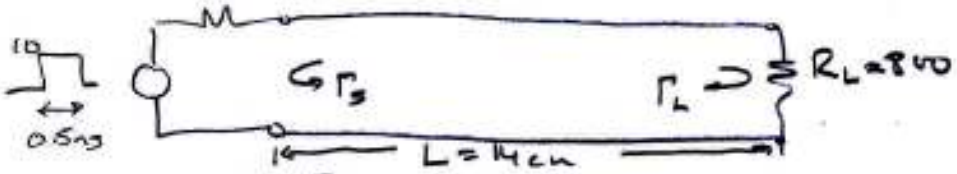
ELEC351 Class Test Solution – 2007

SOLUTION

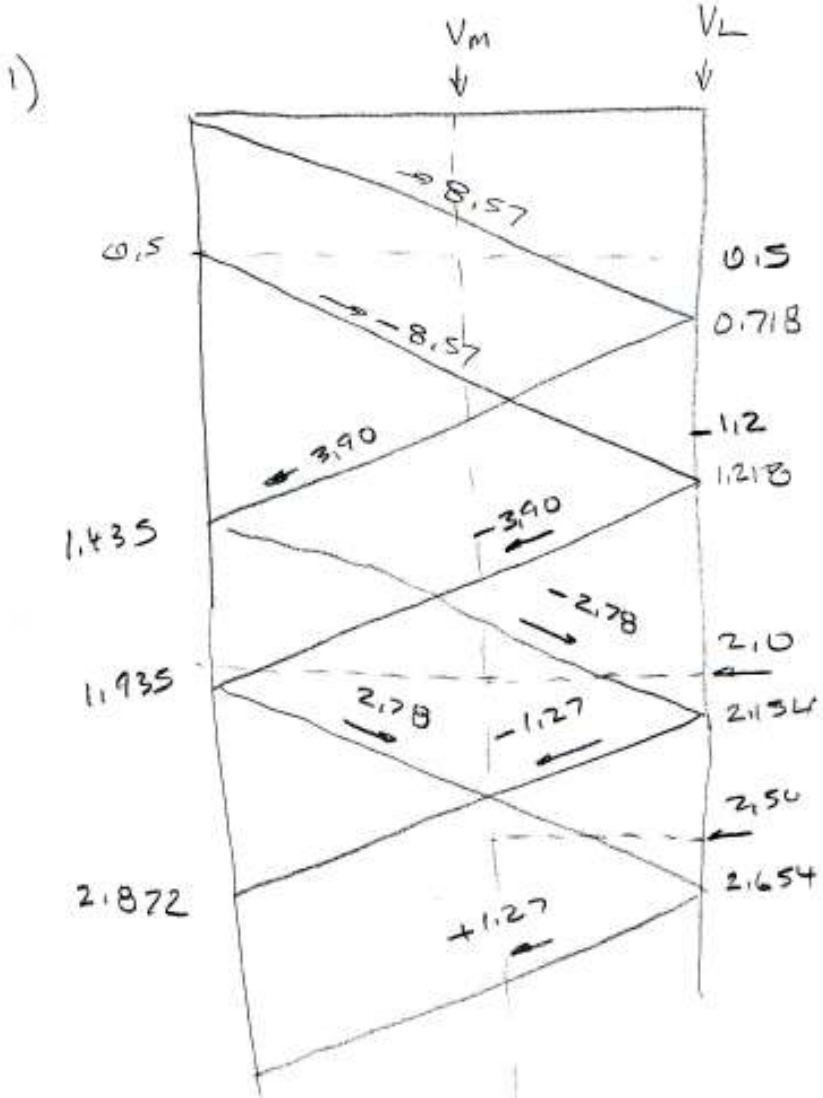
$R_s = 50$ $\quad R_L = 800$, $u = 19.5$ cm/ns ①



$V(0) = \frac{V_s R_c}{R_c + R_s} = \frac{10 \times 300}{300 + 50} = 8.57$ volts $\quad T = \frac{L}{u} = \frac{14}{19.5} = 0.718$ ns

$\Gamma_L = \frac{R_L - R_c}{R_L + R_c} = \frac{800 - 300}{800 + 300} = 0.455$

$\Gamma_s = \frac{R_s - R_c}{R_s + R_c} = \frac{50 - 300}{50 + 300} = \frac{-250}{350} = -0.714$



