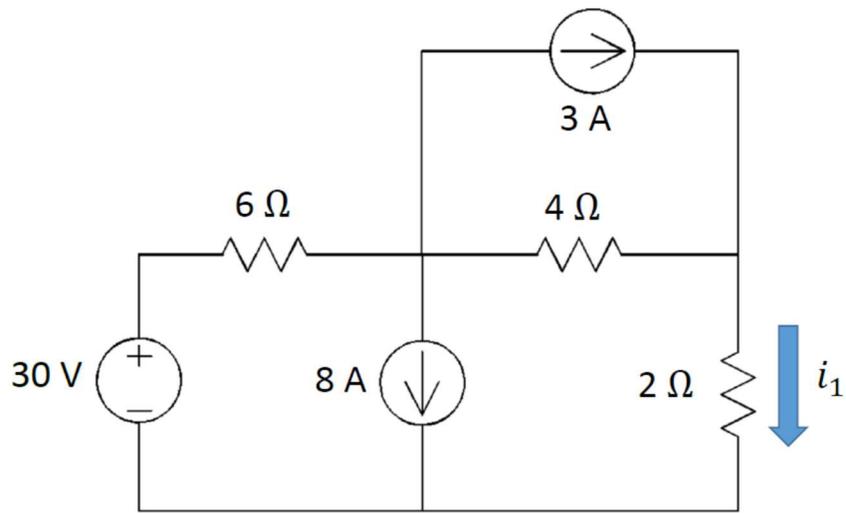
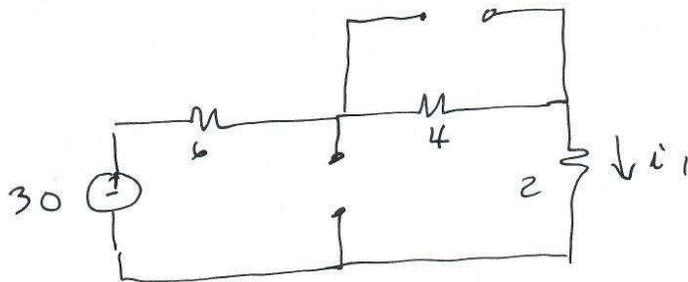


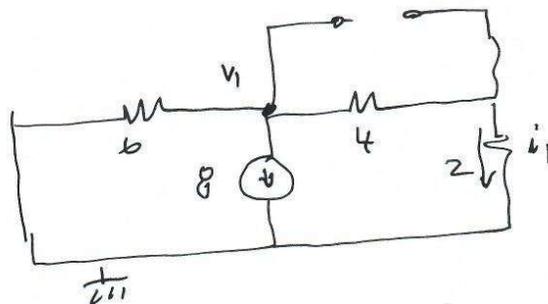
Figure 1b

1b) For the circuit of Figure 1b, calculate current through  $2\Omega$  resistor using superposition theorem. (6 marks)

1b)



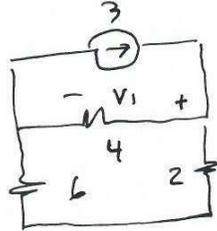
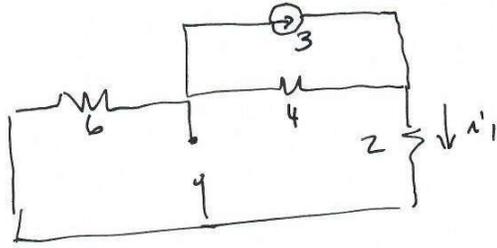
$$i_1 = \frac{30}{2+4+6} = \frac{30}{12} = \frac{5}{2} \text{ A.}$$



$$6 \parallel (2+4) = 6 \parallel 6 = \frac{36}{12} = 3 \Omega$$

$$V_1 = -3 \times 8 = -24 \text{ V.}$$

$$i_1 = \frac{V_1}{6} = \frac{-24}{6} = -4 \text{ A.}$$



$$4 \parallel (2+6) = 4 \parallel 8 = \frac{32}{12} = \frac{8}{3} \Omega$$

$$V_1 = 3 \times \frac{8}{3} = 8 \text{ volts}$$

$$i_1 = \frac{V_1}{8} = 1 \text{ A.}$$

.. ..

