ELEC351 Homework Problems #2

Problem 2.6 Ulaby 6th edition
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Problem 2.6 Ulaby 6th edition

The given parameters are

$a = 0.25$ cm
$b = 0.5$ cm
$\epsilon_r = 4.5$
$\sigma_d = 10^{-3}$ S/m
$\mu = \mu_c = \mu_0$
$\sigma_c = 5.8\times10^{7}$ S/m
$f = 1$ GHz

The intrinsic resistance is

$$R_S = \sqrt{\pi f \mu_c/\sigma_c} = \sqrt{\pi\times10^{9}\times4\pi\times10^{-7}/5.8\times10^{7}} = 0.00825\ \Omega$$

and

$$R' = \frac{R_S}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{0.00825}{2\pi}\left(\frac{1}{2.5\times10^{-3}}+\frac{1}{5\times10^{-3}}\right) = 0.7874\ \Omega/m$$

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times10^{-7}}{2\pi}\ln(2) = 1.386\times10^{-7}\ H/m$$

$$G' = \frac{2\pi\sigma_d}{\ln(b/a)} = \frac{2\pi\times10^{-3}}{\ln(2)} = 0.0090647\ S/m$$

$$C' = \frac{2\pi\epsilon}{\ln(b/a)} = \frac{2\pi\times4.5\times8.854\times10^{-12}}{\ln(2)} = 3.6117\times10^{-10}\ F/m$$

Problem 2.8 6th edition

The complex propagation constant is

$$\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')} = 0.1089 + j44.459$$

The attenuation constant is the real part of γ

$$\alpha = Re\{\gamma\} = 0.1089\ Np/m$$

The phase constant is the imaginary part of γ

$$\beta = \operatorname{Im}\{\gamma\} = 44.459 \quad \text{rad/m}$$

The phase velocity u_p is given by

$$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^{9}}{44.459} = 1.4132 \times 10^{8} \quad \text{m/s}$$

The characteristic impedance is

$$Z_o = \sqrt{\frac{R_o' + j\omega L'}{G' + j\omega C'}} = 19.623 + j0.034 \quad \Omega$$

Problem 2.18 6th edition

a) For a lossless transmission line we have

$$Z_o = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{\ln(b/a)}{2\pi} \quad \Omega$$

Using this formula, we can find the radius of the outer conductor

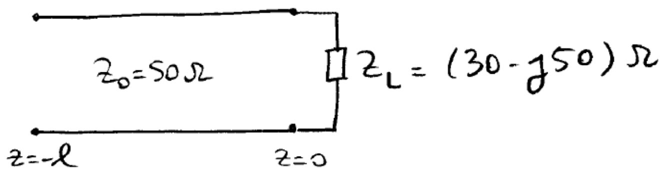
$$b = a\, e^{2\pi Z_o \sqrt{\epsilon/\mu}} = 1.2 \times 10^{-3} \times e^{100\pi \sqrt{\frac{2.25 \times 8.854 \times 10^{-12}}{4\pi \times 10^{-7}}}}$$

$$b = 4.199 \quad \text{mm}$$

b) The phase velocity of the line is

$$u_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^{8}}{\sqrt{2.25}} = 2 \times 10^{8} \quad \text{m/s}$$

Problem 2.19 6th edition



a) The reflection coefficient at the load:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-20 - j50}{80 + j50} = 0.10 - j0.56 = 0.57\angle{-79.8^\circ}$$

b) standing wave ratio:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1.57}{0.43} = 3.65$$

c) The total voltage on the line is given by:

$$V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$
$$= V_0^+ e^{-j\beta z}\left(1 + \Gamma e^{j2\beta z}\right)$$
$$= V_0^+ e^{-j\beta z}\left(1 + |\Gamma| e^{j\theta_\Gamma} e^{j2\beta z}\right)$$

then, $|V(z)|_{max} = |V_0^+|\left(1 + |\Gamma|\right)$

and this occurs when $e^{j(2\beta z + \theta_\Gamma)} = 1$.

$\Rightarrow 2\beta z_{max} + \theta_\Gamma = -2\pi n \qquad (n = 0, 1, 2, 3, \ldots)$

$\Rightarrow z_{max} = \dfrac{-2\pi n - \theta_\Gamma}{2\beta}$

first voltage max. nearest the load corresponds to $n = 1$.

$\Rightarrow z_{max} = \dfrac{-360^\circ + 79.8^\circ}{2 \cdot \frac{360}{8\,cm}} \simeq -3.11\ cm$.

d) The total current on the line is given by:

$$I(z) = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$

$$= \frac{V_0^+}{Z_0} e^{-j\beta z}\left(1 - \Gamma e^{j2\beta z}\right)$$

$$= \frac{V_0^+}{Z_0} e^{-j\beta z}\left(1 - |\Gamma| e^{j(2\beta z + \theta_\Gamma)}\right)$$

then, $|I(z)|_{max} = \left|\frac{V_0^+}{Z_0}\right|(1 + |\Gamma|)$

and this occurs when $e^{j(2\beta z + \theta_\Gamma)} = -1$.

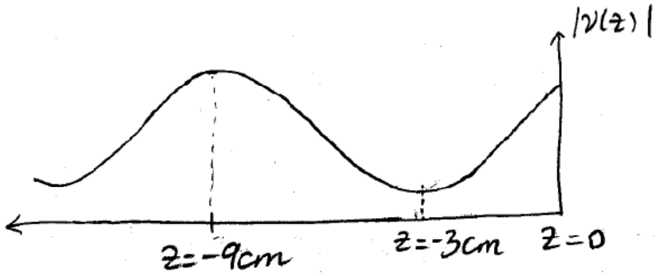
$\Rightarrow 2\beta z_{max} + \theta_\Gamma = -(2n+1)\pi \qquad (n = 0, 1, 2, \ldots)$

$\Rightarrow z_{max} = \dfrac{-(2n+1)\pi - \theta_\Gamma}{2\beta}$

the first current min. nearest the load corresponds to $n = 0$.

$\Rightarrow z_{max} = \dfrac{-180° + 79.8°}{2 \times \frac{360}{8\,cm}} \simeq -1.11\ cm$

Problem 2.21 6th edition



We are given $S = 3$. Voltage standing wave ratio is given by $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$.

Then, $|\Gamma| = \dfrac{S - 1}{S + 1} = 1/2$.

In (2.12-c), we showed that, the distance of the first voltage max. from the load can be obtained using

$$2\beta z_{max} + \theta_\Gamma = -360°$$

$$\Rightarrow \theta_\Gamma = -360° - 2\beta z_{max}$$

the distance between a voltage max. and a voltage min. is $\lambda/4$.

$$\Rightarrow \frac{\lambda}{4} = 6cm \Rightarrow \lambda = 24cm.$$

Then, the phase of the ref. coeff. is:

$$\theta_\Gamma = -360° - 2 \times \frac{360}{24cm} \times (-9cm) = -90°$$

therefore, $\Gamma = 0.5\, e^{-j90°} = -j0.5$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma} = 50 \frac{1-j0.5}{1+j0.5} = (90 - j120)\,\Omega$$