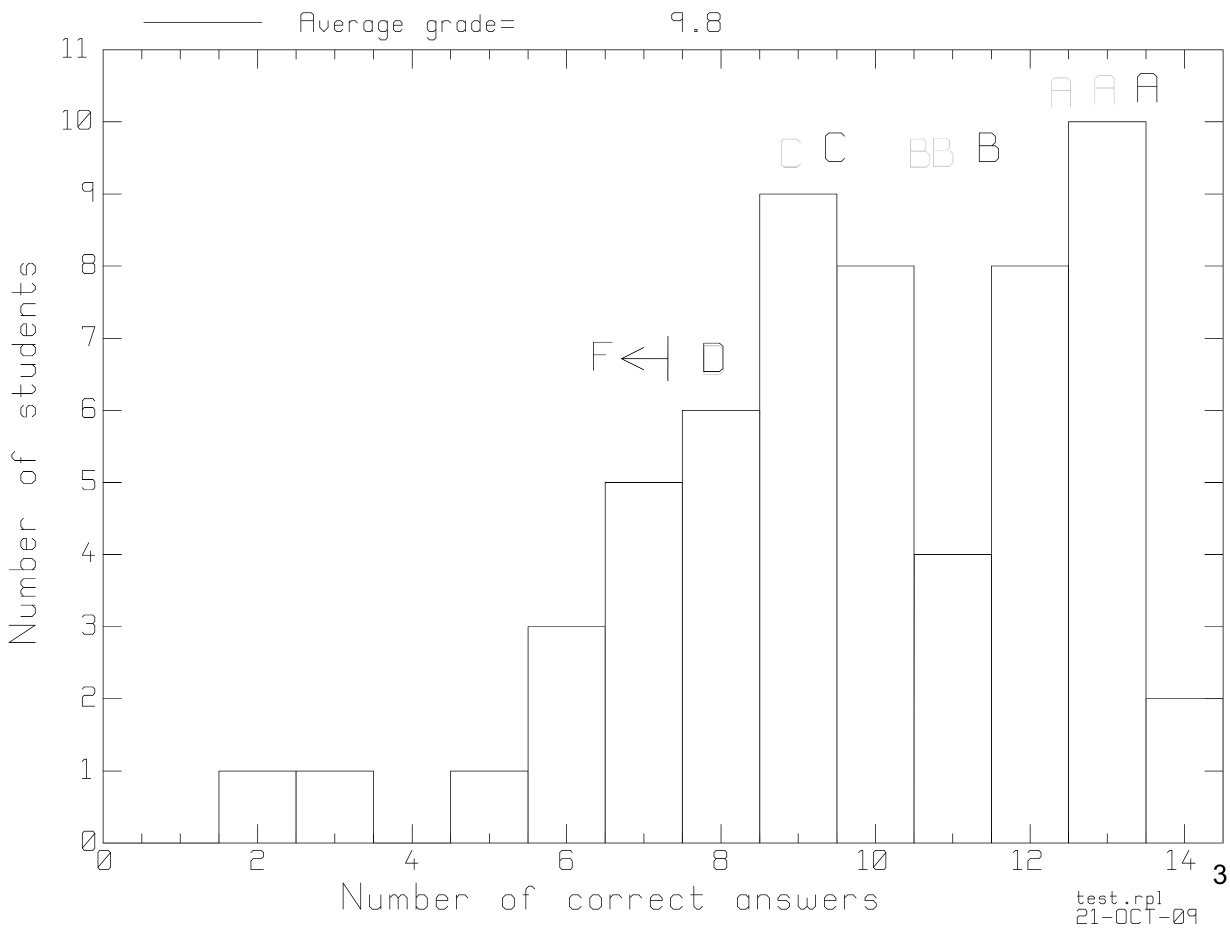
ELEC351 Lecture Notes Set 10

Assignment #3

Ulaby 2.38, 2.39, 2.40, 2.43, 2.44, 2.46, 2.46b
2.46b: Repeat problem 2.46 using the double-stub matching method, using short-circuited stubs. Stub #1 is at the load. Stub #1 should be made as short as possible. Stub #2 is 0.125 λ away from the load.