Super position

$$i_1 = \frac{5}{2} - 4 + 1 = \frac{5-8+2}{2} = -\frac{1}{2}$$

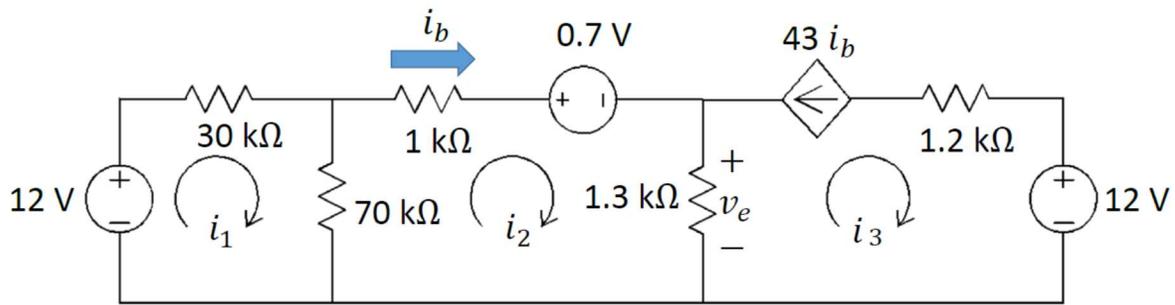


Figure 2

2a) For the circuit of Figure 2, write a set of mesh equations using mesh currents  $i_1$ ,  $i_2$  and  $i_3$ . (6 marks)

2b) Solve the mesh equations to find  $i_b$ . (2 marks)

2c) Find the value of  $v_e$ . (2 marks)

$$2a) \textcircled{I} \quad 12 - 30i_1 - 70(i_1 - i_2) = 0$$

$$\textcircled{II} \quad 70(i_1 - i_2) - i_2 - 0.7 - 1.3(i_2 - i_3) = 0$$

$$\text{Constraint} \quad i_3 = -43i_b$$

$$\text{where} \quad i_b = i_2$$

$$\text{so} \quad i_3 = -43i_2$$

2b) Solve

$$\textcircled{I} \quad -30i_1 - 70i_1 + 70i_2 = -12$$

$$+100i_1 - 70i_2 = 12$$

$$\textcircled{II} \quad 70i_1 - 70i_2 - i_2 - 1.3i_2 + 1.3i_3 = 0.7$$

$$70i_1 - 72.3i_2 + 1.3i_3 = 0.7$$

$$\text{Since } i_3 = -43 i_2$$

$$70 i_1 - 72.3 i_2 + 1.3(-43) i_2 = -0.7$$

$$70 i_1 - 72.3 i_2 - 55.9 i_2 = -0.7$$

$$70 i_1 - 128.2 i_2 = -0.7$$

$$i_1 = \frac{-0.7 + 128.2 i_2}{70}$$

$$\textcircled{I} \quad 100 \left( \frac{-0.7 + 128.2 i_2}{70} \right) - 70 i_2 = 12$$