② 1.1) V_M at 0.5 is

$V_M = 8.57$ V. ⓐ

1.2) V_L at 1.2 ns

$V_L = 8.57 + 3.90$

$= 12.47$ V. ⓓ

1.3) V_M at 2.0 ns

$V_M = (+8.57 - 8.57)$

$(+3.90 - 3.90)$

$- 2.78$

$= -2.78$ volts ⓑ

1.4) V_L at 2.5 ns

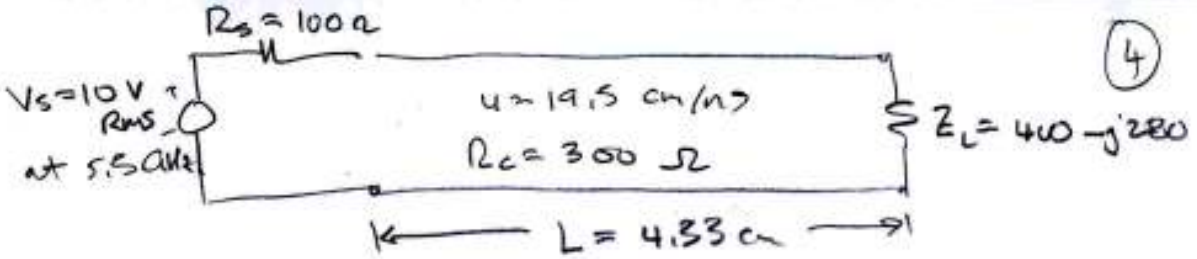
$V_L = (+8.57 + 3.90)$

$+ (-8.57 - 3.90)$

$+ (-2.78 - 1.27)$

$= -4.05$ volts

2) (4)



2.1) $\beta = \frac{2\pi}{\lambda}$ where $\lambda = \frac{u}{f}$ so $\beta = \frac{2\pi f}{u}$

$= \frac{2\pi \times 5.5}{19.5}$

$= 1.772$ rad/cm (b)

2.2) $\tan \beta L = \tan 1.772 \times \frac{180}{\pi} \times 4.33 = 439.6^\circ$

$\tan \beta L = 5.458$ ✓

$$Z_{in} = R_c \frac{Z_L + j R_c \tan \beta L}{R_c + j Z_L \tan \beta L}$$

$$= 300 \frac{(400 - j280) + j\,300 \times 5.458}{300 + j(400 - j280) \times 5.458}$$

$$= 300 \frac{400 + j1357.4}{1828.2 + j2183.2}$$

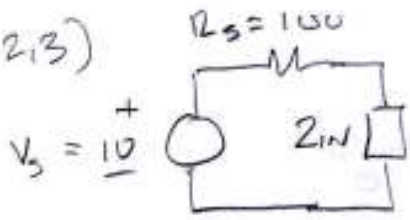
$$= 300 \frac{1415.1 \angle 73.6^\circ}{2847.6 \angle 50.0^\circ}$$

$$= 149.1 \angle 23.6^\circ$$

$$= 136.6 + j59.7\ \Omega$$ (c)

(5)

2.3)



$$V(0) = \frac{Z_{IN} V_S}{Z_{IN} + R_S}$$

$$= \frac{(136.6 + j59.7) \times 10}{136.6 + j59.7 + 100}$$

$$= \frac{(149.1 \angle 23.6°) \times 10}{236.6 + j59.7}$$

$$= \frac{1491.6 \angle 23.6°}{244.0 \angle 14.2°}$$

$$= 6.11 \angle 9.4° \text{ V}$$ (b)

2.4) $$\Gamma_L = \frac{Z_L - R_c}{Z_L + R_c} = \frac{400 - j280 - 300}{400 - j280 + 300}$$

$$= \frac{100 - j280}{700 - j280} = \frac{297.3 \angle -70.3}{753.9 \angle -21.8}$$

$$= 0.394 \angle -48.5$$

$$= 0.261 - j0.295$$ (d)

2.5) $$V^+ = \frac{V(0)}{1 + \Gamma_L e^{-2j\beta L}}$$

$\beta L = 439.6$ so $-2\beta L = -879.2°$

$$1 + \Gamma_L e^{-2j\beta L} = 1 + (0.394 \angle -48.5°)(1 \angle -879.2°)$$

$$= 1 + 0.394 \angle -927.7°$$

$$= 1 + (-0.3488 + j0.1831)$$

$$= 0.6512 + j0.1831 = 0.6765 \angle 15.7°$$

$$V^+ = \frac{V(0)}{1+\Gamma_L e^{2j\beta L}}$$

$$= \frac{6.11 \angle 9.4^\circ}{0.6765 \angle 15.7^\circ} = 9.03 \angle -6.3^\circ$$

$$= 8.98 - 0.99j$$

$$V^- = V^+ \Gamma_L e^{-2j\beta L}$$

$$= (9.03 \angle -6.3^\circ)(0.394 \angle -927.7^\circ)$$

$$= 3.58 \angle -934^\circ = -2.97 + 2.00j$$

$$V(z) = (9.03 \angle -6.3^\circ) e^{-j\beta z} + (3.58 \angle -934^\circ) e^{j\beta z}$$