1

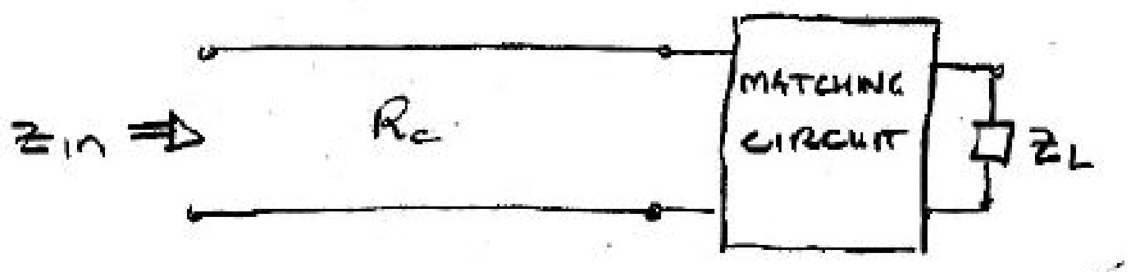
Outcome of the Class Test



3

Impedance Matching in the Sinusoidal Steady State

Ulaby Section 2-10



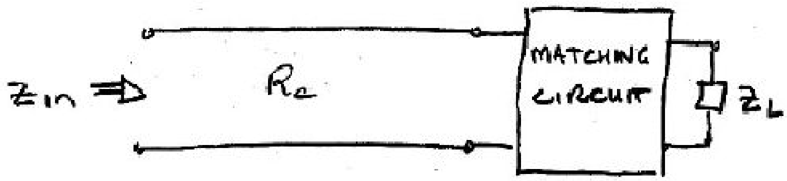
What circuits can we use for the "matching circuit"?

- quarter-wave transformer
- single-stub matching circuit
- double-stub matching circuit

The matching circuit is designed to provide a perfect match at the "center frequency" f_0 .

The **bandwidth** of the match is the range of frequencies over which the match is "sufficiently good".

Return Loss



$$\Gamma_{in} = \frac{Z_{in} - R_c}{Z_{in} + R_c}$$

The "return loss" is defined as

$R.L. = -20\log|\Gamma_{in}|$ in decibels or "dB"

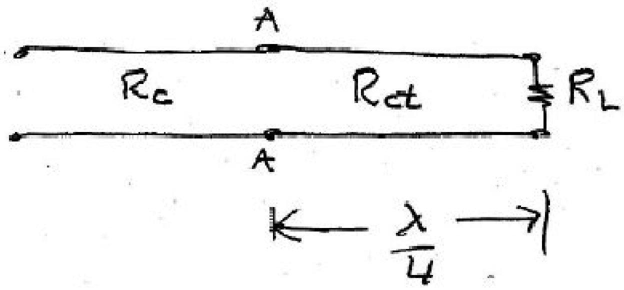
If $|\Gamma_{in}|$<0.1 then the match is good enough for many purposes.

Then $R.L. = -20\log|\Gamma_{in}| = -20\log 0.1 = 20$ dB

The "bandwidth" is often defined as the frequency range over which the return loss is better than, say, 20 dB.
Connectors have a return loss of 30 to 40 dB
A good "matched load" has a return loss of about 40 dB.