$$100(-0.7 + 128.2 i_2) - 4900 i_2 = 840$$

$$-70 + 12820 i_2 - 4900 i_2 = 840$$

$$7920 i_2 = 770$$

$$i_2 = 0.096977 \text{ mA}$$

$$i_n = i_2 = 0.096977 \text{ mA}$$

$$V_e = 1.3(i_2 - i_3)$$

$$= 1.3(i_2 - (-43 i_2))$$

$$= 1.3(44 i_2)$$

$$= 57.2 i_2$$

$$= 57.2 \times 0.096977$$

$$V_e = 5.547 \text{ volts}$$

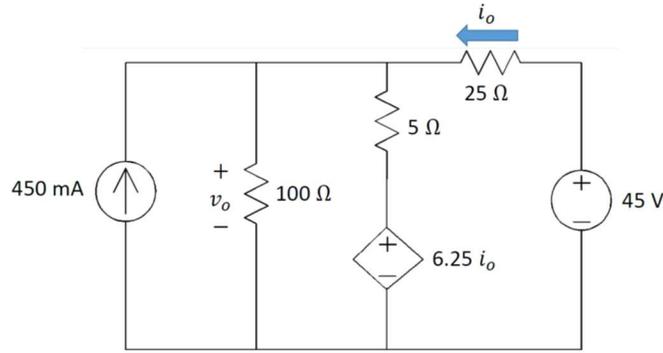
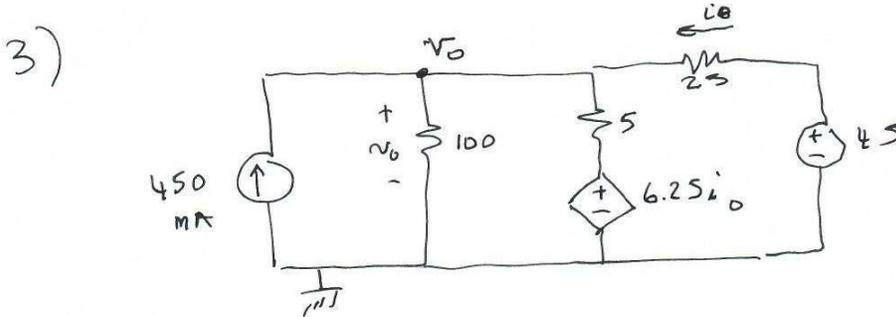


Figure 3

3a) For the circuit of Figure 3, find the value of  $v_o$  using nodal analysis. (6 marks)

3b) Find the value of  $i_o$ . (4 marks)



$$\textcircled{I} \quad 0.45 - \frac{v_o}{100} - \frac{v_o - 6.25i_o}{5} - \frac{v_o - 45}{25} = 0$$

$$\text{Constraint} \quad i_o = \frac{45 - v_o}{25}$$

$$\textcircled{II} \quad 45 - v_o - 20v_o + 125i_o - 4v_o + 180 = 0$$

$$-25v_o + 125i_o = -225$$

$$-25v_o + 125 \left( \frac{45 - v_o}{25} \right) = -225$$

$$-25v_o + 225 - 5v_o = -225$$

$$-30v_o = -450$$

$$v_o = 15 \text{ volts}$$

$$i_o = \frac{45 - v_o}{25} = \frac{45 - 15}{25} = \frac{30}{25} = \frac{6}{5} \text{ A}$$

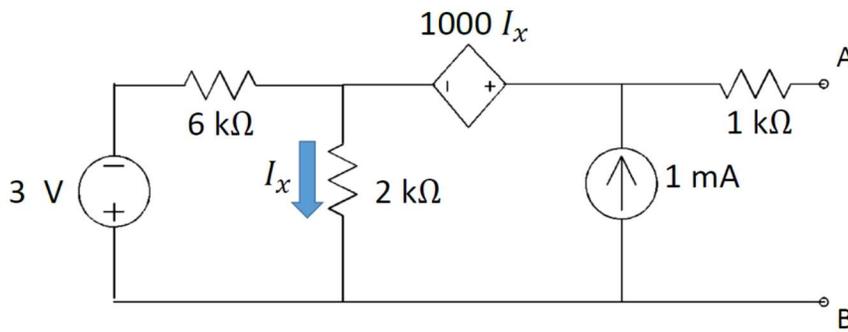
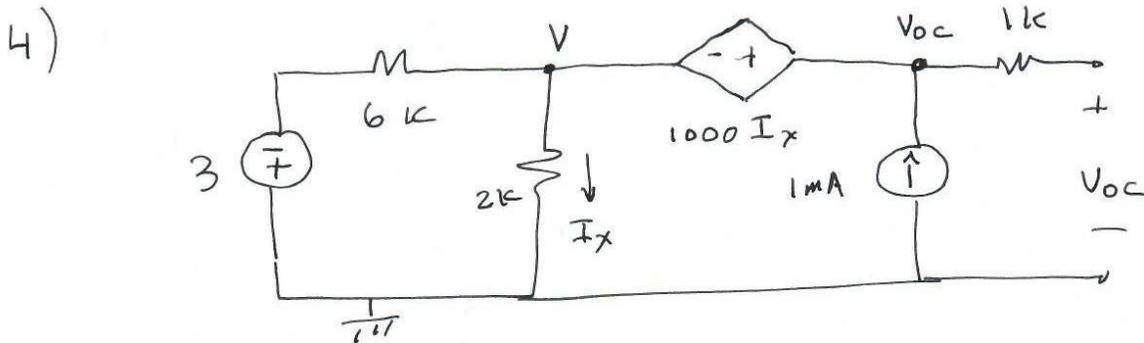