$$V(L) = (9.03 \angle -6.3^\circ) e^{-j\beta L} + (3.58 \angle -934^\circ) e^{j\beta L}$$

$$e^{-j\beta L} = 1 \angle (-\beta L) = 1 \angle -439.6^\circ$$

$$V(L) = (9.03 \angle -6.3^\circ)(1 \angle -439.6^\circ) + (3.58 \angle -934^\circ)(1 \angle 439.6^\circ)$$

$$= 9.03 \angle -445.9^\circ + 3.58 \angle -494.4^\circ$$

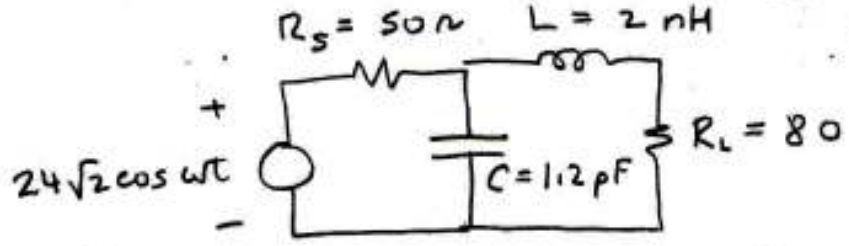
$$= 0.6456 + 9.007j^\circ + (2.505 - j2.558)$$

$$= 11.859 + 11.565j$$

$$= 16.57 \angle 44.8^\circ$$

(b)

3



$f = 2450\ MHz$ so $\omega = 2\pi f = 1.539 \times 10^{10}\ r/s$

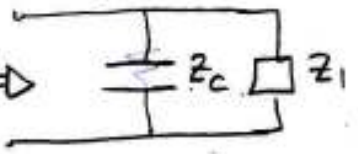
3.1 $Z_L = j\omega L = j \times 1.539 \times 10^{10} \times 2 \times 10^{-9} = j\,30.79$ (b)

3.2 $Z_c = \frac{1}{j\omega C} = \frac{-j}{1.539 \times 10^{10} \times 1.2 \times 10^{-12}} = -j\,54.15$

$Z_1 \Rightarrow$

$Z_1 = Z_L + R_L = 80 + j\,30.79\ \Omega$

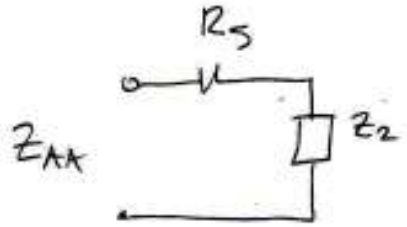
$Z_2 \Rightarrow$



$Z_2 = Z_1 \| Z_c$

$= \frac{(80 + j\,30.79)(-j\,54.15)}{80 + j\,30.79 - j\,54.15}$

$= 33.77 - j\,44.29\ \Omega$

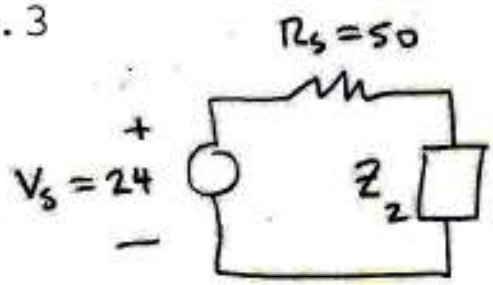


$Z_{AA} = R_s + Z_2$

$= 50 + 33.77 - j\,44.29$

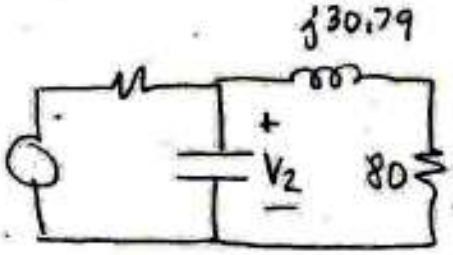
$= 83.77 - j\,44.29\ \Omega$ (c)

3.3 (6)



$$V_2 = \frac{Z_2 V_s}{Z_2 + R_s} = \frac{(33.77 - j44.29) \times 24}{33.77 - j44.29 + 50}$$

$$V_2 = 12.80 - j5.92$$



$$V_L = \frac{80 \cdot \times V_2}{80 + j30.79}$$

$$V_L = \frac{80 \times (12.80 - j5.92)}{80 + j30.79}$$

$$= 9.17 - j9.45$$

$$= 13.17 \angle -45.9^\circ$$ (a)