ELEC351 Homework Problems #4

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Problem 7.2 Ulaby 6^{th} edition

Since the direction of the wave propagation and the polarization are given, we can write the electric field in form of

$$\bar{E}(y,t) = \hat{x} E_0 \cos(\omega t - ky + \phi) = \hat{x}\, 6 \cos(\omega t - ky + \phi)$$

where the wave number is

$$k = \frac{\omega}{u_p} = \frac{\omega\sqrt{\epsilon_r}}{c} = \frac{2\pi \times 10^9 \times \sqrt{9}}{3\times 10^8} = 20\pi$$

The electric field intensity is 4 V/m at y = 2 cm and t = 0. As a result, we can find ϕ as

$$|E(0.02, 0)| = 4 = 6\cos(-0.4\pi + \phi)$$

$$\Rightarrow \phi = 120.2^\circ$$

The expression for the electric field becomes

$$\bar{E}(y,t) = \hat{x}\, 6\cos(2\pi\times 10^9 t - 20\pi y + 120.2^\circ) \quad \text{V/m}$$

By using $\tilde{H} = \frac{1}{\eta}\hat{k}\times\tilde{E}$, the magnetic field can be written as

$$\bar{H}(y,t) = -\hat{z}\,\frac{18}{120\pi}\cos(2\pi\times 10^9 t - 20\pi y + 120.2^\circ) \quad \text{A/m}$$

Problem 7.3 6th edition

a) Since $k = 0.2$, we can find the wavelength by

$$\lambda = \frac{2\pi}{k} = 10\pi \text{ m}$$

b) The frequency is

$$f = \frac{u_p}{\lambda} = \frac{1.5\times10^8}{10\pi} = 4.77 \text{ MHz}$$

c) The relative permittivity of the medium is $\left(u_p = \frac{c}{\sqrt{\epsilon_r \mu_r}}\right)$

$$\sqrt{\epsilon_r} = \frac{c}{u_p\sqrt{\mu_r}} = \frac{3\times10^8}{1.5\times10^8\sqrt{2.4}} = \frac{2}{\sqrt{2.4}} \Rightarrow \epsilon_r = \frac{4}{2.4} = 1.67$$

d) The magnetic field phasor can be written by

$$\tilde{H} = \frac{1}{\eta}\hat{k}\times\tilde{E} = \frac{1}{120\pi\sqrt{\frac{2.4}{1.67}}}(-\hat{z})\times(\hat{y})10e^{j0.2z}$$

$$= \hat{x}\,0.022e^{j0.2z} \quad \text{A/m}$$

and the expression for the magnetic field in time domain becomes

$$\bar{H}(z,t) = \hat{x}\,0.022\cos(2\pi\times4.77\times10^6 t + 0.2z) \quad \text{A/m}$$

Problem 7.6 6th edition

$$\bar{E} = \hat{y}20\cos(6\pi\times10^9 t - kz) \text{ (V/m)}$$

$$\Rightarrow \omega = 6\pi\times10^9 = 2\pi f \Rightarrow f = 3\text{ GHz}.$$

$$k = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r} = \frac{\omega}{c}\sqrt{\epsilon_r} = \frac{6\pi\times10^9}{3\times10^8}\sqrt{2.56} = 100.53 \text{ rad/m}$$

$$\lambda = \frac{2\pi}{k} = 6.25\text{ cm}, \quad u_p = \frac{\omega}{k} = 1.875\times10^8 \text{ m/s}.$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_r}{\epsilon_r}} = \frac{120\pi}{\sqrt{2.56}} = 75\pi$$

direction of propagation is $+\hat{z}$

$$\Rightarrow \bar{H} = \frac{\hat{z}\times\bar{E}}{\eta} = \frac{\hat{z}\times\hat{y}20\cos(\omega t - kz)}{\eta} = -\hat{x}\frac{20}{75\pi}\cos(6\pi\times10^9 t - kz)$$

Problem 7.14 6th edition

$$\bar{E}(z,t) = \hat{x}\sin(\omega t + kz) + \hat{y}2\cos(\omega t + kz)$$

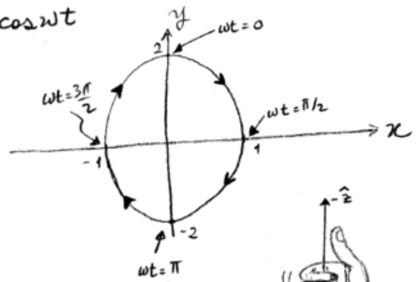
$$\bar{E}(z=0,t) = \hat{x}\sin\omega t + \hat{y}2\cos\omega t$$

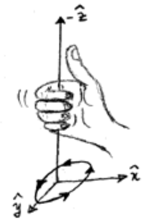
$\omega t = 0 \Rightarrow \bar{E}(0,t) = \hat{y}2$

$\omega t = \pi/2 \Rightarrow \bar{E}(0,t) = \hat{x}$

$\omega t = \pi \Rightarrow \bar{E}(0,t) = -\hat{y}2$

$\omega t = \frac{3\pi}{2} \Rightarrow \bar{E}(0,t) = -\hat{x}$





direction of propagation is $-\hat{z}$.

$\Rightarrow$ right hand elliptical polarization

Problem 7.15 6th edition

a) $\frac{\sigma}{\omega\epsilon} = 3.16\times10^{-3} \Rightarrow$ low-loss dielectric

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}} = \underbrace{8.4\times10^{-11}}_{\alpha} + j\underbrace{468.32}_{\beta}$$

$\lambda = \frac{2\pi}{\beta} = 1.34\text{ cm}$, $u_p = \frac{\omega}{\beta} = 1.34\times10^{8}\text{ m/s}$

low-loss medium $\Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon}} = 168.6\ \Omega$.

b) $\frac{\sigma}{\omega\epsilon} = 4.5 \Rightarrow$ quasi-conductor.

$\gamma = 9.75 + j12.15$, $\lambda = 51.7\text{ cm}$, $u_p = 0.517\times10^{8}\ \frac{m}{s}$

$$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{(\epsilon - j\frac{\sigma}{\omega})}} = 39.54 + j31.78\ \Omega$$

c) $\frac{\sigma}{\omega\epsilon} = 600 \Rightarrow$ good conductor.

$\gamma = 6.28\times10^{-4} + j6.28\times10^{-4}$, $\lambda = 9991.7\text{ m}$, $u_p = 0.0999\times10^{8}\ \frac{m}{s}$

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{6.28\times10^{-4}}{10^{-4}} = 6.28 + j6.28\ \Omega.$$