Figure 4

For the circuit of Figure 4:

- 4a) Find the open circuit voltage across terminals AB. (3 marks)
- 4b) If terminals AB are connected together by a short circuit, find the current flowing in the short circuit. (3 marks)
- 4c) Find the Thevenin Equivalent Circuit at terminals AB. (2 marks)
- 4d) If a load resistor of 2 kΩ is connected across terminals AB, what is the voltage across the load resistor? (2 marks)



Use volts, amps and ohms.

$$-\frac{3}{6000} - \frac{V}{2000} + 0.001 = 0$$

$$\times 6000 \quad -3 - V - 3V + 6 = 0$$

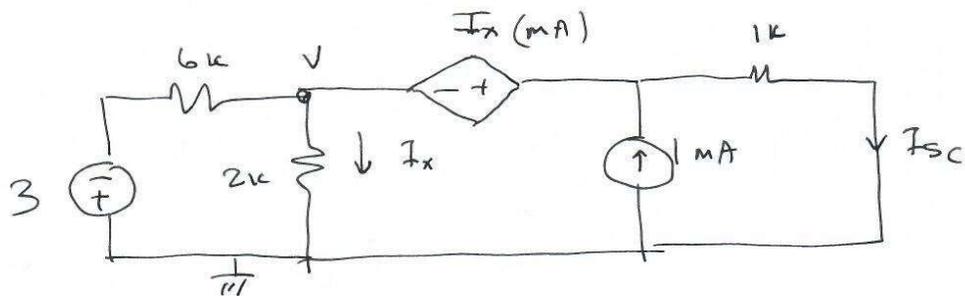
$$-4V = -3$$

$$V = \frac{3}{4} \text{ volts}$$

$$I_x = \frac{V}{2000} = 3.75 \times 10^{-4} \text{ Amps}$$

$$V_{OC} = V + 1000 I_x$$

$$= \frac{3}{4} + 0.375 = 1.125 \text{ volts}$$



$$\frac{-3 - V}{6000} - \frac{V}{2000} + 0.001 - \frac{V + 1000 I_x}{1000} = 0$$

$\times 6000$

$$-3 - V - 3V + 6 - 6V - 6000 I_x = 0$$

$$-10 V - 6000 I_x = -3$$

$$I_x = \frac{V}{2000}$$

$$-10 V - 6000 \frac{V}{2000} = -3$$

$$-10 V - 3V = -3$$

$$V = \frac{3}{13} \text{ volts}$$

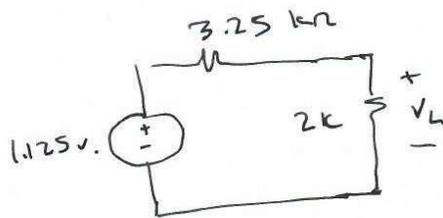
$$= 0.2308 \text{ volts}$$

$$I_{SC} = \frac{V + 1000 I_x}{1000} = \frac{V + 1000 \frac{V}{2000}}{1000}$$

$$I_{sc} = \frac{\frac{3}{2} V}{1000} = \frac{\frac{3}{2} \times \frac{3}{13}}{1000} = 3.46 \times 10^{-4} \text{ Amps}$$