Quarter-Wave Transformer

Ulaby Section 2-7.5 page 68



$$Z_{in} = R_{ct}\frac{Z_L + jR_{ct}\tan\beta L}{R_{ct} + jZ_L\tan\beta L} = R_{ct}\frac{R_L + jR_{ct}\tan\beta L}{R_{ct} + jR_L\tan\beta L}$$

$$L = \frac{\lambda}{4} \qquad \beta L = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \qquad \tan\beta L = \tan\frac{\pi}{2} \to \infty$$

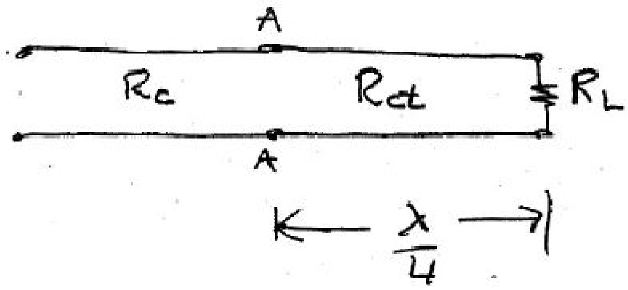
$$Z_{in} = R_{ct}\lim_{\tan\beta L \to \infty}\frac{R_L + jR_{ct}\tan\beta L}{R_{ct} + jR_L\tan\beta L} = R_{ct}\frac{jR_{ct}\tan\beta L}{jR_L\tan\beta L} = \frac{R_{ct}^2}{R_L}$$

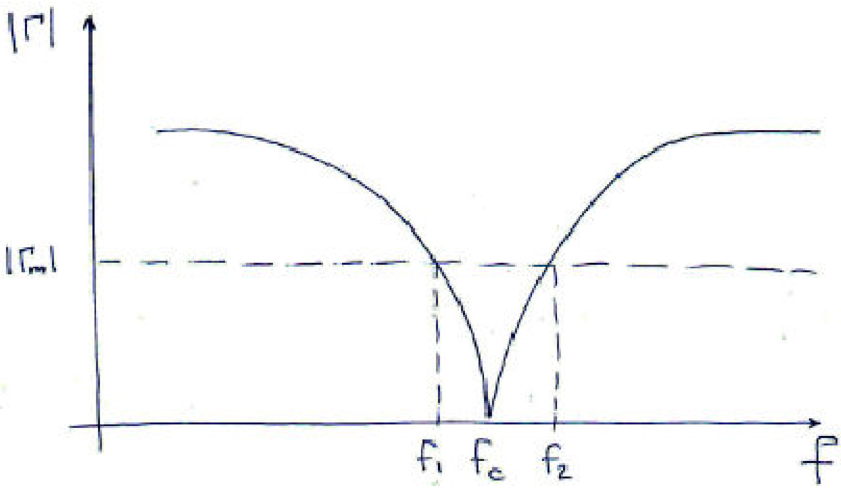
$$Z_{in} = R_c = \frac{R_{ct}^2}{R_L} \quad \text{so choose} \quad R_{ct} = \sqrt{R_c R_L}$$

What is the bandwidth?

At terminals AA:

$$\Gamma(f) = \frac{Z_{in}(f) - R_c}{Z_{in}(f) + R_c}$$

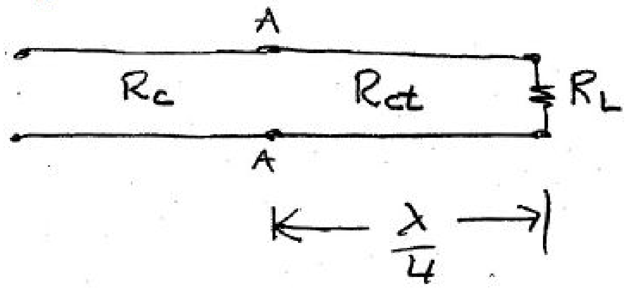




David Pozar in "Microwave Engineering" (Addison-Wesley, 1990) gives us an approximate formula for the bandwidth:

$$f_2 - f_1 \approx f_c\left[2 - \frac{4}{\pi}\cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{R_c R_L}}{|R_L - R_c|}\right)\right]$$

Example: quarter-wave transformer



At 850 MHz, an antenna with an input impedance of $Z_L = 100 + 0j$ ohms must be matched to a transmission line of characteristic impedance $R_c = 50$ ohms. The speed of travel on the transmission line is $u = 30$ cm/ns.