$$= 0.346 \text{ mA}$$

$$R_T = \frac{V_{oc}}{I_{sc}} = \frac{1.125}{0.346} = 3.25 \text{ k}\Omega$$



$$V_L = 2 \frac{1.125}{2 + 3.25} = 0.429 \text{ volts}$$

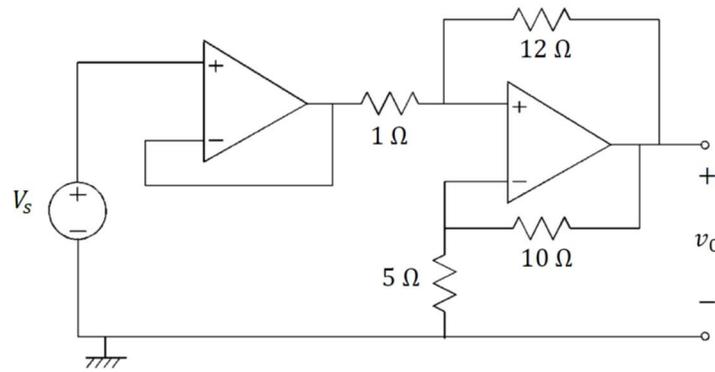
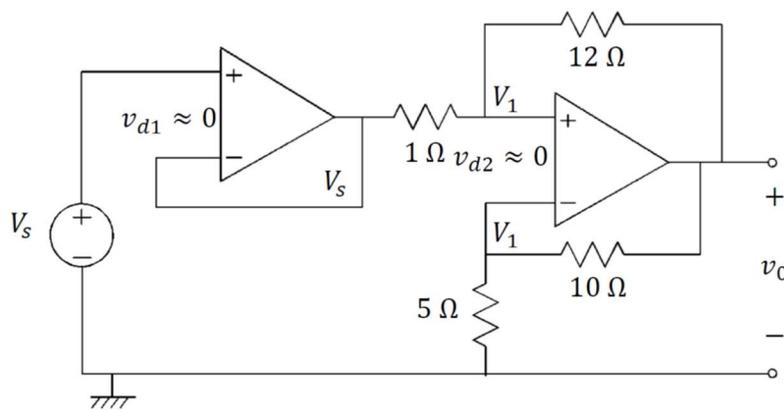


Figure 5

5. Find the output voltage  $v_o$  of the operational amplifier circuit of Figure 5, with  $V_s = 5$  volts. Assume that the op-amps are ideal with infinite gain. (10 marks)

SOLUTION



The first op-amp is a "current blocker" and provides a high input resistance to source  $V_s$ , the output voltage is  $V_s$ .

$$\textcircled{I} \quad \frac{V_s - V_1}{1} - \frac{V_1 - v_o}{12} = 0$$

$$\textcircled{II} \quad -\frac{V_1}{5} - \frac{V_1 - v_o}{10} = 0$$

$$\textcircled{J} \quad 12 V_S - 12 V_1 - V_1 + V_0 = 0$$

$$13 V_1 - V_0 = 12 V_S$$

$$\textcircled{H} \quad -2 V_1 - V_1 + V_0 = 0$$

$$-3 V_1 = -V_0$$

$$V_1 = \frac{V_0}{3}$$

$$\textcircled{I} \quad 13 \left( \frac{V_0}{3} \right) - V_0 = 12 V_S$$

$$13 V_0 - 3 V_0 = 36 V_S$$

$$10 V_0 = 36 V_S$$

$$V_0 = 3.6 V_S$$

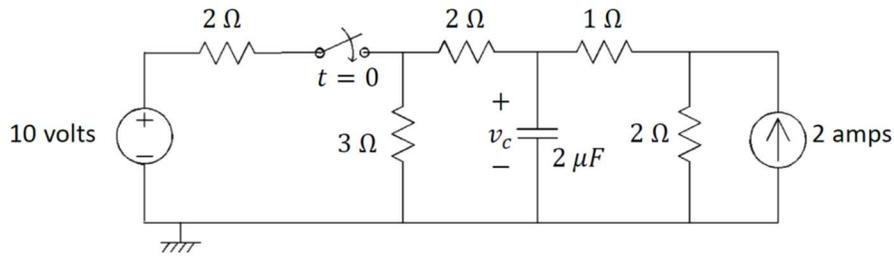
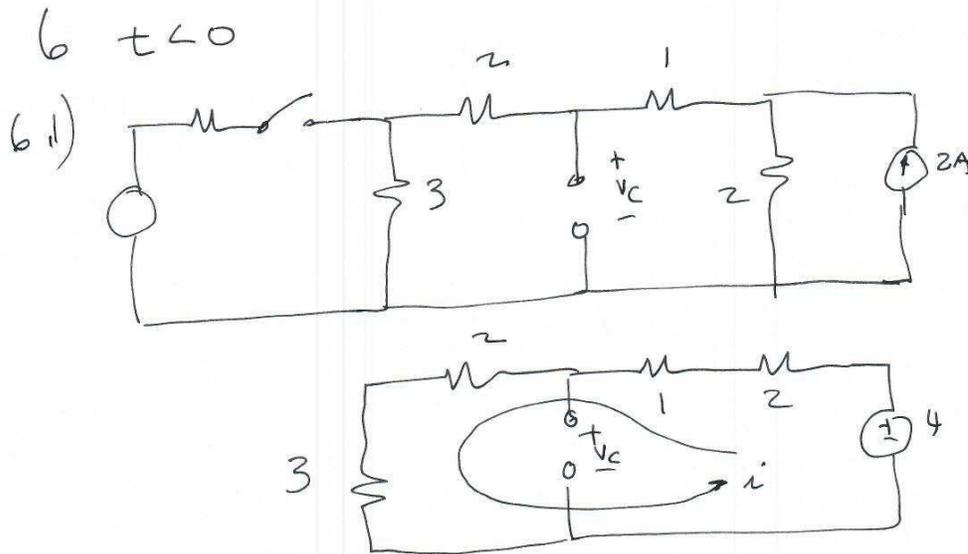


Figure 6

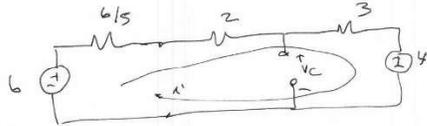
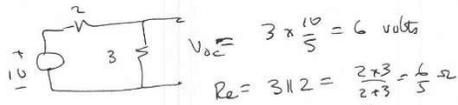
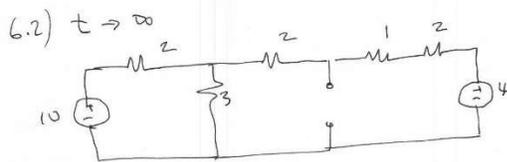
In the circuit of Figure 6, the switch has been open for a long time. At  $t = 0$  the switch closes and remains closed for  $t > 0$ .

- 6a) Find the initial value of the capacitor voltage  $v_c$  just after the switch closes. (2 marks)
- 6b) Find the final value of the capacitor voltage  $v_c$  as  $t \rightarrow \infty$ . (2 marks)
- 6c) Find the time constant  $\tau$ . (2 marks)
- 6d) Write the equation giving the capacitor voltage  $v_c(t)$  as a function of time for  $t > 0$ . (2 marks)
- 6e) What is the value of the capacitor voltage at  $t = 2.2\tau$ ? (2 marks)

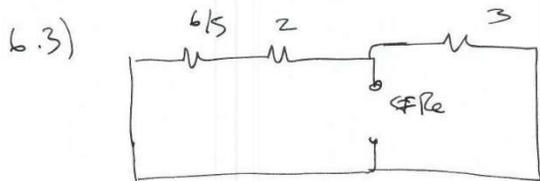


$$i = \frac{4}{8} = \frac{1}{2} \text{ A}$$

$$v_c = 5i = \frac{5}{2} \text{ V} \quad \text{so } v_c(0) = \frac{5}{2} \text{ volts}$$