1. Design a quarter-wave transformer by choosing the length L and the characteristic impedance R_{ct}.
2. Use Pozar's formula to find the bandwidth for a return loss of 20 dB or better.
3. Verify that your design works with TRLINE.
4. Use TRLINE to find the bandwidth of the match and compare with Pozar's value.

Solution

$f_c = 850$ MHz $\qquad \lambda = \frac{u}{f_c} = \frac{300}{850} = 35.29$ cm.

$$L = \frac{\lambda}{4} = \frac{35.29}{4} = 8.82 \text{ cm.}$$

$$R_{ct} = \sqrt{R_c R_L} = \sqrt{50 \cdot 100} = 70.71 \text{ ohms.}$$

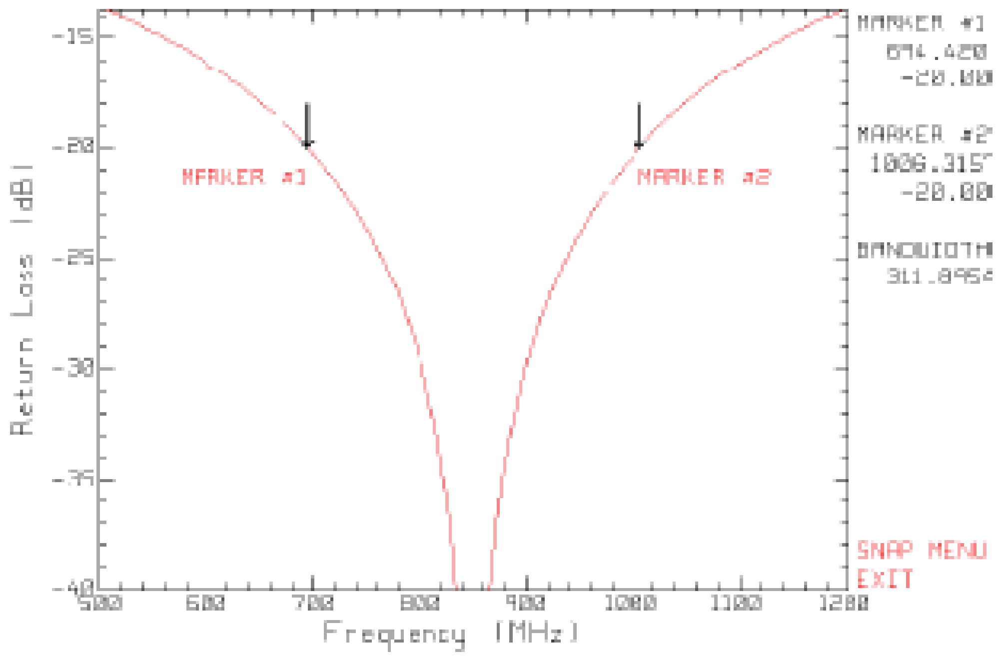
return loss of 20 dB $\qquad -20\log\Gamma_m = 20$ dB $\qquad \Gamma_m = 0.1$

$$f_2 - f_1 \approx f_c\left[2 - \frac{4}{\pi}\cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{R_c R_L}}{|R_L - R_c|}\right)\right]$$

$$f_2 - f_1 \approx 850\left[2 - \frac{4}{\pi}\cos^{-1}\left(\frac{0.1}{\sqrt{1-0.1^2}}\frac{2\cdot 70.71}{|100-50|}\right)\right]$$

$$f_2 - f_1 \approx 850\left[2 - \frac{4}{\pi}\cdot 1.28253\right] = 311.95 \text{ MHz}$$

Use TRLINE to find the bandwidth



This agrees well with the Pozar formula, $f_2 - f_1 \approx 311.95$ MHz.