$i = \frac{6-4}{\frac{6}{5}+2+3} = \frac{2 \times 5}{6+25} = \frac{10}{31} \text{ A}$   
 $V_c = 3i - 4 = \frac{30}{31} - 4 = \frac{30-124}{31} = \frac{-94}{31} \text{ V}$



$R_e = (2 + \frac{6}{5}) || 3 = \frac{16}{5} || 3$   
 $= \frac{\frac{16}{5} \times 3}{\frac{16}{5} + 3} = \frac{48}{31} \Omega$

$\tau = R_e C = \frac{48}{31} \times 2 \mu\text{F}$   
 $= \frac{96}{31} \mu\text{s} = 3.097 \mu\text{s}$

6.4)  $V_c(t) = V_{final} + (V_{initial} - V_{final}) e^{-t/\tau}$

$= \frac{154}{31} - \left( \frac{5}{2} - \frac{154}{31} \right) e^{-t/3.097}$

$= 4.968 - 2.468 e^{-t/3.097}$

6.5)  $t = 3.2 \mu\text{s}$

$V_c(3.2 \mu\text{s}) = 4.968 - 2.468 e^{-\frac{3.2}{3.097}}$   
 $= 4.695 \text{ volts}$

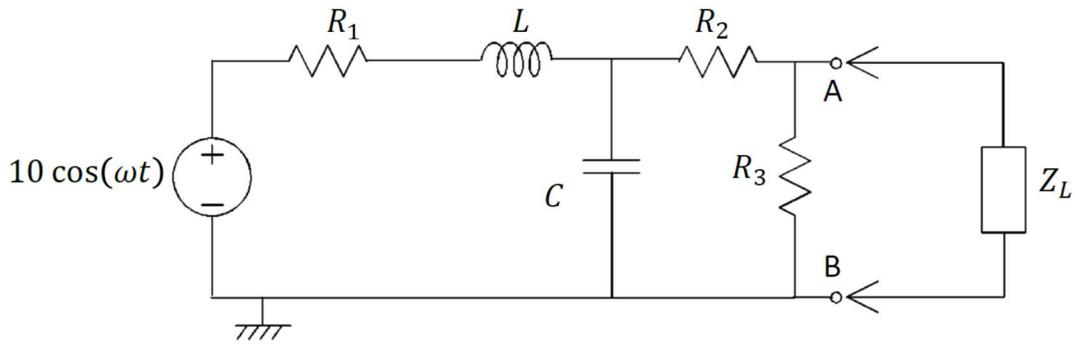


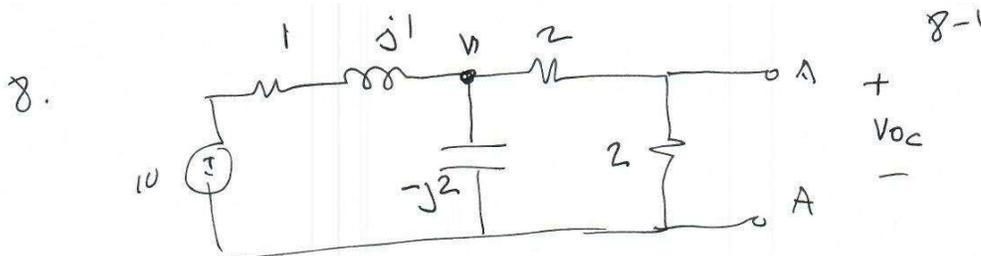
Figure 7

In the circuit of Figure 7, the operating frequency is 60 Hz. The component values are  $R_1 = 1 \Omega$ ,  $R_2 = 2 \Omega$ ,  $R_3 = 2 \Omega$ ,  $L = 2.653$  milliHenries, and  $C = 1,326$  microFarads.

7a) With the load impedance  $Z_L$  removed, find the Thevenin equivalent circuit at terminals AB. (6 marks)

7b) With a load impedance  $Z_L$  connected to terminals AB, what value of  $Z_L$  dissipates the maximum amount of average power? (2 marks)

7c) With  $Z_L$  chosen as in question 7b, how much power does  $Z_L$  dissipate? (2 marks)



$$\omega = 2\pi f = 376.99$$

$$j\omega L = j 376.99 \times 2.653 \times 10^{-3} = j1$$

$$\frac{-j}{\omega C} = \frac{-j}{376.99 \times \frac{1.326 \times 10^{-6}}{1}} = -j0.52$$

$$\frac{10 - V_1}{1 + j} - \frac{V_1}{-j2} - \frac{V_1}{4} = 0$$

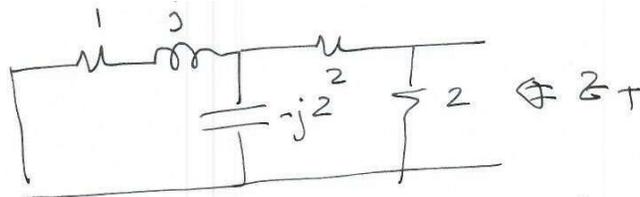
$$V_1 \left( \frac{1}{1+j} + \frac{1}{-j2} + \frac{1}{4} \right) = \frac{10}{1+j}$$

$$V_1 \left( \frac{(-j^2)4 + (1+j)4 + (1+j)(-j^2)}{(1+j)(-j^2)4} \right) = \frac{10}{1+j}$$

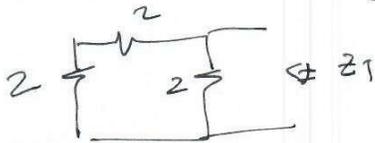
$$V_1 \frac{-8j + 4 + 4j - 2j + 2}{-8j} = 10$$

$$V_1 = \frac{10 \times (-8j)}{6 - 6j} = \frac{-80j}{6(1-j)}$$

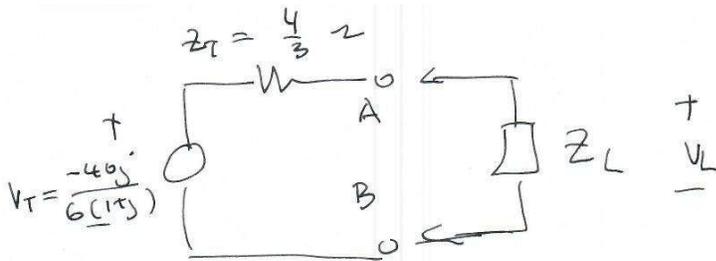
$$V_{bc} = \frac{V_1}{2} = \frac{-40j}{6(1+j)} = 3.33 - 3.33j = 4.71 \angle -45^\circ$$



$$(1+j) \parallel (-j2) = \frac{(1+j) \times (-j2)}{1+j-j2} = \frac{2-j2}{1-j} = \frac{2(1-j)}{1-j} = 2$$



$$2 \parallel 4 = \frac{2 \times 4}{2+4} = \frac{8}{6} = \frac{4}{3} \Omega$$



8-2

$$= 4.71 \angle -45^\circ$$

$$\text{Chose } Z_L = Z_T = \frac{4}{3} \Omega$$

$$\text{Then } I = \frac{V_t}{Z_T + Z_L} = \frac{4.71 \angle -45^\circ}{\frac{4}{3} + \frac{4}{3}}$$

$$= \frac{3 \times 4.71 \angle -45^\circ}{8} = 1.768 \angle -45^\circ$$

$$V_L = z_L I = \frac{4}{3} \times 1.768 \angle -45^\circ$$
$$= \frac{2.357}{1.667} \angle -45^\circ$$

$$P_{av} = \frac{1}{2} \operatorname{Re} (V I^*)$$
$$= \frac{1}{2} \operatorname{Re} (1.667 \angle -45^\circ \times \frac{2.357}{1.768} \angle +45^\circ)$$
$$= \frac{1.667 \times 2.357}{2}$$
$$= 1.965 \text{ watts}$$